A Picture Is Worth a Thousand Words
Understanding Quantities and Their Relationships

Learning Goals
• Understand quantities and their relationships with each other.
• Identify the independent and dependent quantities for a scenario.
• Match a graph with an appropriate scenario.
• Use a reasonable scale for a graph modeling a scenario.
• Identify key characteristics of graphs.
• Describe similarities and differences between pairs of graphs and scenarios.

Key Terms
• dependent quantity
• independent quantity

Warm Up
Emma bought a new video game. The graph shown describes the number of hours Emma spent playing the game over a period of 7 days.

1. Label the axes.
2. What does the highest point on the graph represent with respect to the scenario? The lowest point?

You have analyzed graphs of relationships and identified important features such as intercepts and slopes. How can the key characteristics of a graph tell a story?
What Comes First?

Have you ever planned a party? You may have purchased ice, gone grocery shopping, selected music, made food, or even cleaned in preparation. Many times, these tasks depend on another task being done first. For instance, you wouldn’t make food before grocery shopping, now would you?

Consider the two quantities that are changing in each relationship.

- the number of movie tickets purchased and the total cost
- the number of eggs used and the number of cakes baked
- the number of students in attendance at school and the number of lunches served
- the number of hours driven and the number of miles to a vacation destination
- the number of minutes a swimming pool is filled with water and the number of gallons of water in the swimming pool

1. Circle the independent quantity and underline the dependent quantity in each relationship.

2. Describe how you can determine which quantity is independent and which quantity is dependent in any problem situation.
While a person can describe the monthly cost to operate a business, or talk about a marathon pace a runner ran to break a world record, graphs on a coordinate plane enable people to see the data. Graphs relay information about data in a visual way.

You can use lines or smooth curves to represent relationships between points on a graph. In some problem situations, all the points on the line will make sense. In other problem situations, not all the points will make sense. So, when you model a relationship with a line or a curve, it is up to you to consider the situation and interpret the meaning of the data values shown.

This activity includes eight scenarios and eight graphs that are located at the end of the lesson.

1. Read each scenario. Determine the independent and dependent quantities. Then match each scenario to its corresponding graph. Glue the graph next to the scenario. For each graph, label the x- and y-axis with the appropriate quantity and a reasonable scale, and then interpret the meaning of the origin.

**Daredevil**

Greyson completes a dive from a cliff 75 feet above a river. It takes him only 1.5 seconds to hit the water and then another 0.5 second to descend 10 feet into the river.

- **independent quantity:**

- **dependent quantity:**
Something’s Fishy
Candice is a building manager for the Crowley Enterprise office building. One of her responsibilities is cleaning the office building’s 200-gallon aquarium. For cleaning, she must remove the fish from the aquarium and drain the water. The water drains at a constant rate of 10 gallons per minute.

- independent quantity:
- dependent quantity:

Smart Phone, but Is It a Smart Deal?
You have your eye on an upgraded smart phone. However, you currently do not have the money to purchase it. Your cousin will provide the funding, as long as you pay him back with interest. He tells you that you only need to pay $1 in interest initially, and then the interest will double each week after that. You consider his offer and wonder if this really is a good deal.

- independent quantity:
- dependent quantity:
Can’t Wait to Hit the Slopes!
Andrew loves skiing—he just hates the ski lift. To make matters worse, the ski lift has been acting up today. Andrew is using the GPS on his phone to track the ski lift’s progress as it travels up the mountain. It moves at a steady rate of 400 feet per minute until it stops suddenly. Andrew calls his friends to tell them that he is stuck. They talk on the phone for 10 minutes until finally the ski lift begins moving again.

• independent quantity:

• dependent quantity:

It’s Magic
The Amazing Aloysius is practicing one of his tricks. As part of this trick, he cuts a rope into many pieces and then magically puts the pieces of rope back together. He begins the trick with a 20-foot rope and then cuts it in half. He then takes one of the halves and cuts that piece in half. He repeats this process until he is left with a piece so small he can no longer cut it.

• independent quantity:

• dependent quantity:
Baton Twirling
Jill is a drum major for the Altadena High School marching band. For the finale of the halftime performance, Jill tosses her baton in the air so that it reaches a maximum height of 22 feet. This gives her 2 seconds to twirl around twice and catch the baton when it comes back down.

- independent quantity:

- dependent quantity:

Cold Weather
The number of guests at a ski resort on any given day is related to the day’s high temperature. If the high temperature is −20°F or below, no one comes to the resort. As the temperature increases, so does the number of guests. Once the temperature reaches 0°F and increases through the single digits, the number of guests soars. If the temperature is 10°F or higher, the ski resort is at full capacity with 400 guests.

- independent quantity:

- dependent quantity:
Jelly Bean Challenge
Mr. Wright judges the annual Jelly Bean Challenge at the summer fair. Every year, he encourages the citizens in his town to guess the number of jelly beans in a jar. He records all the possible guesses and the number of jelly beans that each guess was off by.

• independent quantity:

• dependent quantity:
Now that you have matched a graph with the appropriate problem situation, let’s go back and examine all the graphs.

1. What similarities do you notice in the graphs?

2. What differences do you notice in the graphs? What do you see?

3. How did you label the independent and dependent quantities in each graph?

4. Analyze each graph from left to right. Describe any graphical characteristics you notice.
5. Compare the graphs for each pair of scenarios given and describe any similarities and differences you notice.

   a. *Smart Phone, but Is It a Smart Deal?* and *Cold Weather*

   b. *Something’s Fishy* and *It’s Magic*

   c. *Baton Twirling* and *Jelly Bean Challenge*
TALK the TALK 🌐

A Writer and a Mathematician

1. Write a scenario and sketch a graph to describe a possible trip to school.

Scenario

________________________________________
________________________________________
________________________________________
________________________________________
________________________________________

Graph

2. Describe the meaning of the points, or smooth curve, represented by your graph.

3. Compare your scenario and sketch with your classmates’ scenarios and sketches. What similarities do you notice? What differences do you notice?
Graph Cutouts

Graph A

Graph B

Graph C

Graph D

Graph E

Graph F

Graph G

Graph H
Assignment

Write
Describe how you can distinguish between an independent quantity and a dependent quantity. Use an example in your description.

Remember
When one quantity depends on another in a problem situation, it is said to be the dependent quantity. The quantity it depends upon is called the independent quantity. The independent quantity is represented on the x-axis and the dependent quantity is represented on the y-axis.

Practice
1. Read each scenario and identify the independent and dependent quantities. Be sure to include the appropriate units of measure. Then analyze each graph and determine which of the provided scenarios it models. For each graph, label the x- and y-axis with the appropriate quantity and unit of measure.

   a. Endangered Species
      The Elkwood Aquatic Society is working with various reptile species to increase their populations. The initial population of 450 endangered turtles tripled each year for the past five years.

   b. Video Games
      Gillian is playing video games at an arcade. Gillian starts with $40 and is playing games that cost 50 cents per game.

   c. Sales Commission
      Julian works as a salesman. He receives a monthly salary of $3000 as well as a 10% commission on the amount of sales.

   d. Cooling Tea
      A freshly made cup of tea is served at a temperature of about 180°F. The tea cools rapidly at first, and then slows down gradually as it approaches room temperature.

   e. Commuter Flight
      A commuter flight between two cities in Oregon takes about 40 minutes. The plane increases its altitude for the first half of the flight until it gets to 18,000 feet, and then it descends for the second half of the flight. The plane ascends and descends at a constant rate of 900 feet per minute.

   f. Cross Country
      Brady runs for his high school cross country team. His strategy for each 5-kilometer race is always to begin by increasing his speed so that by the time he reaches the first kilometer, he is running at a speed of 0.3 km/min. He maintains that speed for the next 2 km. He then gradually speeds up for the remaining 2 km so that when he crosses the finish line, he is running at a speed of 0.5 km/min.
2. Compare each pair of graphs and describe any similarities and differences you notice.

a. 

b. 

Stretch
Read the scenario and identify the independent and dependent quantities. Be sure to include the appropriate units of measure.

1. A student performs several experiments in which he swings a pendulum for a 20-second duration. He uses a string that is 27 cm long, and he tests pendulum masses of different sizes, varying from 2 to 12 grams. He records the number of swings each pendulum makes in 20 seconds.

2. The student then decides to make a second graph showing the string length (in cm) as the independent quantity. What changes must the student make to his experiment?

Review
1. Solve the equation $-2x + 8 = -3x + 14$.
2. Evaluate the expression $x^2 - 3y + 12$ for $x = -2$ and $y = 5$. 