TEXAS MATH SOLUTION

Accelerated Grade 6

Module 2 Topic 3

Topic Level Materials

Module Overview
Topic Pacing Guide
Family Guide
Topic Overview
Topic Summary

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Module 2 Overview
Relating Quantities

"Facility with proportionality involves much more than setting two ratios equal and solving for a missing term. It involves recognizing quantities that are related proportionally and using numbers, tables, graphs, and equations to think about the quantities and their relationship." (NCTM Principles and Standards, pg. 216)

Why is this Module named Relating Quantities?

Relating Quantities introduces students to ratios, reasoning about proportional relationships, solving problems with ratio reasoning, and special ratios (e.g., percents, unit rates, and conversion rates). Students are expected to think of situations in relative rather than in absolute terms; this is the essence of developing proportional reasoning. Developing proportional thinking requires time and a variety of experiences that make use of multiplicative comparisons.

To deepen students' understanding of ratio and proportional relationships, they are provided with non-examples. For example, as part of **Relating Quantities**, students contrast additive and multiplicative comparisons of quantities, and they compare graphs of proportional and non-proportional relationships. To understand what something *is*, students should also understand what it *is not*.

Relating Quantities focuses on reasoning about ratios and proportional relationships. Because algorithms such as cross multiplication do not develop reasoning, this strategy is not part of the module. However, students relate equivalent ratios and solve for missing values by setting up a proportion. In this course, students should focus on reasoning about the multiplicative relationships in proportions, not on algorithms. As students become more proficient with strategies, they can choose from strategies and develop abbreviated forms of the strategies in order to solve problems. The goal in **Relating Quantities**, as in the rest of the course, is to develop mathematical thinkers rather than merely "answer getters."

What is the mathematics of Relating Quantities?

Relating Quantities contains three topics: *Ratios, Percents*, and *Unit Rates and Conversions*. The focus throughout the module is the development of reasoning with quantities, primarily quantities that exhibit multiplicative relationships. Students consider the different ways in which quantities can be related to each other:

part-to-part or part-to-whole. They reason with tape diagrams, double number lines, ratio tables, and graphs to determine whether ratios are equivalent or to calculate equivalent ratios. Part-to-whole ratios are rewritten as percents, and students solve percent problems. Similarly, special part-to-part ratios are revisited as unit rates and conversion rates, and students solve unit rate and conversion problems.

Ratios builds on students' multiplicative reasoning to establish ratio and ratio reasoning. Students develop the concept of a ratio as a comparison between two quantities. They engage in both quantitative and qualitative reasoning about quantities. Throughout the topic, students use ratio and rate reasoning to solve mathematical and real-world problems. Students use tape diagrams, tables of equivalent ratios, double number lines, and graphs to determine equivalent ratios.

Percents is framed as a special type of ratio: a rate per 100. Students write fraction, decimal, and percent equivalences to see different ways to represent percents. They use ratio reasoning developed in the prior topic to find a percent of a quantity and to solve percent problems involving finding the whole. They use estimation and benchmark percents in computations and to judge the reasonableness of their answers.

Unit Rates and Conversions addresses a special type of ratio, unit rates, and a special type of unit rate, conversion rates. Students convert within and between the customary and metric systems of measurements using proportions and unit rates. Students develop an understanding of unit rate and explore different rates when comparing two quantities. They use the strategies and reasoning developed in *Ratios* to reason about and solve conversion problems and solve unit rate problems, including problems about unit pricing and constant speed.

How is Relating Quantities connected to prior learning?

Relating Quantities builds on students' experiences solving additive and multiplicative word problems. The module also requires students to use their knowledge of equivalent fractions as they reason about and calculate equivalent ratios using strategies such as scaling. Representing real-world and mathematical problems in the first quadrant and interpreting coordinates in the context of a situation is the foundation for students' abilities to graph and interpret ratios and unit rates on the coordinate plane.

Students continue to develop their fluency with decimals and fractions, skills developed in prior grades and in Module 1, **Composing and Decomposing**. Students also use the geometry standards learned in Module 1 in problem-solving contexts throughout **Relating Quantities**.

In elementary school, students converted unit of measure within the same measurement system, either customary or metric units. In this course, students build off of this knowledge to convert units of measure within and between the customary and metric systems using proportions and unit rates.

When will students use knowledge from Relating Quantities in future learning?

This module introduces and formalizes ratios and proportional relationships, which provide the foundation for future work with linear equations and functions. Slope of linear equations will lead to slopes of other functions and the study of derivatives in calculus.

In Module 4, **Determining Unknown Quantities**, students revisit multiplicative versus additive relationships, including writing and graphing equations of the forms $x + p = q$ and $px = q$. Students compare graphs and equations, making generalizations about the forms of equations and characteristics of graphs that correspond to additive and multiplicative relationships between quantities.

In Module 3, **Moving Beyond Positive Quantities**, students attend to magnitude of rational numbers and their placement on the number line, connecting back to the ordering of rational numbers. The practice with interpreting coordinates of graphs in the first quadrant will also prepare students to interpret coordinates in all four quadrants in Module 3.

*1 Day Pacing = 45 min. Session

Module 2: Relating Quantities

Topic 2: Percents

Lesson #	Lesson Title	Lesson Subtitle	Highlights	TEKS	Pacing*
ELPS: 1.A, 1.C, 1.D, 1.E, 2.C, 2.D, 2.G, 2.H, 3.A, 3.B, 3.C, 3.D, 3.E, 3.F, 4.A, 4.B, 4.C, 4.F, 4.K, 5.E, 5.F					
1	We Are Family!	Percent, Fraction, and Decimal Equivalence	Students learn about the relationships between percents, fractions, and decimals. In the first activity, students analyze the results of a survey of one hundred students. They complete a table by writing the ratio, fraction, and decimal equivalences for each result. Students use hundredths grids to model the result, and they then write the percent equivalence. They are reminded that percents are special types of part-to-whole ratios. *Percent* is described as a fraction in which the denominator is 100 and the % symbol represents the phrase "out of 100." Students write numbers in equivalent forms and use number lines to indicate the equivalent fraction, decimal, and percent represented by the markers on the number line. They analyze reasoning about combining ratios into an overall percent. They then play a percentage match game to identify equivalent representations. A chart is provided in the summary to highlight common fraction, decimal, and percent equivalents.	6.4E 6.4F 6.4G 6.5C	2

Lesson #	Lesson Title	Lesson Subtitle	Highlights	TEKS	Pacing*
2	Warming the Bench	Using Estimation and Benchmark Percents	Students begin the lesson building fluency with ordering fractions, decimals, and percents. They then estimate the percent of cylinders, circles, and squares that are partially shaded. Students write estimates as fractions, decimals, and percents. *Benchmark percents* are introduced to help students mentally estimate the value of a percent. They then use calculators to investigate the values of 1% and 10% of several numbers. Students write rules about moving the decimal two places to the left to determine 1% of any number and moving the decimal one place to the left to determine 10%. Various scenarios are presented in which students are asked to estimate and calculate percents.	6.2D 6.4E 6.4F 6.4G	2
3	The Forest for the Trees	Determining the Part and the Whole in Percent Problems	Percent problems involve three quantities: the part, the whole, and the percent. In this lesson, students solve for the percent, given the part and the whole, and solve for the whole, given the percent and the part. They set up a proportion where the percent, if known, is written as a fraction with a denominator of 100. They then determine the unknown, using multiplication.	6.4G 6.5B	2
End of Topic Assessment					1

Module 2: Relating Quantities

TOPIC 2: PERCENTS

In this topic, students transition from thinking about ratio relationships in general to focusing on a special ratio relationship: percent. Students learn that a percent can be defined multiple ways: as a ratio, as a decimal to the hundredths place, and as a part-to-whole relationship in which the whole is 100. Students use their knowledge of fractions and decimals and their intuitive understanding of percents to write and compare rational numbers written in these three different forms. They complete number lines of common fractions, decimals, percent equivalences, connecting to prior work with benchmark fractions and decimals. Throughout this topic, students continue to develop their fluency with whole numbers, fractions, decimals, area, and volume in the context of solving mathematical and real-world problems.

Where have we been?

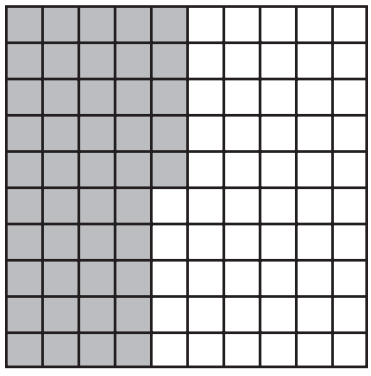
Students have used the relationship between decimals and fractions to write decimals as fractions, and they have used benchmark fractions and decimals to understand ordering of numbers. This topic provides students with similar experiences using this new representation: percents. Because percent is a special ratio, students continue to use the strategies and reasoning developed in the prior topic to solve percent problems.

Where are we going?

Percents are very useful, not only in mathematics, but in everyday life and work. Later in this course, students will use the foundation they establish here to solve more advanced percent problems, including problems involving discounts, tax, interest, percent increase or decrease, and tips.

Using a Hundredths Grid to Represent a Percent

A hundredths grid is a 10 by 10 grid of squares, which are shaded to show different percents. Hundredths grids emphasize that percents are ratios of amounts to 100. When the entire grid is shaded, it represents 1 whole, or 100%.



45%

Myth: Students only use 10% of their brains.

Hollywood is in love with the idea that humans only use a small portion of their brains. This notion formed the basis of the movies *Lucy* (2014) and *Limitless* (2011). Both films ask the audience: Imagine what you could accomplish if you could use 100% of your brain!

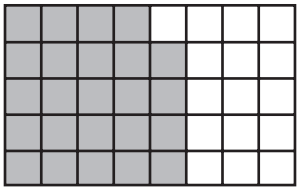
Well, this isn't Hollywood, and you're stuck with an ordinary brain. The good news is that you ***do*** use 100% of your brain. As you look around the room, your *visual cortex* is busy assembling images; your *motor cortex* is busy moving your neck; and all of the *associative areas* recognize the objects that you see. Meanwhile, the *corpus callosum*, which is a thick band of neurons that connect the two hemispheres, ensures that all of this information is kept coordinated. Moreover, the brain does this automatically, which frees up space to ponder deep, abstract concepts like mathematics!

#mathmythbusted

Talking Points

A common error that students make when working with part-to-whole ratios (like percents and fractions) is to forget about the whole. Look for ways to remind your student about this common mistake.

For example, this model shows 24 shaded squares. Students might say that 24% is shaded.



But the whole is not 100, it's 40. So, $\frac{24}{40}$, or 60%, is shaded. Also, more than half is shaded, so it has to be more than 50%.

Key Term

benchmark percent

A benchmark percent is a percent that is commonly used, such as 1%, 5%, 10%, 25%, 50%, and 100%.

Percents
Topic 2 Overview

How is *Percents* organized?

In this topic, students transition from thinking about ratio relationships in general to focusing on a special ratio relationship: percent. Students learn that a percent can be defined multiple ways: as a ratio, as a fraction with a denominator of 100, as a decimal to the hundredths place, and as a part : whole relationship in which the whole is 100.

Students use their knowledge of fractions and decimals and their intuitive understanding of percent to write and compare rational numbers written in these three different forms. From the previous topic, students know how to write a percent as a fraction. In this topic, students generalize how to write a percent as a decimal and a decimal as a percent. They complete number lines of common fraction, decimal, and percent equivalences, connecting to prior work with benchmark fractions and decimals. Students also order non-equivalent fractions, decimals, and percents, using a number line when necessary. Placing numbers written in different representations on a number line broadens the ways in which students can use and relate quantities.

To continue developing their number sense, students use estimation and benchmark percents, which are especially useful in day-to-day life. The theme from the first module is carried over into this topic as students decompose percents and numbers and then compose the components to solve percent problems. Students use ratio reasoning and strategies to solve percent problems, including determining the part of a whole and determining the whole given a part and percent. Throughout this topic, students continue to develop their fluency with whole numbers, fractions, decimals, area, and volume in the context of solving mathematical and real-world problems.

What is the entry point for students?

Students enter grade 6 with extensive experience with fractions and decimals. They have used tape diagrams (sometimes called fraction strips) and number lines to compare and order fractions and decimals. They have used the relationship between decimals and fractions to write decimals as fractions, and they have used benchmark fractions and decimals to understand ordering of numbers. This topic provides students with similar experiences using this new representation: percents.

This topic also employs skills and knowledge that students developed in previous topics of this course. Early activities required

students to recall the basic knowledge of fractions and decimals, including writing decimals as fractions, which they developed in the previous module. Students also recall the basic concept of a percent as a rate per 100, as introduced in the last topic. Because percent is a special ratio, students continue to use the strategies and reasoning developed in the prior topic to solve percent problems. Framing percent as a ratio allows students to see the connected nature of mathematics, rather than viewing the idea of percent as a separate concept.

How does a student demonstrate understanding?

Students will demonstrate understanding of the standards in this topic if they can:

- Write a percent as a ratio of a number to 100.
- Write equivalent fractions, decimals and percents.
- Model percents as rates per 100 on a hundredths grid.
- Determine a percent of a number as a rate per 100.
- Order fractions, decimals and percents.
- Use benchmark percents to calculate common percents of quantities.
- Apply ratio reasoning to solve mathematical and real-world problems to determine the percent of a number or the whole.
- Solve real-world problems that involve finding the whole, given a part and a percent.

Why is *Percents* important?

This topic is useful not only in mathematics, but also in everyday life and work. The ratio and rate reasoning students use to determine a part, percent, or whole in a percent situation allows students to judge the reasonableness of their results when they solve percent problems that cannot easily be solved by scaling up or down or by using double number lines. Later in this course, students will solve more advanced percent problems, including discounts, tax, interest, percent increase or decrease, and tips. The reasoning foundation provided for students in this topic is essential in developing deep conceptual understanding of percent as a ratio, prior to their pursuit of procedural fluency throughout this course and in future courses.

How do the activities in *Percents* promote student expertise in the mathematical process standards?

All Carnegie Learning topics are written with the goal of creating mathematical thinkers who are active participants in class discourse, so elements of the mathematical process standards should be evident

in all lessons. Students are expected to make sense of problems and work toward solutions, reason using concrete and abstract ideas, and communicate their thinking while providing a critical ear to the thinking of others.

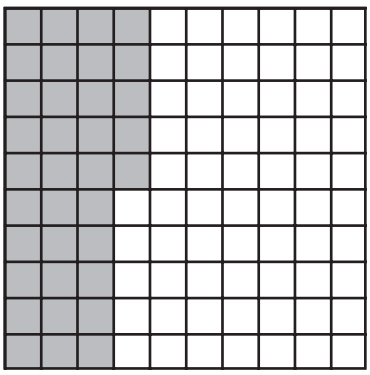
As in the previous topic, precision, reasoning, and the appropriate use of mathematical tools are key mathematical processes being developed in this topic. Students are expected to reason about percents as ratios and develop their own computational shortcuts as they determine the percent value, whole, and part of a percent. They use benchmark percents to estimate percents and calculate simple percent values. Students are also expected to use a variety of tools to develop their understanding and calculation of percent values, selecting the most efficient strategy as they complete the topic. Throughout this topic, students are expected to attend to specific levels of precision and label quantities and number lines with the appropriate units.

Materials Needed

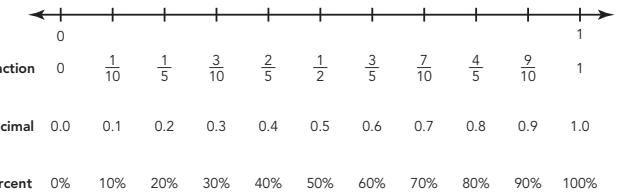
- Scissors
- Note cards
- Calculators

Concrete and Visual Representations Used

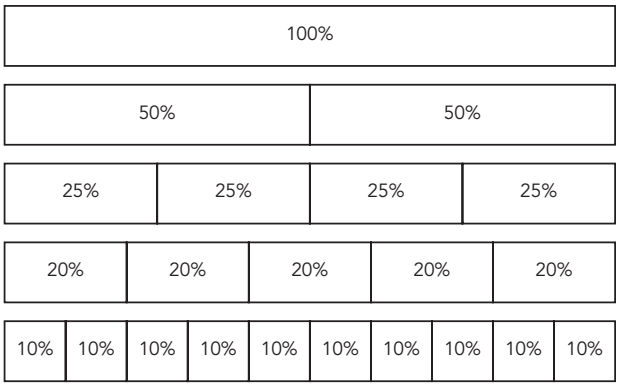
Hundredths Grids



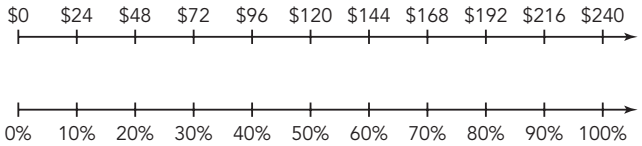
Number Lines to Display Equivalences and Magnitude 45%



Tape Diagrams to Understand Benchmark Percents



Double Number Lines to Solve Problems



Learning Together

ELPS: 1.A, 1.C, 1.D, 1.E, 2.C, 2.D, 2.G, 2.H, 3.A, 3.B, 3.C, 3.D, 3.E, 3.F, 4.A, 4.B, 4.C, 4.F, 4.K, 5.E, 5.F

Lesson	Lesson Name	TEKS	Days	Highlights
1	We Are Family!: Percent, Fraction, and Decimal Equivalence	6.4E 6.4F 6.4G 6.5C	2	Students learn about the relationships between percents, fractions, and decimals. In the first activity, students analyze the results of a survey of one hundred students. They complete a table by writing the ratio, fraction, and decimal equivalences for each result. Students use hundredths grids to model the result, and they then write the percent equivalence. They are reminded that percents are special types of part-to-whole ratios. Percent is described as a fraction in which the denominator is 100 and the % symbol represents the phrase "out of 100." Students write numbers in equivalent forms and use number lines to indicate the equivalent fraction, decimal, and percent represented by the markers on the number line. They analyze reasoning about combining ratios into an overall percent. They then play a percentage match game to identify equivalent representations. A chart is provided in the summary to highlight common fraction, decimal, and percent equivalents.
2	Warming the Bench: Using Estimation and Benchmark Percents	6.2D 6.4E 6.4F 6.4G	2	Students begin the lesson building fluency with ordering fractions, decimals, and percents. They then estimate the percent of cylinders, circles, and squares that are partially shaded. Students write estimates as fractions, decimals, and percents. *Benchmark percents* are introduced to help students mentally estimate the value of a percent. They then use calculators to investigate the values of 1% and 10% of several numbers. Students write rules about moving the decimal two places to the left to determine 1% of any number and moving the decimal one place to the left to determine 10%. Various scenarios are presented in which students are asked to estimate and calculate percents.
3	The Forest for the Trees: Determining the Part and the Whole in Percent Problems	6.4G 6.5B	2	Percent problems involve three quantities: the part, the whole, and the percent. In this lesson, students solve for the percent, given the part and the whole, and solve for the whole, given the percent and the part. They set up a proportion where the percent, if known, is written as a fraction with a denominator of 100. They then determine the unknown, using multiplication.

Suggested Topic Plan

*1 Day Pacing = 45 min. Session

Day 1	Day 2	Day 3	Day 4	Day 5
TEKS: 6.4E, 6.4F, 6.4G, 6.5C	**LESSON 1** continued	TEKS: 6.2D, 6.4E, 6.4F	**LESSON 2** continued	TEKS: 6.5B
LESSON 1 **We Are Family!** GETTING STARTED ACTIVITY 1	ACTIVITY 2 ACTIVITY 3 TALK THE TALK	**LESSON 2** **Warming the Bench** GETTING STARTED ACTIVITY 1 ACTIVITY 2	ACTIVITY 3 ACTIVITY 4 TALK THE TALK	**LESSON 3** **The Forest for the Trees** GETTING STARTED ACTIVITY 1 ACTIVITY 2 ACTIVITY 3

Day 6	Day 7
LESSON 3 continued ACTIVITY 4 ACTIVITY 5 TALK THE TALK	**END OF TOPIC ASSESSMENT**

Assessments

There is one assessment aligned to this topic: End of Topic Assessment.

Percents Summary

KEY TERM

- benchmark percent

LESSON 1

We Are Family!

Percent can be used to represent a part-to-whole relationship with a whole of 100. The symbol "%" means "out of 100." You can think of a percent as a fraction in which the denominator is 100.

Percents, fractions, and decimals can be used interchangeably.

For example, you can write 15 out of 100 as the fraction $\frac{15}{100}$, or $\frac{3}{20}$. Written as a decimal, 15 out of 100 is 0.15. Because percent means "out of 100," 15 out of 100 can also be written as 15%.

When the denominator is a factor of 100, scale up the fraction to write it as a percent.

$$\frac{4}{5} \overset{\times 20}{=} \frac{80}{100}$$

$$\frac{80}{100} = 80\%$$

When the denominator is not a factor of 100, you can divide the numerator by the denominator to write the fraction as a decimal, which you can then write as a percent.

$$\begin{array}{r} 0.625 \\ 8\overline{)5.000} \\ -\,48 \\ \hline 20 \\ -\,16 \\ \hline 40 \\ -\,40 \\ \hline 0 \end{array}$$

$\frac{5}{8} = 5 \div 8$

$0.625 = 62.5\%$

Common Equivalent Fractions, Decimals, and Percents									
Fraction	$\frac{1}{5}$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{2}{5}$	$\frac{1}{2}$	$\frac{3}{5}$	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{4}{5}$
Decimal	0.2	0.25	$0.\overline{3}$	0.4	0.5	0.6	$0.\overline{6}$	0.75	0.8
Percent	20%	25%	$33\frac{1}{3}\%$	40%	50%	60%	$66\frac{2}{3}\%$	75%	80%

LESSON 2

Warming the Bench

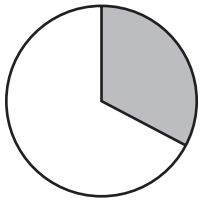
When ordering numbers expressed as fractions, decimals, and percents, you can first write the numbers in the same form before comparing.

For example, to order the numbers 0.88, 90%, and $\frac{17}{20}$ from least to greatest, you can write each number as a percent.

$$0.88 = \frac{88}{100} = 88\% \qquad \frac{17}{20} = \frac{85}{100} = 85\%$$

The numbers in order from least to greatest are $\frac{17}{20}$, 0.88, and 90%.

You can estimate percents when using a visual model.



For example, the shaded part appears to be about $\frac{1}{3}$ of the whole circle, and $\frac{1}{3} \approx 33\%$.

A **benchmark percent** is a percent that is commonly used, such as 1%, 5%, 10%, 25%, 50%, and 100%. With fractions and decimals, benchmarks can be used to make estimations. With percents, however, you can use benchmarks to calculate any whole percent of a number.

For example, determine each value if 400 is 100%. There is more than one way to use benchmark percents to determine the values.

a. 50%	50% is half of 100%. $400 \times \frac{1}{2} = 200$
b. 25%	25% is half of 50%. $200 \times \frac{1}{2} = 100$
c. 10%	10% is one-fifth of 50%. $200 \times \frac{1}{5} = 40$
d. 5%	5% is half of 10%. $40 \times \frac{1}{2} = 20$
e. 1%	1% is one-fifth of 5%. $20 \times \frac{1}{5} = 4$

You can determine any whole percent of a number by using 10%, 5%, and 1%.

For example, what is 28% of 500?

$28\% = 10\% + 10\% + 5\% + 1\% + 1\% + 1\%$

10% of 500 is $500 \times \frac{1}{10}$, or 50.

5% of 500 is $50 \times \frac{1}{2}$, or 25.

1% of 500 is $25 \times \frac{1}{5}$, or 5.

$50 + 50 + 25 + 5 + 5 + 5 = 140$

28% of 500 is 140.

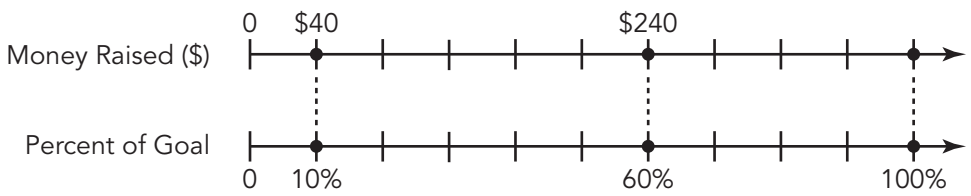
LESSON 3

The Forest for the Trees

Percent problems often have a part, a percent, and a whole. When you know the part and the percent, you can use a variety of strategies to determine the whole.

One strategy is a double number line.

For example, Karla's homeroom raised $240 for charity, which is 60% of their goal. Karla uses a double number line to record the amount of money raised and the percent of the goal raised.



Karla's homeroom has raised $240, which is 60% of the goal.

To determine the value that corresponds to 10%, Karla divided the amount raised so far by 6: $240 ÷ 6 = $40.

Since 10% × 10 = 100%, she can multiply $40 by 10 to determine the homeroom's goal: $40 × 10 = $400.

You can also use proportions to determine the whole in percent problems.

For example, Carlos is told that 65% of the students, or 78 students, prefer pizza for lunch according to a recent survey. He wants to know how many students were surveyed.

He wrote a proportion and determined that 120 students were surveyed.

$\frac{\text{part}}{\text{whole}}$

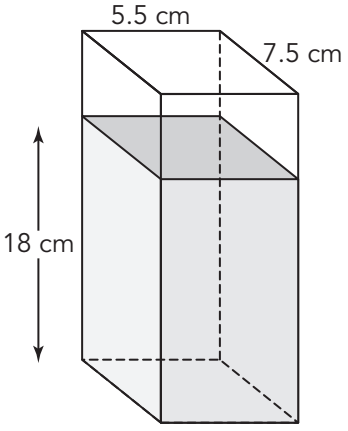
$$\frac{78}{?} = \frac{65}{100}$$

$$\frac{65}{100} \overset{\div 5}{=} \frac{13}{20} \overset{\times 6}{=} \frac{78}{?}$$

$$\frac{78}{120} = \frac{65}{100}$$

These strategies can be used to solve geometry problems as well.

For example, the tank shown is 75% full of water. What is the height of the tank?



You can set up a proportion to solve for the height of the water tank. Since 18 cm represents the height of the tank when it is 75% full of water, you can determine the height of the tank in centimeters when it is 100% full of water.

The height of the tank is 24 cm.