TEXAS MATH SOLUTION

Accelerated Grade 6

Module 1 Topic 4

Topic Level Materials

Module Overview
Topic Pacing Guide
Family Guide
Topic Overview
Topic Summary

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Module 1 Overview
Composing and Decomposing

"Understanding of and proficiency with measurement should flourish in the middle grades, especially in conjunction with other parts of the mathematics curriculum."—Navigating through Measurement, page 4

Why is this Module named Composing and Decomposing?

Throughout Grade 6, students reason, look for structure, and identify similarities across mathematical domains: number and operations, proportionality, expressions, equations and relationships, measurement and data, and personal financial literacy. **Composing and Decomposing** begins this work by deepening student understanding of numbers and shapes and exploring their relationships. Students compose familiar numbers and shapes into less familiar or more complicated ones. They decompose large numbers and complex shapes into smaller numbers and simpler shapes to perform calculations. As students become more flexible with how they see shapes and numbers, they will better understand their structure, which in turn will enable them to develop strategies for solving problems across mathematical domains.

Students learn to approach a problem by decomposing (taking apart) or composing (putting together) objects or numbers already understood. By grade 6, students have developed some number sense; they have broken down numbers into sums, differences, products, and quotients. Now, students discover that numbers are composed of numerical expressions, and learn to make use of the distributive property. In previous grades, students studied basic shapes and determined their areas. Now, they calculate the area of complex shapes by composing them from familiar shapes. Students apply these skills as they calculate surface area of rectangular and triangular prisms and pyramids from their nets. They also calculate volume of rectangular prisms with rational number dimensions. Students have studied whole numbers, fractions, and decimals. Now, they learn to see these forms of numbers as composing a single set that can all be plotted on the same number line. They will continue to build fluency with operating with rational numbers throughout this module and course.

Module 1 sets the stage for the habits of mind students will develop as they grow in their mathematical understanding. To see structure across domains, students

must be able to shift perspective and see objects and numbers as entities on which to operate, not just as isolated objects or numbers. When students encounter a new idea, they should ask themselves, "Is this problem or idea similar to another problem or idea I've learned in the past?" "Are there similarities between this topic and a previously learned topic?"

What is the mathematics of Composing and Decomposing?

Composing and Decomposing contains four topics: *Factors and Multiples*, *Positive Rational Numbers*, *Angles and Shapes*, and *Decimals and Volume*. Students begin this module by examining the relationships between numbers and shapes, using areas models to solve problems. They strengthen their skills with fraction and decimal operations, then use these skills to solve problems involving area of various shapes and volume of rectangular prisms.

Factors and Multiples connects number properties and the areas of two-dimensional shapes. Students use the distributive property and prime factorization to write equivalent numeric expressions and calculate greatest common factors and least common multiples. Throughout this topic, connections are drawn between area models and factors, using properties of arithmetic as tools for exploration.

Positive Rational Numbers merges the sets of numbers that students have previously studied into a single set of numbers. Students focus on representing, comparing, and creating equivalence of fractions using a physical model. They move from the concrete model of fractions to a more abstract model on the number line. To understand relative size of fractions, students compare them to benchmark fractions and investigate the relationship of the numerator and denominator of fractions. They review multiplication of whole numbers with fractions using area models to develop understanding of fraction by fraction multiplication. Student draw on the inverse relationship between multiplication and division to develop an understanding of fraction division.

In *Angles and Shapes*, students begin with studying the relationships of angles and side lengths of triangles before they develop the area formula for a triangle by decomposing rectangles to calculate area. Students model the area formulas for parallelograms, trapezoids, and triangles by decomposing and composing parts of shapes due to the additive nature of area. Finally, students calculate the area of composite figures by

decomposing a complex figure into familiar shapes to calculate their areas.

Decimals and Volume builds off of Grade 5 mathematics knowledge of volume of cubes and rectangular prisms. Students deepen their understanding of volume of rectangular prisms with positive rational number dimensions. The work with volume formulas serves to build fluency of operating with rational numbers, specifically decimal operations in this topic. Students use their knowledge of area of composite figures from the previous topic to determine the surface area of rectangular and triangular prisms and pyramids. They calculate the surface area of the three-dimensional solids by determining the area of their two-dimensional nets which are composed of rectangles and/or triangles.

How is Composing and Decomposing connected to prior learning?

In **Composing and Decomposing**, students use what they already know about area, number properties, and volume to access grade 6 mathematics. Their prior knowledge is formalized and used as a basis for the fluency number standards.

The course begins with familiar shapes and numbers, and students move from an intuitive understanding of shape and how to compose and decompose numbers to a formalization of the ideas studied in elementary school. This module develops students' mathematical language around number properties and shape, putting formal language around operations, properties, and strategies students have used throughout elementary school.

When will students use knowledge from Composing and Decomposing in future learning?

Fluency develops over time. This module supports future learning by establishing the fluency standards at the beginning of the course, allowing students to practice these skills throughout the rest of the course. **Composing and Decomposing** sets the stage for seeing structure in numbers and shapes. Students will continue to use fractions and decimals in their work with geometric shapes, percents, expressions, equations, graphs, and statistics.

*1 Day Pacing = 45 min. Session

Module 1: Composing and Decomposing

Topic 4: Decimals and Volume

Lesson #	Lesson Title	Lesson Subtitle	Highlights	TEKS	Pacing*
ELPS: 1.A, 1.B, 1.C, 1.D, 1.E, 1.F, 1.G, 1.H, 2.C, 2.D, 2.E, 2.G, 2.H, 2.I, 3.A, 3.B, 3.C, 3.D, 3.E, 3.F, 3.G, 3.J, 4.A, 4.B, 4.C, 4.D, 4.F, 4.G, 4.I, 4.K, 5.A, 5.B, 5.C, 5.D, 5.E, 5.F, 5.G					
1	Depth, Width and Length	Deepening Understanding of Volume	In this lesson, students are introduced to geometric solids. Students will investigate various figures and sort them based on the definition of a polygon or a polyhedron. The intent of this lesson is for students to determine the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unti fraction edge lengths. In addition, they will review and practice decimal multiplication by calculating volumes of right rectangular prisms.	6.8C 6.8D	2
2	Which Warehouse?	Volume Composition and Decomposition	A scenario about building a bench is provided. Students review estimating sums and differences of decimals and how to add and subtract decimals by adding or subtracting the digits in like place values. They then determine the volume of the bench, a composite solid, using decomposition into smaller rectangular prisms and composition into a larger rectangular prism. The two different strategies require either addition or subtraction of decimals. Students practice solving problems requiring addition and subtraction of decimal volumes.	6.3E 6.8D	2

Lesson #	Lesson Title	Lesson Subtitle	Highlights	TEKS	Pacing*
3	Breaking the Fourth Wall	Surface Area of Rectangular Prisms and Pyramids	Students apply mathematical and spatial reasoning to determine the surface areas of prisms and pyramids using nets, drawings, and measurements. Students solve a variety of surface area problems and distinguish between volume and surface area measurements.	7.9D	2
4	Dividend in the House	Dividing Whole Numbers and Decimals	In this lesson, students use the standard algorithm for long division with whole numbers. They demonstrate how the algorithm works for decimal dividends by relating it to a model and make sense of why the algorithm is modified to accommodate decimal divisors. Students solve area and volume problems requiring decimal division.	6.3E 6.8D	2
End of Topic Assessment					1

Module 1: Composing and Decomposing

TOPIC 4: DECIMALS AND VOLUME

This topic builds on students' prior knowledge of volume, area, and decimal operations. Students are introduced to the language of prisms and pyramids so that distinctions can be made as they solve volume and surface area problems. Through problem-solving activities with volume, students review addition and subtraction of decimal numbers and continue operating with decimals, with the eventual goal of fluency. Students decompose three-dimensional solids into two-dimensional nets and compose solids from nets. Students review whole-number and decimal multiplication and division as they solve area and volume problems.

Where have we been?

Students began learning about decimals in grade 4 and 5. They have experience using concrete models and place-value strategies to operate with decimals to the hundredths place. In grade 5, students learned how to calculate the volume of a right rectangular prism by filling it with cubes and eventually by using the formulas $V = lwh$ and $V = Bh$.

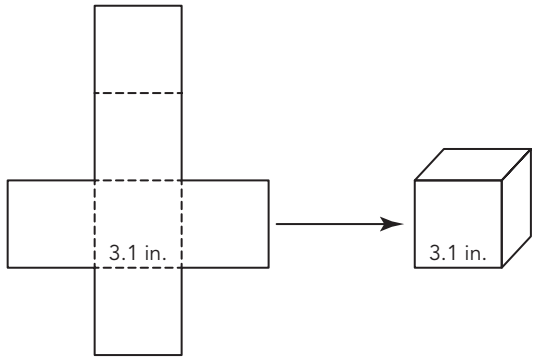
Where are we going?

Students will use decimal operations to solve real-world and mathematical problems throughout the remaining modules of this course. Fractions and decimals are encountered more frequently than whole numbers in daily life, so students should be comfortable and confident solving problems that require operating with such numbers.

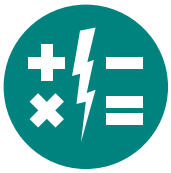
Using Nets to Construct Models of Solid Figures

A net is a two-dimensional model that can be folded into a three-dimensional solid.

The net shown is the net of a cube, which is a rectangular prism that has 6 square faces that are the same size. The net helps to show the entire surface area of the cube.



Myth: Some students are "right-brain" learners while other students are "left-brain" learners.



As you probably know, the brain is divided into two hemispheres: the left and the right. Some categorize people by their preferred or dominant mode of thinking. "Right-brain" thinkers are considered to be more intuitive, creative, and imaginative. "Left-brain" thinkers are more logical, verbal, and mathematical.

The brain can also be broken down into *lobes*. The *occipital lobe* can be found in back of the brain, and it is responsible for processing visual information. The *temporal lobes*, which sit above your ears, process language and sensory information. A band across the top of your head is the *parietal lobe*, and it controls movement. Finally, the *frontal lobe* is where planning and learning occurs. Another way to think about the brain is from the back to the front, where information goes from highly concrete to abstract.

Why don't we claim that some people are "back of the brain" thinkers who are highly concrete; whereas, others are "frontal thinkers" who are more abstract? The reason is that the brain is a highly interconnected organ. Each lobe hands off information to be processed by other lobes, and they are constantly talking to each other. All of us are *whole-brain thinkers!*

#mathmythbusted

Talking Points

You can further support your student's learning by asking questions about the work they do in class or at home. Your student is becoming fluent with decimal operations and gaining experience with two- and three-dimensional measures such as square and cubic units.

Questions to Ask

- How does this problem look like something you did in class?
- Can you show me the strategy you used to solve this problem? Do you know another way to solve it?
- Does your answer make sense? Why?
- Is there anything you don't understand? How can you use today's lesson to help?

Key Terms

polygon
A polygon is a closed figure that is formed by joining three or more line segments at their endpoints.

polyhedron
A polyhedron is a three-dimensional figure that has polygons as faces.

volume
Volume is the amount of space occupied by an object.

surface area
The surface area of a polyhedron is the total area of all its two-dimensional faces.

Decimals and Volume

Topic 4 Overview

How is *Decimals and Volume* organized?

This topic builds on students' prior knowledge of volume, area, and decimal operations. Students are introduced to the language of prisms and pyramids so that distinctions can be made as they engage in discussions of volume and surface area.

Students begin this topic by building on their prior knowledge of volume of right rectangular prisms with whole-number side lengths in order to calculate volumes of right rectangular prisms with rational number dimensions. Next, students use their knowledge of area of composite figures from the previous topic to determine the surface area of three-dimensional solids by determining the area of their two-dimensional nets. Through the problem-solving activities with area and volume, students review operating with decimals with the eventual goal of fluency.

What is the entry point for students?

Students began learning about decimals in grades 4 and 5. They have experience using concrete models and place-value strategies to operate with decimals to the hundredths place.

In grade 5, students learned how to calculate the volume of a right rectangular prism by filling it with cubes and eventually by using the formulas $V = lwh$ and $V = Bh$. Students continue to build fluency in operating with positive rational numbers by solving area and volume problems with positive rational number dimensions.

How does a student demonstrate understanding?

Students will demonstrate understanding of the standards in this topic if they can:

- Fluently add, subtract, multiply, and divide positive rational numbers.
- Write equations that represent problems related to volume of right rectangular prisms where dimensions are positive rational numbers.
- Determine volume of right rectangular prisms where dimensions are positive rational numbers.
- Decompose rectangular and triangular prisms and pyramids into their two-dimensional nets composed of rectangles and/or triangles.
- Solve problems involving total surface area of a rectangular prism or pyramid, or a triangular prism or pyramid by determining the area of the shape's net.

Why is *Decimals and Volume* important?

This topic focuses on the fluency standards for grade 6 that students will practice

throughout the course. The remaining modules require decimal operations, particularly as students work through problem-solving scenarios. As part of distributed practice, students will revisit decimal operations when they solve equations in a later module. Fractions and decimals are encountered more frequently than whole numbers in daily life, so students should be comfortable and confident solving problems that require operating with these numbers.

How do the activities in *Decimals and Volume* promote student expertise in the mathematical process standards?

All Carnegie Learning topics are written with the goal of creating mathematical thinkers who are active participants in class discourse, so elements of the mathematical process standards should be evident in all lessons. Students are expected to make sense of problems and work toward solutions, reason using concrete and abstract ideas, and communicate their thinking while providing a critical ear to the thinking of others.

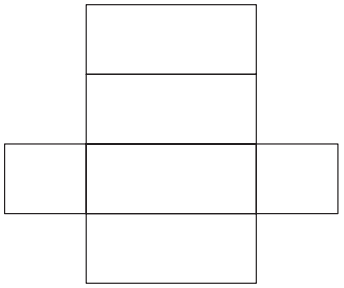
Throughout this topic, students develop their attention to precision as they compute with whole numbers and decimals. They draw from strategies for computing with decimals as they make estimates and judge the reasonableness of their answers. Students calculate volume and surface area of three-dimensional solids with positive rational number dimensions. They also attend to precision in language as they describe three-dimensional solids. Students also understand structure as they compose and decompose three-dimensional solids and their two-dimensional nets.

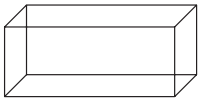
Materials Needed

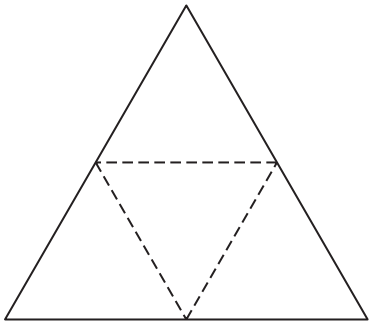
- Scissors
- Tape or glue sticks

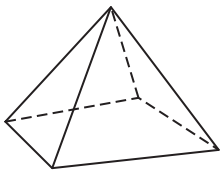
Concrete and Visual Representations Used

Nets of Right Rectangular Prisms and Pyramids









Learning Together

ELPS: 1.A, 1.B, 1.C, 1.D, 1.E, 1.F, 1.G, 1.H, 2.C, 2.D, 2.E, 2.G, 2.H, 2.I, 3.A, 3.B, 3.C, 3.D, 3.E, 3.F, 3.G, 3.J, 4.A, 4.B, 4.C, 4.D, 4.F, 4.G, 4.I, 4.K, 5.A, 5.B, 5.C, 5.D, 5.E, 5.F, 5.G

Lesson	Lesson Name	TEKS	Days	Highlights
1	Depth, Width and Length: Deepening Understanding of Volume	6.8C 6.8D	2	In this lesson, students are introduced to geometric solids. Students will investigate various figures and sort them based on the definition of a polygon or a polyhedron. The intent of this lesson is for students to determine the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths. In addition, they will review and practice decimal multiplication by calculating volumes of right rectangular prisms.
2	Which Warehouse?: Volume Composition and Decomposition	6.3E 6.8D	2	A scenario about building a bench is provided. Students review estimating sums and differences of decimals and how to add and subtract decimals by adding or subtracting the digits in like place values. They then determine the volume of the bench, a composite solid, using decomposition into smaller rectangular prisms and composition into a larger rectangular prism. The two different strategies require either addition or subtraction of decimals. Students practice solving problems requiring addition and subtraction of decimal volumes.
3	Breaking the Fourth Wall: Surface Area of Rectangular Prisms and Pyramids	7.9D	2	Students apply mathematical and spatial reasoning to determine the surface areas of prisms and pyramids using nets, drawings, and measurements. Students solve a variety of surface area problems and distinguish between volume and surface area measurements.
4	Dividend in the House: Dividing Whole Numbers and Decimals	6.3E 6.8D	2	In this lesson, students use the standard algorithm for long division with whole numbers. They demonstrate how the algorithm works for decimal dividends by relating it to a model and make sense of why the algorithm is modified to accommodate decimal divisors. Students solve area and volume problems requiring decimal division.

Suggested Topic Plan

*1 Day Pacing = 45 min. Session

Day 1	Day 2	Day 3	Day 4	Day 5
TEKS: 6.8C, 6.8D **LESSON 1** **Depth, Width, and Length** GETTING STARTED ACTIVITY 1	**LESSON 1** continued ACTIVITY 2 ACTIVITY 3 TALK THE TALK	TEKS: 6.3E, 6.8D **LESSON 2** **Which Warehouse?** GETTING STARTED ACTIVITY 1	**LESSON 2** continued ACTIVITY 2 TALK THE TALK	TEKS: 7.9D **LESSON 3** **Breaking the Fourth Wall** GETTING STARTED ACTIVITY 1
Day 6	**Day 7**	**Day 8**	**Day 9**	
LESSON 3 continued ACTIVITY 2 ACTIVITY 3 ACTIVITY 4 TALK THE TALK	TEKS: 7.9D **LESSON 4** **Dividend in the House** GETTING STARTED ACTIVITY 1 ACTIVITY 2 ACTIVITY 3 ACTIVITY 4	**LESSON 4** continued ACTIVITY 5 ACTIVITY 6 TALK THE TALK	**END OF TOPIC ASSESSMENT**	

Assessments

There is one assessment aligned to this topic: End of Topic Assessment.

Decimals and Volume Summary

KEY TERMS

- point
- line segment
- polygon
- geometric solid
- polyhedron
- face
- edge
- vertex
- right rectangular prism
- cube
- pyramid
- volume
- composite solid
- trailing zeros
- net
- surface area
- slant height

LESSON 1

Depth, Width, and Length

The mathematical definition of **point** is a location in space, often represented using a dot and named by a capital letter. A **line segment** is a portion of a line that includes two points and the points between those two points.

A **polygon** is a closed figure formed by three or more line segments. A **geometric solid** is a bounded three-dimensional geometric figure. A **polyhedron** is a three-dimensional solid figure that is made up of polygons that are called **faces**. An **edge** is the intersection of two faces and a **vertex** is the point where the edges meet.

For example, Figure A is a **right rectangular prism**, which is a polyhedron with three pairs of congruent and parallel faces.

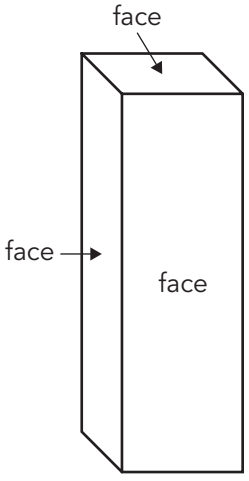


Figure A

Figure B is a **cube**, which is a polyhedron that has six congruent squares as faces.

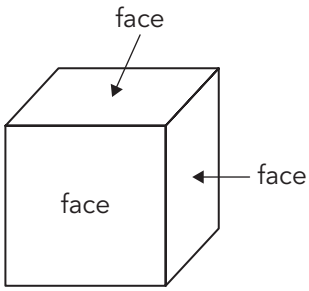


Figure B

Figure C is a rectangular pyramid. A **pyramid** is a polyhedron with one base and the same number of triangular faces as there are sides of the base.

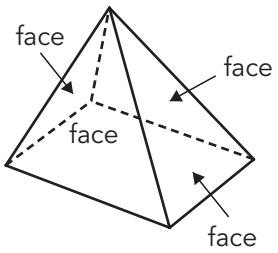
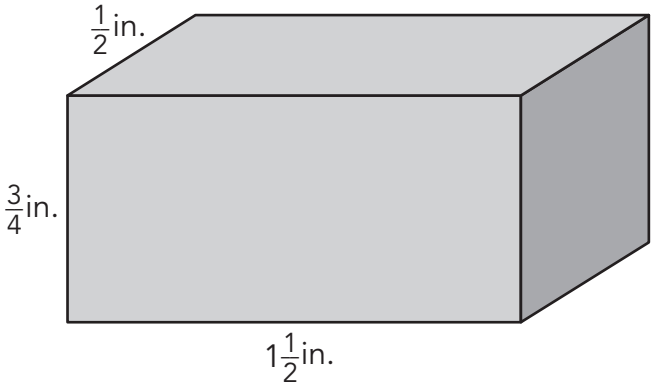


Figure C

Volume is the amount of space occupied by an object. The volume of an object is measured in cubic units. A unit cube is a cube whose sides are all 1 unit long.

The volume of a rectangular prism is a product of its length, width and height: $V = l \cdot w \cdot h$.

For example, to determine the volume of the right rectangular prism shown with the given dimensions, you can fill the prism with cubes, but smaller unit cubes with fractional side lengths are required.



Assign a unit fraction to the dimensions of each cube. Use the least common multiple (LCM) of the fraction denominators to determine the unit fraction.	LCM(2, 4) = 4 So, each cube will measure $\frac{1}{4}$ in. $\times$ $\frac{1}{4}$ in. $\times$ $\frac{1}{4}$ in. The volume of each unit cube is $\frac{1}{64}$ cubic inches.
Determine the number of cubes needed to pack the prism in each dimension.	length: $1\frac{1}{2} \div \frac{1}{4} = 6$ width: $\frac{1}{2} \div \frac{1}{4} = 2$ height: $\frac{3}{4} \div \frac{1}{4} = 3$
Determine the number of cubes that make up the right rectangular prism.	$6 \times 2 \times 3 = 36$
Multiply the number of cubes by the volume of each cube to determine the volume of the right rectangular prism.	$36 \times \frac{1}{64} = \frac{36}{64}$ $= \frac{9}{16}$

The volume of the right rectangular prism is $\frac{9}{16}$ cubic inches.

You can use the formula $V = Bh$ to calculate the volume of any prism. However, the formula for calculating the value of B will change depending on the shape of the base. In a rectangular prism, $B = l \cdot w$.

LESSON 2

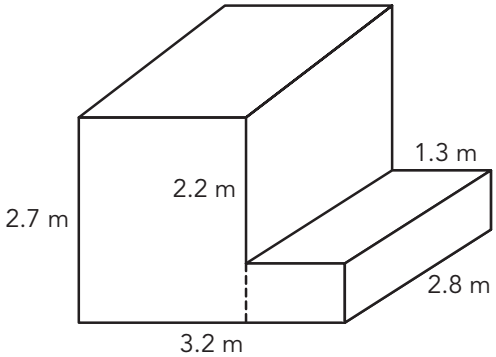
Which Warehouse?

A **composite solid** is made up of more than one geometric solid. You can decompose a composite solid into more than one polyhedron in order to determine its volume.

For example, to determine the volume of the composite solid shown, you can decompose the solid into two rectangular prisms and calculate the volume of each.

Volume of larger prism $= 1.9 \times 2.8 \times 2.7 = 14.364$ m^3

Volume of smaller prism $= 1.3 \times 2.8 \times 0.5 = 1.82$ m^3



To calculate the sum or difference of decimals, line up the decimals so that like place values are in the same column. Use the decimal point to help you correctly align.

$$\begin{array}{r} 14.364 \\ +1.820 \\ \hline 16.184 \end{array}$$

A *trailing zero* was added to 1.82. **Trailing zeros** are a sequence of 0s in a decimal representation of a number, after which no non-zero digits follow. Trailing zeros do not affect the value of a number.

The volume of the composite solid is 16.184 cubic meters.

LESSON 3

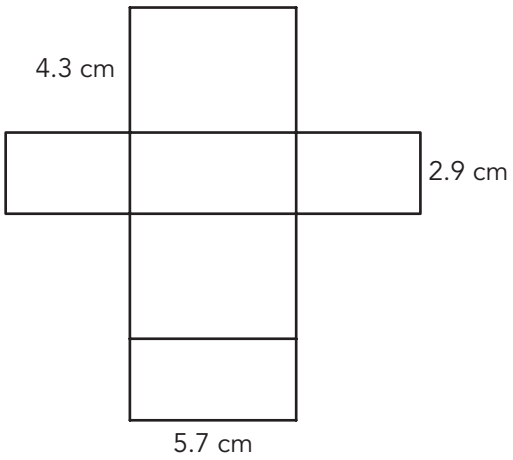
Breaking the Fourth Wall

A **net** is a two-dimensional representation of a three-dimensional geometric figure. A net has all of these properties:

- The net is cut out as a single piece.
- All of the faces of the geometric solid are represented in the net.
- The faces of the geometric solid are drawn so that they share common edges.

The **surface area** of a polyhedron is the total area of all its two-dimensional faces.

For example, you can use the net to calculate the surface area of the right rectangular prism.



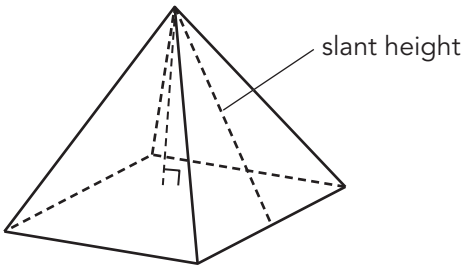
Determine the area of each unique face.

$4.3 \text{ cm} \times 5.7 \text{ cm} = 24.51 \text{ cm}^2$
$2.9 \text{ cm} \times 5.7 \text{ cm} = 16.53 \text{ cm}^2$
$4.3 \text{ cm} \times 2.9 \text{ cm} = 12.47 \text{ cm}^2$

Determine the sum of all faces of the right rectangular prism.

$2(24.51) + 2(16.53) + 2(12.47)$
$= 49.02 + 33.06 + 24.94$
$= 107.02$

The surface area of the right rectangular prism is 107.02 cm^2.



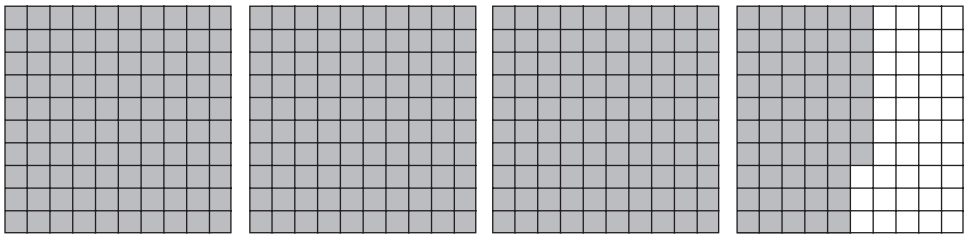
Remember, the vertex of a pyramid is the point at which all the triangular faces of the pyramid intersect. A **slant height** of a pyramid is the distance measured along a triangular face from the vertex of the pyramid to the midpoint, or center, of the base.

LESSON 4

Dividend in the House

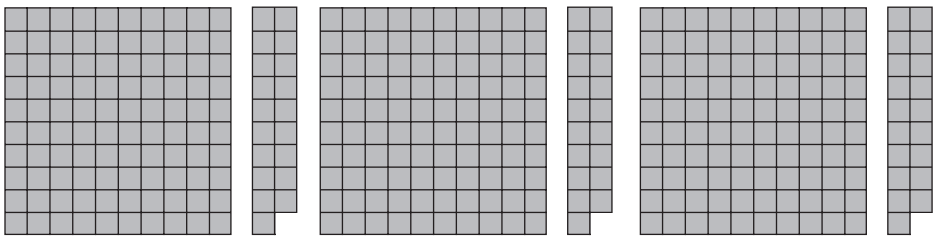
You can use a hundredths grid to model dividing decimals, such as 3.57 ÷ 3.

First, shade hundredths grids to represent 3.57.



Next, divide the shaded model into 3 equal groups.

One whole grid and 19 small squares are in each group. So, 3.57 ÷ 3 = 1.19.



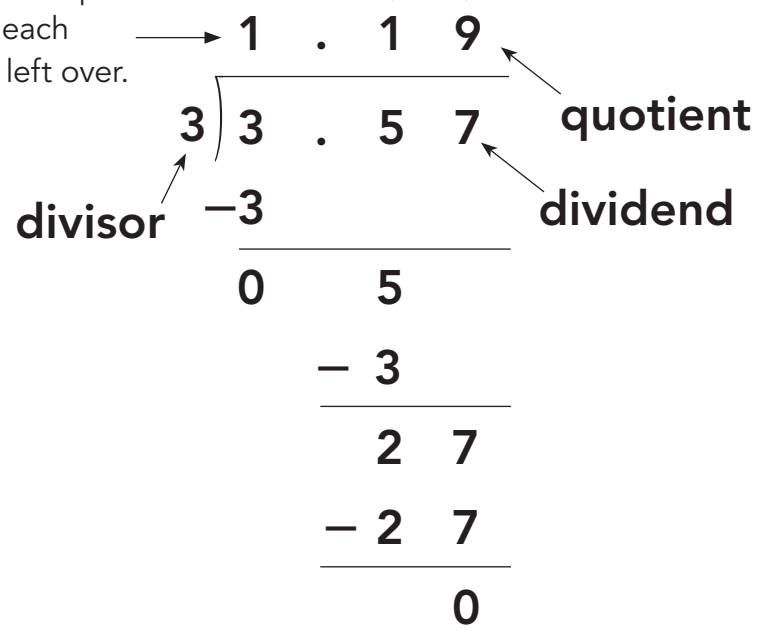
Group 1 Group 2 Group 3

You can also use a standard algorithm to divide 3.57 ÷ 3.

5 tenths divided into 3 equal groups is 1 tenth in each group with 2 tenths left over.

2 tenths and 7 hundredths is 27 hundredths. 27 hundredths divided into 3 equal groups is 9 hundredths in each group with 0 hundredths left over.

3 ones divided into 3 equal groups is 1 one in each group with 0 ones left over.



If you multiply or divide both the dividend and divisor by the same number, the quotient remains the same.

$$7.7 \div 3.5 = 77 \div 35$$

$$\frac{7.7}{3.5} = \frac{77}{35}$$

You can use what you know about dividing with decimals to solve problems about volume and surface area. For example, suppose the surface area of a cube is 48.24 square inches. Calculate the area of each face of the cube.

$$6\overline{)\,48.24}\qquad 8.04$$

Since a cube has six congruent square faces, each face has an area of 8.04 square inches.