The A-Ha! Moment: Encouraging Student Mathematical Discourse

Sandy Bartle Finocchi
Senior Academic Officer
The discourse of a classroom – the ways of representing, thinking, talking, agreeing and disagreeing – is central to what students learn about mathematics as a domain of human inquiry with characteristic ways of knowing.

NCTM 2000
Making the Case for Meaningful Discourse
### The OLD versus the NEW

<table>
<thead>
<tr>
<th>The 20th Century Classroom</th>
<th>The 21st Century Classroom</th>
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</thead>
<tbody>
<tr>
<td>Time-based</td>
<td>Outcome-based</td>
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<tr>
<td>Focus: memorization of discrete facts</td>
<td>Focus: what students know, can do and are like after all the details are forgotten.</td>
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<tr>
<td>Lessons focus on the lower level of Bloom’s Taxonomy – knowledge, comprehension and application</td>
<td>Learning is designed on upper levels of Blooms’ – synthesis, analysis and evaluation (although it includes lower levels as curriculum is designed down from the top)</td>
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<tr>
<td>Textbook-driven</td>
<td>Research-driven</td>
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<tr>
<td>Passive learning</td>
<td>Active Learning</td>
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<tr>
<td>Learners work in isolation</td>
<td>Learners work collaboratively</td>
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<tr>
<td>Teacher-centered: teacher is center of attention and provider of information</td>
<td>Student-centered: teacher is facilitator/coach</td>
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<tr>
<td>Little to no student freedom</td>
<td>Great deal of student freedom</td>
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<tr>
<td>“Discipline problems” – educators do not trust students and vice versa. No student motivation.</td>
<td>No “discipline problems” – students and teachers have mutually respectful relationship as co-learners; students are highly motivated.</td>
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<tr>
<td>Teacher is judge. No one else sees student work.</td>
<td>Self, Peer and Other assessments. Public audience, authentic assessments.</td>
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<tr>
<td>Curriculum/School is irrelevant and meaningless to the students.</td>
<td>Curriculum is connected to students’ interests, experiences, talents and the real world.</td>
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<tr>
<td>Driven by the NCLB and standardized testing mania.</td>
<td>Driven by student future success in the ever-changing workplace.</td>
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adapted from www.21stcenturyschools.com
“21st Century teachers and learners alike must realize that education is no longer about what we’ve memorized, but about how we learn to evaluate and utilize information!”

--Anonymous
We cannot know what students will need to know in their future lives.

But, we do know at least one thing that students will need to know in the future: *how to learn*. We need to shift from facilitating learning to developing learners.
Making the Case for Meaningful Discourse

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning
Carnegie Learning’s Three Big Ideas

1. Engage and Motivate
2. Promote Deep Conceptual Understanding
3. Powerful Ongoing Formative Assessment
Instructional Design

- Lessons are structured to provide students with various opportunities to reason, to model and to expand on explanations about mathematical ideas.

- Within each lesson, questions, instructions and worked examples are interleaved to engage students as they develop their own mathematical understanding.
Discussion

- In depth accountable talk
- Two-way interactions

Self-Evaluation

- Seek information
- Share what you know

Mathematical Representations

Introduction

During this course, you will solve problems and work with many different representations of mathematical concepts, ideas, and processes to better understand the world. Each lesson will provide you with opportunities to discuss your ideas, work within groups, and share your solutions and methods with your class. These process icons are placed throughout the text.

Discuss to Understand

- Read the problem carefully.
- What is the context of the problem? Do we understand it?
- What is the question that we are being asked? Does it make sense?
- Is this problem similar to some other problem we know?

Think for Yourself

- Do I need any additional information to answer the question?
- Is this problem similar to some other problem that I know?
- How can I represent the problem using a picture, a diagram, symbols, or some other representation?

Work with Your Partner

- How did you do the problem?
- Show me your representation.
- This is the way I thought about the problem—how did you think about it?
- What else do we need to solve the problem?
- Does our reasoning and our answer make sense to each other?
- How will we explain our solution to the class?

Share with the Class

- Here is our solution and the methods we used.
- Are we communicating our strategies clearly?
- We could only get this far with our solution. How can we finish?
- Could we have used a different strategy to solve the problem?
Resources


Table 1

<table>
<thead>
<tr>
<th>Levels</th>
<th>Characteristics of Discourse</th>
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</thead>
<tbody>
<tr>
<td>0</td>
<td>The teacher asks questions and affirms the accuracy of answers or introduces and explains mathematical ideas. Students listen and give short answers to the teacher’s questions.</td>
</tr>
<tr>
<td>1</td>
<td>The teacher asks students direct questions about their thinking while other students listen. The teacher explains student strategies, filling in any gaps before continuing to present mathematical ideas. The teacher may ask one student to help another by showing how to do a problem.</td>
</tr>
<tr>
<td>2</td>
<td>The teacher asks open-ended questions to elicit student thinking and asks students to comment on one another’s work. Students answer the questions posed to them and voluntarily provide additional information about their thinking.</td>
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<tr>
<td>3</td>
<td>The teacher facilitates the discussion by encouraging students to ask questions of one another to clarify ideas. Ideas from the community build on one another as students thoroughly explain their thinking and listen to the explanations of others.</td>
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Adapted from Hufferd-Ackles, Fuson, and Sherin (2004)
The Five Practices (+)

1. **Anticipating** (e.g., Fernandez & Yoshida, 2004; Schoenfeld, 1998)

2. **Monitoring** (e.g., Hodge & Cobb, 2003; Nelson, 2001; Shifter, 2001)

3. **Selecting** (e.g., Lampert, 2001; Stigler & Hiebert, 1999)

4. **Sequencing** (e.g., Schoenfeld, 1998)

5. **Connecting** (e.g., Ball, 2001; Brendehur & Frykholm, 2000)
Getting Started...Setting Goals

• Create a classroom environment that supports and encourages conversation

• Identify learning goals and what students are to understand as a result of doing the lesson

• Think about what students will come to know and understand rather than only on what they will do

• Work in collaboration with other teachers
The teacher’s role in classroom discourse may signal to students whether teachers think that they are capable of learning and whether they are succeeding in meeting the teacher’s expectations. If students perceive teachers as supporting their learning through what they say, the students may be less likely to adopt defensive measures such as avoidance strategies. Conversely, if students perceive teacher discourse as nonsupportive—as suggesting that they cannot or will not meet such expectations—they may then adopt avoidance strategies.

“The Classroom Environment and Students’ Reports of Avoidance Strategies in Mathematics: A Multimethod Study,” Journal of Educational Psychology
U.S. Shirts
Problem 1 **Cost Analysis**

This past summer you were hired to work at a custom T-shirt shop, U.S. Shirts. One of your responsibilities is to calculate the total cost of customers’ orders. The shop charges $8 per shirt plus a one-time charge of $15 to set up a T-Shirt design.
2. What is the total cost of an order for:
   a. 3 shirts?
   b. 10 shirts?
   c. 100 shirts?

   If the order doubles, does the total cost double?

   Your answers should include the number of shirts and the total cost.

3. Explain how you calculated each total cost.
1. **Anticipating** likely student responses

It involves considering:

- The array of strategies that students might use to approach or solve a challenging mathematical task
- How to respond to what students produce
- Which strategies will be most useful in addressing the mathematics to be learned

It is supported by:

- Doing the problem in as many ways as possible
- Doing so with other teachers
- Documenting student responses year to year
2. **Monitoring** students actual responses during independent work

**It involves:**

- Circulating while students work on the problem and watching and listening
- Recording interpretations, strategies, and points of confusion
- Asking questions to get students back “on track” or to advance their understanding

**It is supported by:**

- Anticipating student responses beforehand
- Using recording tools
3. **Selecting** student responses to feature during discussion

**It involves:**

- Choosing particular students to present because of the mathematics available in their responses
- Making sure that over time all students are seen as authors of mathematical ideas and have the opportunity to demonstrate competence
- Gaining some control over the content of the discussion (no more “who wants to present next”)

**It is supported by:**

- Anticipating and monitoring
- Planning in advance which types of responses to select
Monitoring

Teacher
Sandy Bartle

Carnegie Learning Answer
Show Carnegie Learning answers

Students
Bartle Student 1
Bartle Student 2
Bartle Student 3
Bartle Student 4
Bartle Student 5
1. Describe the problem situation and your responsibility in your...

2. What is the total cost of an order for:

3. Explain how you calculated each total cost.

4. How many shirts can a customer buy if they have:

5. Explain how you calculated the number of shirts that each custo...

6. Complete the table of values for the problem situation.

7. Analyze the problem situation.

8. Create a graph of the data from your table.

9. Define the domain and range for this problem situation.

10. Define the variables and write an algebraic equation for the pro...
4. **Sequencing** student responses during the discussion

It involves:

- Purposefully ordering presentations so as to make the mathematics accessible to all students
- Building a mathematically coherent story line

It is supported by:

- Anticipating, monitoring, and selecting
- During anticipation work, considering how possible student responses are mathematically related
Sequencing

3(8) + 15 = 39
3 shirts cost $39

Drop here to Present  Drop here to Present  Drop here to Present  Drop here to Present
Total cost in dollars: \(3(8) + 15 = 39\)
An order of 3 shirts will cost $39.
Sequencing

9 + 8 + 8 + 15

3 shirts?

# 39

3(8) + 15 = 39

3 shirts cost #39

3 × 8 = 24

24 + 15 = 39

# 39

15 + 3(8)

15 + 24 = 39

3 shirts, #39
5. **Connecting** student responses during the discussion

**It involves:**
- Encouraging students to make mathematical connections between different student responses
- Making the key mathematical ideas that are the focus of the lesson salient

**It is supported by:**
- Anticipating, monitoring, selecting, and sequencing
- During planning, considering how students might be prompted to recognize mathematical relationships between responses
Connecting

- $9 + 8 + 8 + 15$
  - 3 shirts?
  - $\#39$

- $3(8) - 15 = 39$
  - 3 shirts, $\#39$

- $3 \times 8 = 24$
  - $24 + 15 = 39$
  - $\#39$

- $15 + 3(8)$
  - $15 + 24 = 39$
  - 3 shirts, $\#39$
Connecting

$9 + 8 + 8 + 15$

3 shirts?

$\#39$

$3(8) - 15 = 39$

3 shirts cost $39

$3 \times 8 = 24$

$24 + 15 = 39$

$\#39$

$15 + 3(8)$

$15 + 24 = 39$

3 shirts, $39$

Carnegie Learning
Connecting
digital ACE™

9 + 8 + 8 + 15

3 shirts?
#39

3(8) + 15 = 39
3 shirts cost $39

3x8 = 24
24 + 15 = 39
#39

15 + 3(8) = 39
15 + 24 = 39
3 shirts, $39

Carnegie Learning
“Teachers need to develop a range of ways of interacting with and engaging students as they work on tasks and share their thinking with other students. This includes having a repertoire of specific kinds of questions that can push students’ thinking toward core mathematical ideas as well as methods for holding students accountable to rigorous, discipline-based norms for communicating their thinking and reasoning.”

(Smith and Stein, 2011)
What is a Question?

“A question is any sentence which has an interrogative form or function.”

“Teacher questions are instructional cues or stimuli that convey to students the content elements to be learned and the directions for what they are to do and how they are to do it.”
Powerful Questioning

• Creating a climate of discovery
• Exploring underlying assumptions and beliefs
• Listening for connections
• Articulating shared understanding
• Facilitating conversations that enhance trust and reduce fear
• Shifting the mathematical authority to the class
A few questions to consider as you reflect on your teaching practice...
• Do you emphasize mathematical thinking and process, or is the goal to demonstrate procedures and obtain right answers?
• Are you providing opportunities for students to demonstrate and communicate their knowledge?
• Do you help students **see** connections between big ideas and concepts in mathematics?
• Do you encourage students to **understand** the connections between big ideas and concepts in mathematics?
• Do you encourage students to **make** the connections between big ideas and concepts in mathematics?

Reflecting on our Practice
Do you know the mathematical standards for the grade levels below the grade you teach and what is next for your students?

How are you thinking deeper about mathematics and teaching mathematics?

Reflecting on our Practice
• What do you do when students get frustrated or confused?
• What do you do when a student makes a mistake?
• Is your classroom a safe place to learn?
• Do you believe in your students?

Reflecting on our Practice
Learning Is **Not** a Spectator Sport

Students must:

- Talk about it
- Write about it
- Relate it to past experiences
- Apply it to their daily lives

• **DO THE MATH!**
“If I supply you a thought, you may remember it and you may not. But if I can make you think a thought for yourself, I have indeed added to your stature.”

Elbert Hubbard (1856 – 1915)

American writer and printer

Thank You!

Sandy Bartle Finocchi
sandy@carnegielearning.com
Related Resources
