TEXAS MATH SOLUTION

Accelerated Grade 7

Module 2 Topic 3

Topic Level Materials

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Topic Summary

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Module 2 Overview
Developing Function Foundations

"Functions are situations that covary. Yet functions and function notation are very abstract and difficult for students. Experiences with function situations must begin with meaning-making experiences. . . . Linear (and nonlinear) situations should be analyzed across representations." (*Teaching Student-Centered Mathematics: Developmentally Appropriate Instruction for Grades 6–8, p.* 242, 248)

Why is this Module named Developing Function Foundations?

The understanding of functions is a predominant concept studied in algebra courses. As students in high school math courses learn new notation and new function families, they often lose sight of the structural similarities among the function families and forget that each family has the same essential characteristics because they are all functions. **Developing Function Foundations** provides students with a deep conceptual understanding of functions. Students define a function as a rule that assigns each input in a relation to exactly one output. They explore linear functions specifically: how they are represented, how they build from prior knowledge, and how they can be analyzed to answer questions for the situation they represent. By the conclusion of this module, students will have a strong conceptual understanding of functions, particularly linear functions.

What is the mathematics of Developing Function Foundations?

Developing Function Foundations contains five topics: *From Proportions to Linear Relationships*, *Two-Step Equations and Inequalities*, *Multiple Representations of Equations*, *Linear Relationships*, and *Introduction to Functions*.

From Proportions to Linear Relationships connects to students' prior knowledge of proportional relationships, familiarity with linear equations, and transformations. Students review the constant of proportionality and multiple representations of proportional relationships, differentiating between proportional and non-proportional relationships. They connect the equation of a proportional relationship to the equation of a line that passes through the origin, $y = kx$ and $y = mx$, respectively, using similar triangles. Students reason with similar triangles, proving that the steepness

and direction (slope) of a line is constant between any two points on the line. Finally, students use transformations of linear functions to verify properties of translations, rotations, and reflections with respect to lines, line segments, and parallel lines.

Two-Step Equations and Inequalities uses variables to represent quantities and construct equations and inequalities to solve by reasoning about the quantities. Building on prior knowledge, students write equations and inequalities and reason about the magnitudes of the quantities to determine solutions and solution sets. They use a type of bar model to decompose problem situations and double number lines to reason about solving two-step equations. After they have reasoned through solutions and solution sets, students use inverse operations to solve equations and inequalities.

Multiple Representations of Equations requires students to analyze relationships between variables on a coordinate plane. Students engage in problem solving, writing equations and inequalities for problem situations, interpreting the meanings of quantities in the problems, creating tables of values, graphing problem situations, and making connections across the representations. They solve word problems beginning with a table, graph, equation, or verbal description of the problem.

Building on the first topic of this module, *Linear Relationships* analyzes linear relationships. Students learn to construct models of linear relationships, formally calculate slope and initial values from verbal descriptions, pairs of coordinates, tables, and graphs. They construct linear equations and graph linear relationships expressed in slope-intercept form, $y = mx + b$.

Introduction to Functions provides students with connections from prior knowledge (e.g., sequences and describing graphs) to the basic ideas around the concept of a function. Students define and compare functions using multiple representations. They learn to analyze verbal descriptions, mappings, tables, graphs, and equations to determine if the relationships represent functions. They formalize the use of qualitative descriptions to explain the functional relationship between two quantities on a graph.

How is *Developing Function Foundations* connected to prior learning?

Developing Function Foundations relies on students' understanding of proportional relationships and the work

they did in **Transforming Geometric Objects** (Module 1). Students should begin this module with a strong foundation in multiple representations of proportional relationships to include tables, graphs, and equations. The second topic builds on students' experience in the previous course with expressions, equations, and inequalities. Students are expected to recall ideas of equivalent expressions, inverse operations, writing and solving one-step equations, along with graphing equations on a four-quadrant graph and apply these ideas using the set of rational numbers to model the more complex two-step equations and inequalities.

Students use their prior knowledge of multiple representations of proportional relationships to make connections to the slope-intercept form of a linear equation. Students also use their understanding of similar triangles to illustrate why the slope of a line is the same between any two points on the line. They use their new knowledge of effects of transformations on coordinates as they transition from equations of the form $y = mx$ to $y = mx + b$ and as they analyze parallel lines and their relationships.

When will students use knowledge from *Developing Function Foundations* in future learning?

The content in this module provides the basis for Algebra 1 and their study of other function families in future math courses. Students will build upon their conceptual understanding of functions as they learn and use function notation and combine functions to build new functions in advanced algebra courses. They will differentiate between linear and nonlinear functions, including exponential and quadratic functions. Students will be able to transform any functions, in a way similar to the transformation of lines in *From Proportions to Linear Relationships*.

*1 Day Pacing = 45 min. Session

Module 2: Developing Function Foundations

Topic 3: Multiple Representations of Equations

Lesson #	Lesson Title	Lesson Subtitle	Highlights	TEKS	Pacing*
ELPS: 1.A, 1.D, 1.E, 1.G, 1.F, 2.C, 2.D, 2.G, 2.H, 2.I, 3.A, 3.B, 3.C, 3.D, 3.F, 4.A, 4.B, 4.C, 4.G, 4.K, 5.E					
1	Put It on the Plane	Representing Equations with Tables and Graphs	Students analyze linear equations using tables and graphs. Given situations written as sentences, students identify the quantities that change, the quantities that remain constant, and the quantity that depends on the other. They write and solve equations. Students then create a table of values related to the situation. Using the table of values, they define the upper bounds, lower bounds, and intervals of the graph and create a graph of the situation. Students then answer questions related to possible ordered pairs and use the equation and the graph to justify their reasoning. In one situation, students answer questions regarding events that occurred before a specified time, and the timing of those events are represented using negative numbers. Throughout the lesson, students explain if the linear situations represent proportional relationships using the tables, equations, and graphs.	7.7A 7.10A 7.11A	2

Lesson #	Lesson Title	Lesson Subtitle	Highlights	TEKS	Pacing*
2	Stretches, Stacks, and Structure	Structure of Linear Equations	Students write and solve equations for more complicated contexts. They use tables to create equations that require the use of the expression $(n - 1)$ to represent the quantity of the independent variable except for the initial value. They compare the two forms of the same equation and write possible scenarios that could be modeled by the new equations. Students use different representations—the two different forms of the equation and the graph—to answer questions about each context. They also relate the equations to the graphs. Students analyze the different forms of linear equations to reveal aspects on the problems and how the quantities are related. They generalize interpretations for each form of the equations in terms of the equations and in terms of the graphs.	7.7A 7.10A 7.11A	3
3	Deep Flight I	Building Inequalities and Equations to Solve Problems	Students work with a negative rate of change. They use negative values to create a table and graph a problem situation. Students write an equation that represents the situation with a negative value for the unit rate of change, answer several questions, and enter the results in a table which is used to graph the situation. Students analyze the graph to write inequalities based on constraints provided in the scenario. Students write and solve inequalities to answer questions about the scenario.	7.4A 7.7A 7.10A 7.11A	2

Lesson #	Lesson Title	Lesson Subtitle	Highlights	TEKS	Pacing*
4	Texas Tea and Temperature	Using Multiple Represetnations to Solve Problems	Students put together all that they have learned about the different representations of a linear relationship. Throughout these activities, students are given one of the representations—a verbal description, an equation, a table, or a graph—and they have to use what they know from that representation to create the other representations. They connect these different representations to model each situation.	7.4A 7.7A 7.10C	2
End of Topic Assessment					1

Module 2: Developing Function Foundations

TOPIC 3: MULTIPLE REPRESENTATIONS OF EQUATIONS

This topic broadens students' perspective on solving and interpreting linear equations and inequalities through the use of tables and graphs. Students write and solve two-step equations using positive and negative numbers on four-quadrant graphs. Students then compare graphs of linear equations in different forms. Finally, students practice solving problems by writing equations and inequalities for problem situations, analyzing tables and graphs to solve the equations or inequalities, and interpreting the quantities in each problem situation.

Where have we been?

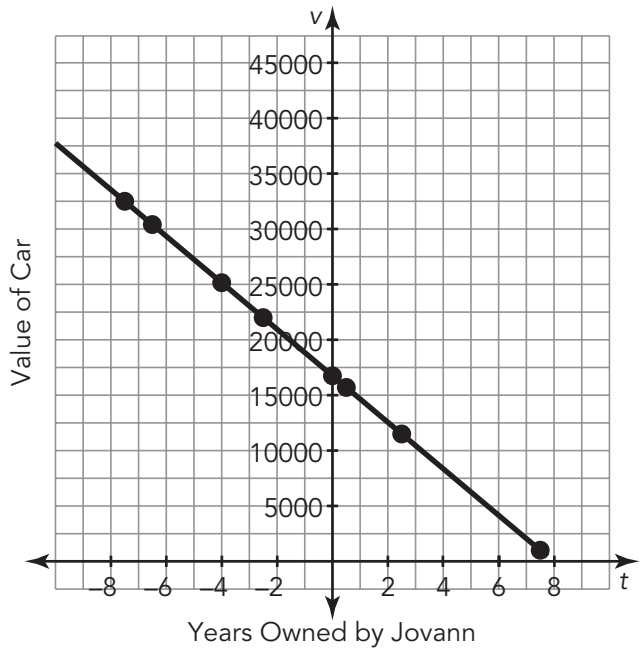
In the previous course, students used multiple representations to model and solve problems, primarily one-step equations. They learned that quantities can vary in relation to each other and are often classified as independent and dependent quantities.

Where are we going?

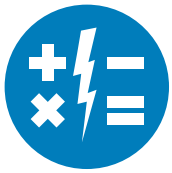
Students' ability to use symbolic algebra can be supported through the use of visual representations. Using and connecting symbolic and graphical representations of equations and inequalities occurs throughout the study of functions in this course and in Algebra 1 and beyond.

Interpreting Situations in More Than One Quadrant

This graph shows the relationship between the time someone has owned a car, *t*, and the value of the car, *v*. We only have information on the values to the right of the vertical axis, but if we assume that the relationship is linear, we can use an equation to determine car values for negative time values.



Myth: Memory is like an audio or video recording.



Let's play a game. Memorize the following list of words: strawberry, grape, watermelon, banana, orange, peach, cherry, blueberry, raspberry. Got it? Good. Some believe that the brain stores memories in pristine form. Memories last for a long time and do not change—like a recording. Without looking back at the original list, was apple on it?

If you answered "yes," then go back and look at the list. You'll see that apple does not appear, even though it seems like it should. In other words, memory is an active, reconstructive process that takes additional information, like the category of words (e.g., fruit), and makes assumptions about the stored information.

This simple demonstration suggests memory is not like a recording. Instead, it is influenced by prior knowledge and decays over time. Therefore, students need to see and engage with the same information multiple times to minimize forgetting (and distortions).

#mathmythbusted

Talking Points

You can further support your student's learning by asking questions about the work they do in class or at home. Your student is learning to represent relationships involving the equivalence of values in a variety of ways.

Questions to Ask

- How does this problem look like something you did in class?
- Can you show me the strategy you used to solve this problem? Do you know another way to solve it?
- Does your answer make sense? How do you know?
- Is there anything you don't understand? How can you use today's lesson to help?

Key Term

unit rate of change

The unit rate of change is the amount that the dependent value changes for every one unit that the independent value changes.

Multiple Representations of Equations

Topic 3 Overview

How is *Multiple Representations of Equations* organized?

This topic broadens students' perspective on solving and interpreting linear equations and inequalities through the use of tables and graphs. Students write, analyze, and solve two-step equations using positive and negative numbers on four-quadrant graphs. In the process of problem solving, students identify independent and dependent variables and interpret negative solutions to problem situations.

Students then compare graphs of linear equations in different forms. In both cases, the rate of change is visible but the initial value may not be evident from the equation. Students analyze the equations and explain how the quantities in each equation are evident on the graphs of the equations.

Next, students practice solving problems by writing equations and inequalities for problem situations, analyzing tables and graphs to solve the equation or inequality, and interpret the quantities in each problem situation. In the last lesson, students solve problems, starting from a different representation for each problem.

What is the entry point for students?

In the previous course, students used multiple representations to model and solve problems, primarily one-step equations. They learned that quantities can vary in relation to each other and are often classified as independent and dependent quantities. For all types of scenarios, students can solve for unknown values of a variable by analyzing a graph. This topic begins by asking students to analyze dependence and independence and, as they previously did with one-step equations, make a table of values, define bounds, graph the problem situation, and then write an equation.

How does a student demonstrate understanding?

Students will demonstrate understanding of the standards in this topic if they can:

- Use variables to represent numbers in real-world or mathematical problems.
- Write and solve two-step equations that represent real-world problems.
- Write and solve two-step inequalities that represent real-world problems.
- Represent linear relationships using verbal descriptions, tables, graphs and equations.
- Make connections between verbal descriptions, tables, graphs, and equations of problem situations.

- Represent constant rates of change in mathematical and real-world problems given verbal descriptions, tables, graphs and equations.
- Use a graph to solve a two-step equation or inequality with rational coefficients.

Why is *Multiple Representations of Equations* important?

As the title suggests, the focus of this topic is on using multiple representations to analyze equations. Students must use prior knowledge of graphing, as well as their knowledge from the previous topic. Historically, students struggle to retain knowledge of basic symbolic computation and calculations, but students' ability to use symbolic algebra can be supported through the use of visual representations. Using and connecting symbolic and graphical representations of equations and inequalities occurs throughout the study of functions in high school mathematics.

How do the activities in *Multiple Representations of Equations* promote student expertise in the mathematical process standards?

All Carnegie Learning topics are written with the goal of creating mathematical thinkers who are active participants in class discourse, so elements of the mathematical process standards should be evident in all lessons. Students are expected to make sense of problems and work toward solutions and to reason using concrete and abstract ideas. They should communicate their thinking while providing a critical ear to the thinking of others.

This topic requires students to analyze problem situations in terms of tables, graphs, and equations and to make connections across the representations. This topic focuses students' attention on the use of mathematics to solve problems that arise in everyday life and to use tools such as graphs, tables, and equations to analyze and draw conclusions about the problem situations. Students are routinely expected to interpret the meanings of values in equations and interpret results in the context of the situation. When they encounter multiple tools—scenarios, tables, graphs, and equations—they are expected to reason about the relationships among them and the affordances and limitations of each. Throughout the topic, students are also expected to notice similarities and differences among graphs and generalize about the relationships between the quantities.

Materials Needed

- None

Learning Together

ELPS: 1.A, 1.D, 1.E, 1.G, 2.C, 2.D, 2.G, 2.H, 2.I, 3.A, 3.B, 3.C, 3.D, 3.F, 4.A, 4.B, 4.C, 4.G, 4.K, 5.E

Lesson	Lesson Name	TEKS	Days	Highlights
1	Put It on the Plane: Representing Equations with Tables and Graphs	7.7A 7.10A 7.11A	2	Students analyze linear equations using tables and graphs. Given situations written as sentences, students identify the quantities that change, the quantities that remain constant, and the quantity that depends on the other. They write and solve equations. Students then create a table of values related to the situation. Using the table of values, they define the upper bounds, lower bounds, and intervals of the graph and create a graph of the situation. Students then answer questions related to possible ordered pairs and use the equation and the graph to justify their reasoning. In one situation, students answer questions regarding events that occurred before a specified time, and the timing of those events are represented using negative numbers. Throughout the lesson, students explain if the linear situations represent proportional relationships using the tables, equations, and graphs.
2	Stretches, Stacks, and Structure: Structure of Linear Equations	7.7A 7.10A 7.11A	3	Students write and solve equations for more complicated contexts. They use tables to create equations that require the use of the expression $(n - 1)$ to represent the quantity of the independent variable except for the initial value. They compare the two forms of the same equation and write possible scenarios that could be modeled by the new equations. Students use different representations—the two different forms of the equation and the graph—to answer questions about each context. They also relate the equations to the graphs. Students analyze the different forms of linear equations to reveal aspects on the problems and how the quantities are related. They generalize interpretations for each form of the equations in terms of the equations and in terms of the graphs.

Lesson	Lesson Name	TEKS	Days	Highlights
3	Deep Flight I: Building Inequalities and Equations to Solve Problems	7.4A 7.7A 7.10A 7.11A	2	Students work with a negative rate of change. They use negative values to create a table and graph a problem situation. Students write an equation that represents the situation with a negative value for the unit rate of change, answer several questions, and enter the results in a table which is used to graph the situation. Students analyze the graph to write inequalities based on constraints provided in the scenario. Students write and solve inequalities to answer questions about the scenario.
4	Texas Tea and Temperature: Using Multiple Representations to Solve Problems	7.4A 7.7A 7.10C	2	Students put together all that they have learned about the different representations of a linear relationship. Throughout these activities, students are given one of the representations—a verbal description, an equation, a table, or a graph—and they have to use what they know from that representation to create the other representations. They connect these different representations to model each situation.

Suggested Topic Plan

*1 Day Pacing = 45 min. Session

Day 1	Day 2	Day 3	Day 4	Day 5
TEKS: 7.7A, 7.10A, 7.11A **LESSON 1** **Put It on the Plane** GETTING STARTED ACTIVITY 1	**LESSON 1** continued ACTIVITY 2 TALK THE TALK	TEKS: 7.7A, 7.10A, 7.11A **LESSON 2** **Stretches, Stacks,and Structure** GETTING STARTED ACTIVITY 1	**LESSON 2** continued ACTIVITY 2	**LESSON 2** continued ACTIVITY 3 TALK THE TALK
Day 6	**Day 7**	**Day 8**	**Day 9**	**Day 10**
TEKS: 7.4A, 7.7A, 7.10A, 7.11A **LESSON 3** **Deep Flight 1** GETTING STARTED ACTIVITY 1	**LESSON 3** continued ACTIVITY 2 ACTIVITY 3 TALK THE TALK	TEKS: 7.4A, 7.7A, 7.10C **LESSON 4** **Texas Tea and Temperature** GETTING STARTED ACTIVITY 1 ACTIVITY 2	**LESSON 4** continued ACTIVITY 3 ACTIVITY 4 TALK THE TALK	**END OF TOPIC ASSESSMENT**

Assessments

There is one assessment aligned to this topic: End of Topic Assessment.

Multiple Representations of Equations Summary

KEY TERM

- unit rate of change

LESSON 1

Put It on the Plane

You can represent a problem situation in many ways.

For example, Ms. Jackson translates books for a living. Her earnings can be represented by a verbal description, table, graph, and equation.

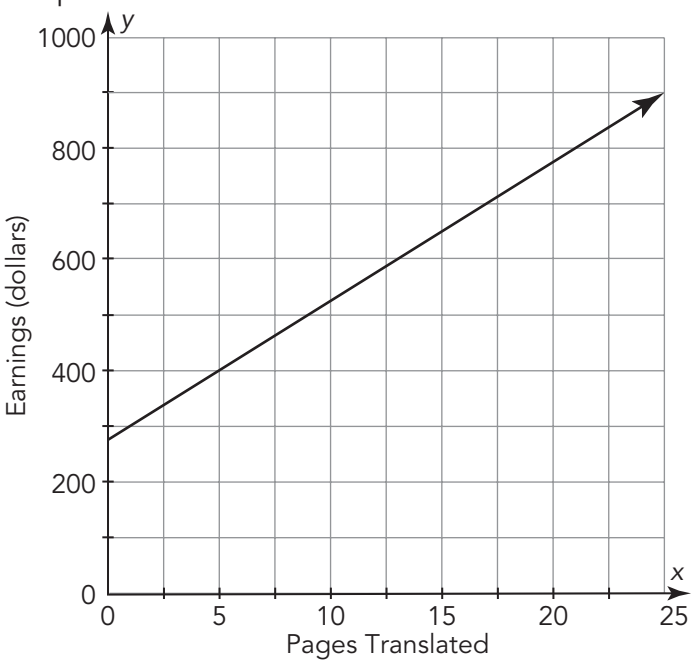
Verbal description: Ms. Jackson charges an initial fee of $275 to manage a project and $25 per page of translated text.

Table:

Number of Pages	Total Earnings for the Project (dollars)
1	300
3	350
10	525
25	900

Equation: $y = 275 + 25x$

Graph:



To solve a linear equation from a graph, locate the value of the given variable, independent or dependent, and determine the exact, if possible, or estimated point corresponding to that variable.

For example, you can use the graph to determine that Ms. Jackson will earn $400 if she translates 5 pages for a customers. She will earn approximately $775 for translating 20 pages.

LESSON 2

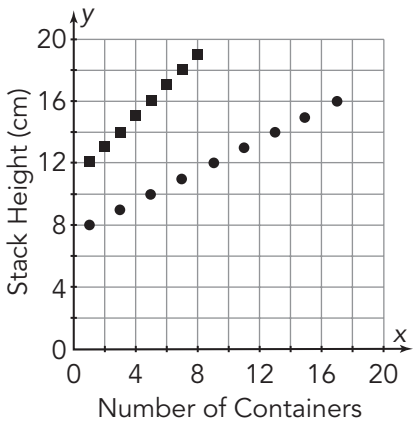
Stretches, Stacks, and Structure

Different forms of an equation reveal different information about a problem situation and about other representations of the problem situation. An equation can have different forms, but it is still the same equation.

An equation in the form $y = c + d(x - 1)$ can be rewritten in the form $y = dx + (c - d)$. Consider this example. A limousine rental company charges $250 for the first two hours the limousine is rented and $30 per hour after that. The equation that represents this scenario can be written as $y = 250 + 30(x - 2)$. The equation can be simplified and rewritten as $y = 190 + 30x$. The structure of each equation reveals different aspects of the scenario. Although the two equations are equivalent, the first one reveals that the customer is going to pay $250 upfront for the first two hours and then $30 for any additional hours.

You can compare the graphs of linear equations.

For example, the graph shown represents the relationship between the number of containers of a certain shape to their height when stacked within each other. You can tell from the graph that the height of a round container is 8 centimeters, while the height of a square container is 12 centimeters. Each additional round container raises the total height by 0.5 centimeter, while each additional square container raises the total height by 1 centimeter.



Legend:
- Round container
- Square container

LESSON
3

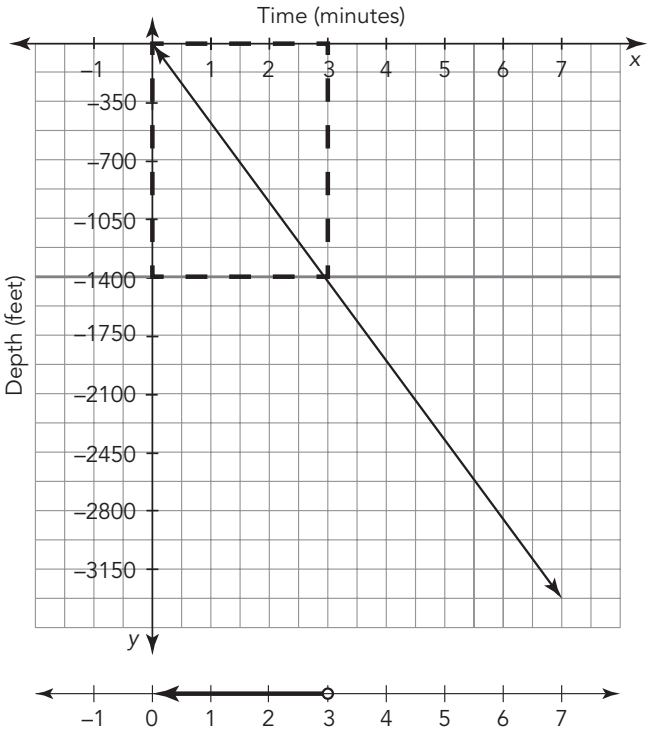
Deep Flight I

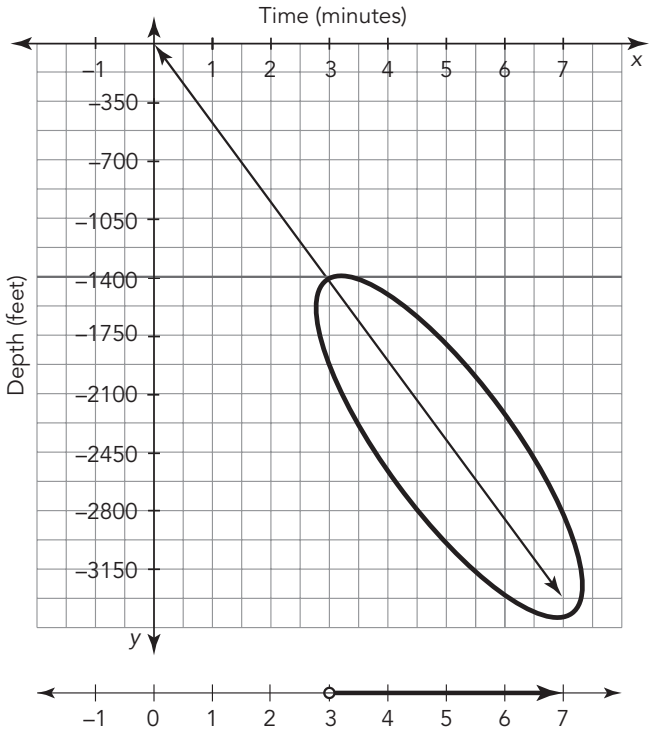
The **unit rate of change** is the amount that the dependent value changes for every one unit that the independent value changes.

For example, suppose the submarine Deep Flight I is going to do a dive starting at sea level, descending 480 feet every minute. The unit rate of change is −480 feet per minute. You can use a graph to estimate solutions to inequality problems.

Estimate the times Deep Flight I will be more than 1400 feet below sea level and the times Deep Flight I will be less than 1400 feet below sea level.

Each of these graphs shows the relationship between the time in minutes and the depth of Deep Flight I. The rectangle on the left graph shows the set of all depths for Deep Flight I less than 1400 feet below sea level. The oval on the right graph shows the set of all depths for Deep Flight I more than 1400 feet below sea level.





Deep Flight I will be less than 1400 feet below sea level for times less than 3 minutes. The submarine will be more than 1400 feet below sea level for times greater than 3 minutes.

LESSON
4

Texas Tea and Temperature

Multiple representations such as a table, an equation, and a graph can be used to represent a problem situation. You may start with any of these representations to solve a problem and move from one to another by studying their forms and determining unit rates of change.

For example, suppose you are given this table of values.

Time	Height of the Water in the Beaker
days	inches
0	6
1	5.5
4	4
8	2

You can use the values in the table to represent the problem situation with a graph, equation, and verbal description.

Equation:
$y = 6 - 0.5x$

Verbal description:
The height of the water in the beaker begins at 6 inches. The height of the water decreases by 0.5 inches each day.

Graph: