Pi: The Ultimate Ratio

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Exploring the Ratio of Circle Circumference to Diameter

WARM UP

Scale up or down to determine an equivalent ratio.

1. $\dfrac{18 \text{ miles}}{3 \text{ hours}} = \dfrac{?}{1 \text{ hour}}$

2. $\dfrac{\$750}{4 \text{ days}} = \dfrac{?}{1 \text{ day}}$

3. $\dfrac{12 \text{ in.}}{1 \text{ ft}} = \dfrac{?}{5 \text{ ft}}$

4. $\dfrac{48 \text{ oz}}{3 \text{ lb}} = \dfrac{?}{1 \text{ lb}}$

LEARNING GOALS

- Identify pi (π) as the ratio of the circumference of a circle to its diameter.
- Construct circles using a compass and identify various parts of circles.
- Know and write the formula for the circumference of a circle, and use the formula to solve problems.
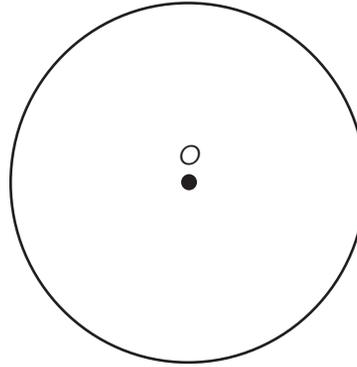
KEY TERMS

- congruent
- circle
- radius
- diameter
- circumference
- pi

You have learned about ratios. How can you use ratios to analyze the properties of geometric figures such as circles?

Across and Around

A circle is shown with a point drawn at the center of the circle. The name of the point is *O*, so let's call this Circle *O*.

O

> Be sure to include units when you record your measurements.

1. Analyze the distance around the circle.

   a. Use a string and a centimeter ruler to determine the distance around the circle.

   b. How does your measurement compare to your classmates' measurements? Summarize the similarities and differences.

2. Draw a line from a point on the circle to the center of the circle, point *O*.

   a. Measure your line using your centimeter ruler.

   b. How does your measurement compare to your classmates' measurements? Summarize the similarities and differences.
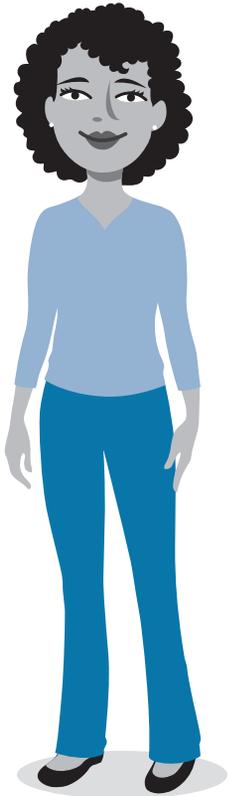
Analyzing the Parts of a Circle

Everyone can identify a circle when they see it, but defining a circle is a bit harder. Can you define a circle without using the word *round*? Investigating how a circle is formed will help you mathematically define a circle.

1. Follow the given steps to investigate how a circle is formed.

2. How many other points could be located exactly 5 cm from point *A*? How would you describe this collection of points in relation to point *A*?

3. Define the term *circle* without using the word *round*.

Step 1: In the space provided, draw a point and label the point *A*.

Step 2: Use a centimeter ruler to locate and draw a second point that is exactly 5 cm from point *A*. Label this point *B*.

Step 3: Locate a third point that is exactly 5 cm from point *A*. Label this point *C*.

Step 4: Repeat this process until you have drawn at least ten distinct points that are each exactly 5 cm from point *A*.

A **circle** is a collection of points on the same plane equidistant from the same point. The center of a circle is the point from which all points on the circle are equidistant. Circles are named by their center point.
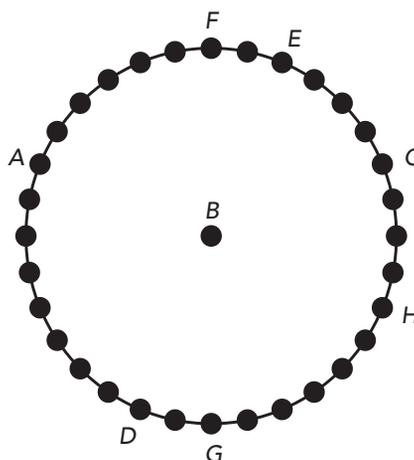
4. Use the circle shown to answer each question.

   a. Name the circle.

The **radius** of a circle is a line segment formed by connecting a point on the circle and the center of the circle. The distance across a circle through the center is the diameter of the circle. The **diameter** of a circle is a line segment formed by connecting two points on the circle such that the line segment passes through the center point. The distance around a circle is called the **circumference** of the circle.

   b. Identify a radius of the circle.

   c. Identify a diameter of the circle.

   d. Are all radii of this circle the same length? Explain your reasoning.

> The plural of *radius* is *radii*.
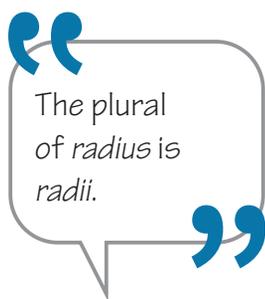
5. What is the relationship between the length of a radius and the length of a diameter?

Measuring the Distance Around a Circle

Let's explore circles. Use circles *A, B, D, E,* and *O* provided at the end of the lesson. Circle *O* is the same as the circle from the activity *Across and Around.*

1. **Use a string and a centimeter ruler to measure the distance from a point on the circle to the center and the distance around each circle. Record your measurements in the table. In the last column, write the ratio of *Circumference* : *Diameter* in fractional form.**

Circle	Circumference	Radius	Diameter	$\frac{Circumference}{Diameter}$
Circle *A*				
Circle *B*				
Circle *O*				
Circle *D*				
Circle *E*				

2. **Average the ratios recorded for $\frac{Circumference}{Diameter}$. What is the approximate ratio for the circumference to the diameter for the set of circles? Write the approximate ratio as a fraction and as a decimal.**

3. **How does your answer to Question 2 compare to your classmates' answers?**

4. **Average all of your classmates' answers to Question 3. Write the approximate ratio of circumference to the diameter as a fraction and as a decimal.**

NOTES

The number **pi** (π) is the ratio of the circumference of a circle to its diameter. That is pi $= \frac{\text{circumference of a circle}}{\text{diameter of a circle}}$, or $\pi = \frac{C}{d}$, where C is the circumference of the circle, and d is the diameter of the circle. The number π has an infinite number of decimal digits that never repeat. Some approximations used for the value π are 3.14 and $\frac{22}{7}$.

1. Use this information to write a formula for the circumference of a circle, where d represents the diameter of a circle and C represents the circumference of a circle.

2. Rewrite the formula for the circumference of a circle, where r represents the radius of a circle and C represents the circumference of a circle.

3. Use different representations for π to calculate the circumference of a circle.

   a. Calculate the circumference of a circle with a diameter of 4.5 centimeters and a circle with a radius of 6 inches. Round your answer to the nearest ten-thousandths, if necessary.

| Value for π	d = 4.5 centimeters	r = 6 inches
π		
Use the π Key on a Calculator		
Use 3.14 for π		
Use $\frac{22}{7}$ for π		

b. Compare your circumference calculations. How do the different values of π affect your calculations?

> When you use 3.14 for pi, your answers are approximations. But an answer like 12π is exact.

4. Use the circumference of a circle formula to determine each unknown. Use 3.14 for π.

a. Compute the diameter of the circle with a circumference of 65.94 feet.

b. Compute the radius of the circle with a circumference of 109.9 millimeters.
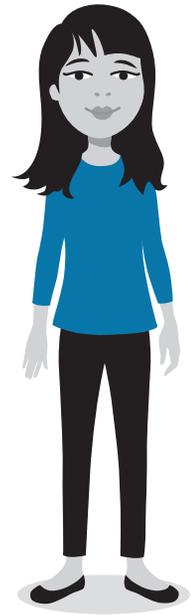
5. What is the minimum amount of information needed to compute the circumference of a circle?

TALK the TALK

Twice

Use what you have learned to compare circles by their characteristics.

1. Draw each circle.

   a. radius length of 3 centimeters

   b. diameter length of 3 centimeters

2. Describe the similarities and differences between your two circles.

3. Describe the relationship between the circumferences of the two circles.

4. Describe the circumference-to-diameter ratio of all circles.

Measuring the Distance Around a Circle

A

B

O

D

E
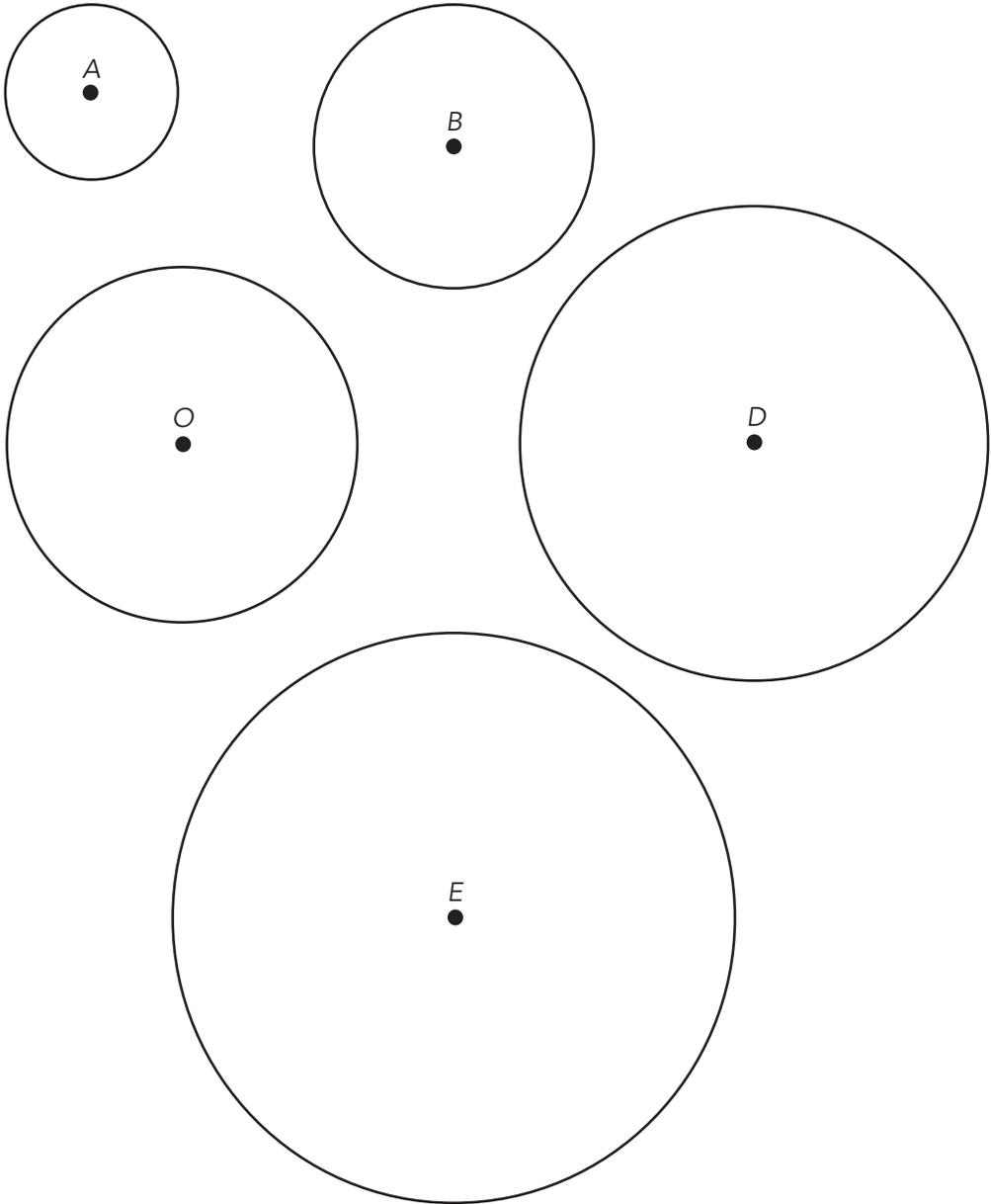
Assignment

Write

Define each term in your own words.

1. circle
2. radius
3. diameter
4. pi

Remember

The circumference of a circle is the distance around the circle. The formulas to determine the circumference of a circle are $C = \pi d$ or $C = 2\pi r$, where d represents the diameter, r represents the radius, and π is a constant value equal to approximately 3.14 or $\frac{22}{7}$.

The constant pi (π) represents the ratio of the circumference of a circle to its diameter.

Practice

Answer each question. Use 3.14 for π. Round your answer to the nearest hundredth, if necessary.

1. Although she's only in middle school, Tameka loves to drive go-carts! Her favorite place to drive go-carts, Driver's Delight, has 3 circular tracks. Track 1 has a radius of 60 feet. Track 2 has a radius of 85 feet. Track 3 has a radius of 110 feet.
   a. Compute the circumference of Track 1.
   b. Compute the circumference of Track 2.
   c. Compute the circumference of Track 3.
   d. Driver's Delight is considering building a new track. They have a circular space with a diameter of 150 feet. Compute the circumference of the circular space.
2. Tameka wants to build a circular go-cart track in her backyard.
   a. If she wants the track to have a circumference of 150 feet, what does the radius of the track need to be?
   b. If she wants the track to have a circumference of 200 feet, what does the radius of the track need to be?
   c. If she wants the track to have a circumference of 400 feet, what does the diameter of the track need to be?

Stretch

A rope is arranged using three semi-circles to form the pattern shown. Determine the length of the rope.

8 in.   3 in.   8 in.
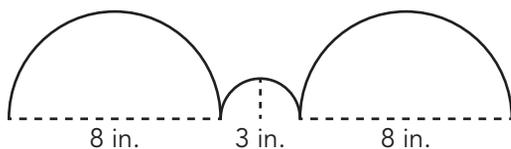
Review

1. Ethan and Corinne are training for a marathon.

    a. Corinne runs 13.5 miles in 2 hours. What is her rate?

    b. Ethan wants to run the 26.2 miles of the marathon in 4.5 hours. At about what rate will he have to run to reach this goal? Round to the nearest tenth.

2. Fifteen seventh graders were randomly selected to see how many pushups in a row they could do. Their data are shown.

    45, 40, 36, 38, 42, 48, 40, 40, 70, 45, 42, 43, 48, 36

    a. Determine the mean of this data set.

    b. Determine the median of this data set.

3. Convert each measurement.

    a. $4\frac{1}{2}$ pounds = _____ oz

    b. 22.86 cm = _____ in.