TEXAS MATH SOLUTION

Accelerated Grade ɔ

Module 2 Topic 3 Lesson 2

Stretches, Stacks, and Structure

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Stretches, Stacks, and Structure

2

Structure of Linear Equations

MATERIALS

None

Lesson Overview

Students write and solve equations for more complicated contexts. They use tables to create equations that require the use of the expression ($n - 1$) to represent the quantity of the independent variable except for the initial value. They compare the two forms of the same equation and write possible scenarios that could be modeled by the new equations. Students use different representations—the two different forms of the equation and the graph—to answer questions about each context. They also relate the equations to the graphs. Students analyze the different forms of linear equations to reveal aspects on the problems and how the quantities are related. They generalize interpretations for each form of the equations in terms of the equations and in terms of the graphs.

Grade 7

Expressions, Equations, and Relationships

(7) The student applies mathematical process standards to represent linear relationships using multiple representations. The student is expected to:

(A) represent linear relationships using verbal descriptions, tables, graphs, and equations that simplify to the form $y = mx + b$.

(10) The student applies mathematical process standards to use one-variable equations and inequalities to represent situations. The student is expected to:

(A) write one-variable, two-step equations and inequalities to represent constraints or conditions within problems.

(11) The student applies mathematical process standards to solve one-variable equations and inequalities. The student is expected to:

(A) model and solve one-variable, two-step equations and inequalities.

ELPS

1.A, 1.D, 1.F, 2.C, 2.D, 2.G, 2.H, 3.A, 3.B, 3.C, 3.D, 4.A, 4.B, 4.C, 4.K, 5.E

Essential Ideas

- More complex equations may require use of the Distributive Property and/or combining like terms in order to simplify an equation to a two-step equation.
- Writing a linear equation in a different form can reveal information about the problem situation.

Lesson Structure and Pacing: 3 Days

Day 1

Engage

Getting Started: Learning the Limo Business

Students examine the cost structures for two different limousine companies in order to create a competitive cost structure for a third company. Students will use this table in Activity 2.1.

Develop

Activity 2.1: Different Forms, Same Equation

Students write and solve equations for more complicated contexts. They use a table to create equations that require the use of the expression $(x - 1)$ to represent the quantity of the independent variable except for the initial value. The resulting equation, $y = a + b(x - 1)$ requires the use of the Distributive Property and combining like terms prior to solving the resulting two-step equation. Students compare the two forms of the same equation and write possible scenarios that could be modeled by the new equations.

Day 2

Activity 2.2: Comparing Graphs of Linear Equations

Students compare the heights of stacks of round and square containers. They identify the variable quantities of the situation and which quantity depends on the other quantity. Students use a table of values to graph the heights of both stacks on the same coordinate plane. Then students write equations that require the use of the expression $(c - 1)$ for each stack. They use the various representations to answer questions about each stack. Students rewrite the equations in the form $y = ax + b$, interpret the values in the new equations, and relate the equations to the graphs.

Day 3

Activity 2.3: Interpreting Forms of Equations

Students analyze the different forms of linear equations they wrote in Activities 2.1 and 2.2 to reveal aspects on the problems and how the quantities are related. They generalize interpretations for each form of the equations in terms of the equations and in terms of the graphs. Students then write the equations from Activity 2.2 in an additional form and interpret the form in terms of the problem situation.

Demonstrate

Talk the Talk: Back to the Limos!

Students use the equations from Activity 2.1 to determine which of two linear graphs represents each equation.

Getting Started: Learning the Limo Business

ENGAGE

Facilitation Notes

In this activity, students examine the cost structures for two different limousine companies in order to create a competitive cost structure for a third company. Students will use this table in Activity 2.1.

Ask a student to read the introductory paragraph aloud. Discuss both the paragraph and table and complete Question 1 as a class.

Questions to ask

- What patterns do you notice in the table?
- When renting a limousine, why do you think companies charge more money for the first hour than the other hours?
- If you planned on renting a limousine for 4 hours, what company would you choose?
- If you planned on renting a limousine for 2.5 hours, what company would you choose?
- If you planned on renting a limousine for 1 hour, what company would you choose?
- If you planned on renting a limousine for an evening, what company would you choose?
- If wanted to know the cost for 10 hours, why is doubling the cost for 5 hours an incorrect strategy?
- Why might the response, "It depends" be the best response for Question 1?
- Does this table represent a proportional relationship? How can you tell?

Differentiation strategies

To extend the activity, have students:

- Write inequalities for the time frame when each plan is the less expensive plan.
- Explain what constant in their price structure each company should emphasize when advertising.

DEVELOP

Activity 2.1
Different Forms, Same Equation

Facilitation Notes

In this activity, students write and solve equations for more complicated contexts. They use a table to create equations that require the use of the expression $(x - 1)$ to represent the quantity of the independent variable except for the initial value. The resulting equation, $y = a + b(x - 1)$ requires the use of the Distributive property and combining like terms prior to solving the resulting two-step equation. Students compare the two forms of the same equation and write possible scenarios that could be modeled by the new equations.

In high school, students will use piecewise equations to model these same types of situations; however, that is beyond the scope of grade 7.

Differentiation strategy

Split the class, with half of the class answering questions regarding Limousines by Lilly and the other half answering questions regarding Transportation with Class. Share responses as a class so that everyone experiences a summary regarding both companies. Note that Questions 1 through 4, which refer to Limousines by Lilly, have more scaffolding than Question 5, which refers to Transportation with Class. For this reason, direct the students only working on Question 5 to answer Questions 1 through 3 in place of parts (a) through (e), but with the Transportation with Class given information, and then answer Question 5 part (f).

Have students work with a partner or in groups to complete Questions 1 through 4. Share responses as a class.

As students work, look for

- Errors demonstrating the arithmetic to answer Question 1 part (c) and part (d). Work may look like this: $23.75 \times 9 = 213.75 + 99.99 = 313.74$, showing a series of operations that do not demonstrate equalities. Have students rewrite the series of operations accurately as: $23.75 \times 9 = 213.75$; $213.75 + 99.99 = 313.74$.
- A generalization of the process, rather than using specific values, in Question 1 part (e). Encourage students to use as few numbers as possible to move towards responses such as, "I took the number of hours and subtracted 1 before I multiplied by the rate (23.75), then I added that product to the cost for the first hour (99.99)." This will help students see the structure of the problem and write an equation to model it.

Differentiation strategies

To scaffold support with writing or understanding the equation,

- Ask students why the following equations do or do not model the situation:
  $t = 23.75h + 99.99$
  $t = 23.75h + 99.99 - 23.75$
  $t = 23.75(h - 1) + 99.99$
  Then, discuss how the second and third equations are equivalent.
- Encourage students to use phrases first, then math symbols.
- Have students refer to their process in Question 1 parts (c), (d), and (e).

Questions to ask

- When calculating the cost of a ten hour rental, why did you multiply 9 times the cost per hour rather than 10 times the cost per hour?
- Why is the expression $(h - 1)$ used in your equation?
- Is there another possible way to write the equation?
- What is the meaning of each component of your equation?
- What is the meaning of the value 76.24 in the problem situation?

Have students work with a partner or in groups to complete Question 5. Share responses as a class.

Questions to ask

- Why is the expression $(h - 1)$ used in your equation?
- Is there another possible way to write the equation?
- What is the meaning of each component of your equation?
- What is the meaning of the value 53.44 in the problem situation?

Have students work with a partner or in groups to complete Question 6. Share responses as a class.

Questions to ask

- When providing advice to Katie, did you think it was more important to charge less than the competitors for the first hour, each additional hour, both or neither? Explain.
- What other pricing framework could Katie use rather than charging a set price for the first hour and another rate for the additional hours? Do you think setting a pricing framework different from her competitors would help or hinder Katie's business?

Differentiation strategy

To extend the activity, encourage students to develop their own limousine rental fee schedule. They will revisit this activity and see Katie's fee schedule at the end of this lesson.

Summary

For situations that have a constant rate except for the initial value, the expression $(n - 1)$ is used to represent the quantity of the independent variable except for the initial value.

Activity 2.2

Comparing Graphs of Linear Equations

Facilitation Notes

In this activity, students compare the heights of stacks of round and square containers. They identify the variable quantities of the situation and which quantity depends on the other quantity. Students use a table of values to graph the heights of both stacks on the same coordinate plane. Then students write equations that require the use of the expression $c - 1$ for each stack. They use the various representations to answer questions about each stack. Students rewrite the equations in the form $y = ax + b$, interpret the values in the new equations, and relate the equations to the graphs.

Ask a student to read the scenario aloud. Discuss the scenario and table as a class.

Differentiation strategy

Have manipulatives that can be stacked, such as storage containers or styrofoam cups, for a demonstration or individual group use. It is important that the manipulatives have a consistent length that can be easily seen when the objects are stacked.

Questions to ask

- What is another example that demonstrates this stacking or nesting concept?
- What patterns do you notice in the table?
- What is the height of one round container?
- Why isn't 18 centimeters the height of two round containers?
- How is this situation similar in structure to the limousine rental problem?
- Does this table represent a proportional relationship? How can you tell?

Have students work with a partner or in groups to complete Questions 1 through 3. Share responses as a class.

Differentiation strategy

To scaffold support with graphing, provide students scaled graphs.

Questions to ask

- How were the upper and lower bounds determined?
- How do you know which quantity to put on which axis when graphing the situation?
- What is a good rule of thumb to use when determining the intervals for the graph?
- How can you tell if the data points are discrete or continuous?
- If a question is asked about a data point that is beyond your graph, how will you answer the question?
- Can there be a point between two of the graphed data points?
- Is (0, 0) a data point?
- Did you place (0, 0) on the graph?
- Does the point (0, 0) line up with the other points on the graph? Why or why not?
- How do the graphs of the round and square containers compare to each other?
- Which graph is steeper? Why do you think that is the case?
- Which graph starts out higher than the other? Why do you think that is the case?

Have students work with a partner or in groups to complete Questions 4 through 8. Share responses as a class.

Questions to ask

- How did you determine the difference in height when you added another container to the stack?
- Why will the difference be the same each time an additional container is stacked?
- How did you determine your equation?
- What is another equation that models this situation?
- Why does it make sense to use $(c - 1)$ rather than c in this problem situation?
- How can you tell when a problem situation will require the expression $(c - 1)$ rather than c?
- What property did you use to write your equations in the form $y = ax + b$?
- How can you tell from your equation which stack started with a greater height?
- How can you tell from your equation which stack gets taller faster (if you are adding the same number of containers to both stacks at the same time)?
- How can you tell from your graph which stack started with a greater height?

- How can you tell from your graph which stack gets taller faster (if you are adding the same number of containers to both stacks at the same time)?
- Do you think both stacks will ever be the same height with the same amount of containers? How can you tell?

Have students work with a partner or in groups to complete Questions 9 and 10. Share responses as a class.

Questions to ask

- How did you determine the height of two dozen containers?
- What height should Storage Pro make its shipping boxes?
- How did you determine how many containers can fit in the boxes that are 45 centimeters tall?

Summary

For situations that have a constant rate except for the initial value, the expression ($n - 1$) is used to represent the quantity of the independent variable except for the initial value. This relationship can be interpreted through a graph, as well as through the situation, table and equation.

Activity 2.3

Interpreting Forms of Equations

Facilitation Notes

In this activity, students analyze the different forms of linear equations they wrote in Activities 2.1 and 2.2 to reveal aspects on the problems and how the quantities are related. They generalize interpretations for each form of the equations in terms of the equations and in terms of the graphs. Students then write the equations from Activity 2.2 in an additional form and interpret the form in terms of the problem situation.

Have students work with a partner or in groups to complete Questions 1 through 5. Share responses as a class.

Differentiation strategies

- Use a discussion format rather than having students write their responses to the questions. Students may need guidance generalizing their answers and forming explanations.
- Encourage the use of examples from the previous activities to confirm students' thinking.

Misconception

Students may misunderstand the questions and explain the meaning of *d* instead of the *d* term and *a* instead of the *a* term. Have students refer back to the definition of a term to understand their error.

Questions to ask

- What is the result of *d* times $(x - 1)$?
- How does the value of *b* compare to the value of *c*?
- Which form makes more sense to you? Why?
- In what situations is using the structure $y = ax + b$ especially useful?
- In what situations is using an alternate structure more useful than the $y = ax + b$ structure?

Summary

Situations can be demonstrated by equivalent equations with different structures.

Talk the Talk: Back to the Limos!

DEMONSTRATE

Facilitation Notes

In this activity, students use the equations from Activity 2.1 to determine which of two linear graphs represents each equation.

Have students work with a partner or in groups to complete Questions 1 and 2. Share responses as a class.

Questions to ask

- What are the constants in each equation?
- How can the constants in the equations help you figure out which line represents which situation?
- How does the structure of Katie's fee schedule compare to the two other limousine services?
- How do Katie's fees compare to the two other limousine services?
- For what number of hours would Katie's fee be the best option?
- Why do you think Katie set up her fee schedule this way?

Summary

Recognizing the structure of a problem and the meaning of constants helps to interpret different representations of the problem.

NOTES

Stretches, Stacks, and Structure

2

Structure of Linear Equations

Warm Up Answers

1. $3x - 3$
2. $18 - 9x$
3. $\frac{1}{2}x - 3$
4. $3x + 18$

WARM UP

Use properties to rewrite.

1. $3(x - 1)$
2. $-9(-2 + x)$
3. $\frac{1}{2}(x - 6)$
4. $6 + 3(x + 4)$

LEARNING GOALS

- Write and solve two-step equations.
- Compare two linear problem situations.
- Rewrite expressions in different forms in problem contexts in order to interpret how quantities are related.
- Compare graphs of linear problem situations.
- Compare and interpret forms of linear equations.

All of the linear equations you have written for problem situations have been in the form $y = ax + b$. Are there other common forms of equations used to express linear problem situations?

Answers

1. Answers will vary. Students may look only at the first row and state that they would rent from Transportation with Class because the first hour is less than the first hour with Limousines by Lilly. Most will say that they would want the limousine for more than one hour. After the first hour, Limousines by Lilly is always less expensive than Transportation with Class, so they would choose Limousines by Lilly.

Getting Started

Learning the Limo Business

Katie is starting her own limousine rental company. She wisely decides to check her competitors' pricing plans before setting her own plan. The table shows the fees from two rival limousine rental companies.

Examine the fee schedule for the two limousine companies provided in the table.

Number of Hours Rented	Limousines by Lilly Fees (in dollars)	Transportation with Class Fees (in dollars)
1	99.99	89.99
2	123.74	126.54
3	147.49	163.09
4	171.24	199.64
5	194.99	236.19

1. Which company would you choose if you were renting a limousine? Support your answer with information from the table.

ACTIVITY 2.1

Different Forms, Same Equation

Katie starts by analyzing the cost structure of Limousines by Lilly.

1. Consider the cost of renting a limousine from Limousines by Lilly.

a. What does the first hour of a rental from Limousines by Lilly cost?

b. What does each additional rental hour cost from Limousines by Lilly after the first hour?

c. What would it cost to rent a limo from Limousines by Lilly for 10 hours? Explain your reasoning.

d. What would it cost to rent a limo from Limousines by Lilly for 13 hours? Explain your reasoning.

e. Explain how you calculated each cost.

Answers

1a. The first hour costs $99.99.

1b. Each additional hour costs $23.75.

1c. It would cost $313.74. It costs $99.99 for the first hour and $23.75 for each of the next 9 hours.

1d. It would cost $384.99. It costs $99.99 for the first hour and $23.75 for each of the next 12 hours.

1e. I subtracted one from the number of hours, multiplied that answer by $23.75, and then added $99.99 to that product.

Answers

2. $t = 23.75(h - 1) + 99.99$

3a. I will need the Distributive Property so that I can rewrite the equation without parentheses. Then, I can combine like terms. I can use the Commutative Property to reorder the terms being added.

3b. $t = 23.75h + 76.24$ This equation is a two-step equation because two operations must be reversed to isolate the variable.

3c. Same: the coefficient of the term that includes the variable is 23.75

Different: constants, 99.99 or 76.24 and variable expressions, h or $(h - 1)$

3d. Answers will vary. Limousines by Lilly charges a booking fee of $76.24 and $23.75 per hour.

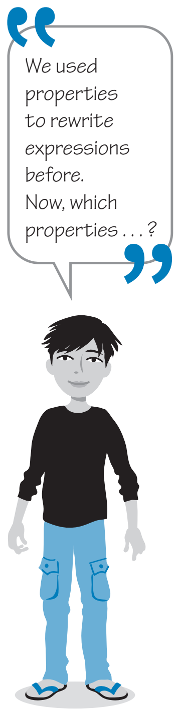
4. $h = 8$. The limousine is rented for 8 hours for a charge of $266.24.

2. Write an equation for the total cost, *t*, of renting from Limousines by Lilly for any given number of rental hours, *h*.

You can rewrite your equation for Limousines by Lilly before using it to solve problems. Previously, you have learned to simplify algebraic expressions using a variety of strategies.

3. Rewrite your equation in the form $ax + b = c$.

a. Name the strategies necessary to rewrite the equation you wrote.



b. Rewrite the equation you wrote for Limousines by Lilly. Explain why the resulting equation is a two-step equation.

c. Compare the two equations you wrote for this company. What is the same? What is different?

d. Write a possible fee scenario for Limousines by Lilly to match the rewritten equation.

4. Use your equation to calculate how many hours you rented from Limousines by Lilly if the total cost is $266.24.

4 • TOPIC 3: Multiple Representations of Equations

5. Consider the cost of renting a limousine from Transportation with Class.

   a. What does the first hour of a rental from Transportation with Class cost?

   b. What does each additional rental hour cost from Transportation with Class after the first hour?

   c. Write an equation for the total cost, t, of renting from Transportation with Class for any given number of rental hours, h.

   d. Rewrite your equation in the form $ax + b = c$.

   e. Write a possible fee scenario for Transportation with Class to match the rewritten equation.

   f. Use your equation to determine the number of hours that cost $309.29 from Transportation with Class.

6. What suggestions would you provide to Katie on the fees she should charge for her limo rental business? Explain your reasoning.

Answers

5a. The first hour costs $89.99.

5b. Each additional hour costs $36.55.

5c. $t = 89.99 + 36.55(h - 1)$

5d. $36.55h + 53.44 = t$

5e. Answers will vary. Transportation with Class charges a booking fee of $53.44 and $36.55 per hour.

5f. $h = 7$. The limousine is rented for 7 hours.

6. Answers will vary.

Answers

1. The variable quantities in this problem situation are the number of containers and the stack height for both round and square containers.
2. The stack height depends on the number of containers.

ACTIVITY 2.2

Comparing Graphs of Linear Equations

Your job at Storage Pros is to create new boxes to ship the company's plastic containers. Storage Pros makes all different shapes and sizes of plastic containers. To ship the containers, the lids are removed, allowing the containers to be stacked. Storage Pros wants to design its shipping boxes so that they will hold two dozen stacks of the plastic containers without lids in stacks of two dozen, regardless of the size or shape of the container.

The table shows the data gathered from measuring the heights of different-sized stacks of the various plastic containers.

Number of Containers	Stack Height (centimeters)	
	Round	Square
1	9	15
2	9.8	15.4
3	10.6	15.8
4	11.4	16.2
5	12.2	16.6
6		
7		
13		

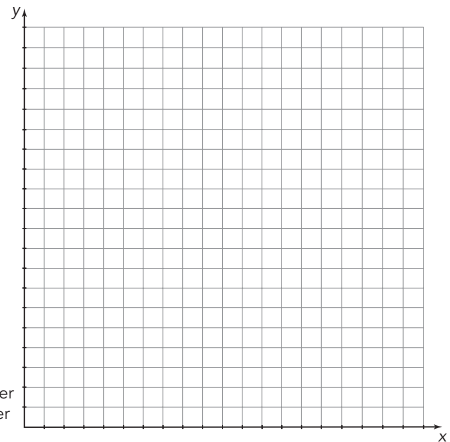
1. What are the variable quantities in this problem situation?

2. What quantity depends on the other?

6 • TOPIC 3: Multiple Representations of Equations

3. Create a graph for each container shape's stack height in terms of the number of containers used. Determine the bounds and intervals, complete the table, and label your graph clearly. Use the symbols in the legend shown when graphing.

Variable Quantity	Lower Bound	Upper Bound	Interval
Number of Containers			
Stack Height			



Legend:
▪ Square container
• Round container

4. Consider the stack of round containers.

a. What is the height of the first round container?

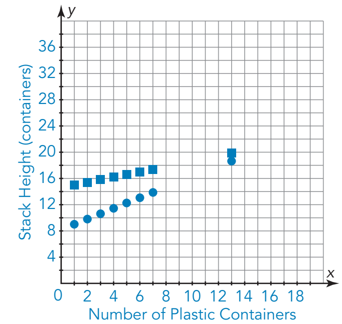
b. How does the height change when one round container is added to a stack of round containers?

c. Let h represent the stack height. Write an equation that represents the stack height of the round containers in terms of the number of round containers, c, in the stack.

d. Use your table, graph, or equation to determine the stack height of 6, 7, and 13 round containers. Add these values to your table and graph.

Answers

3. Sample Answers.



Legend:
▪ Square container
• Round container

Lower Bound	Upper Bound	Interval
0	20	1
0	40	2

4a. The height of 1 round container is 9 centimeters.

4b. Adding one round container to the stack adds 0.8 centimeter to the stack height.

4c. $h = 9 + 0.8(c - 1)$

4d.

Number of Containers	Stack Height (centimeters)	
	Round	Square
6	13	
7	13.8	
13	18.6	

See graph in Question 3

Answers

5a. $h = 15 + 0.4(c - 1)$

5b.

Number of Containers	Stack Height (centimeters)	
	Round	Square
6		17
7		17.4
13		19.8

See graph in Question 3

6a. Both equations have the same structure of a constant added to a product of two factors. One of the factors in both equations is $c - 1$. The two equations are similar because they both model containers of a fixed height that are placed in a stack.

6b. The constants in the equations are different because the different types of containers result in different heights. Also, each different shape of container changes the stack height by a different amount when added to the stack.

7a. Round containers: $h = 0.8c + 8.2$

Square containers: $h = 0.4c + 14.6$

7b. In each equation, the coefficient of the variable represents the "extra" height of each container that is visible when the containers are stacked. This includes the first, or bottom, container. The constant term represents the height of the first container without the height of the part of each container that shows.

5. Consider the stack of square containers.

   a. Let *c* represent the number of containers in a stack of square containers, and let *h* represent the stack height. Write an equation that gives the stack height in terms of the number of containers in the stack.

   b. Use your table, graph, or equation to determine the stack height of 6, 7, and 13 square containers. Add these values to your table and graph.

6. Analyze the equations you wrote for round and square containers.

   a. How are the two equations you wrote similar? Why are these equations similar? Explain your reasoning.

   b. How are the two equations you wrote different? Why are these equations different? Explain your reasoning.

7. The equations you wrote for the heights of the containers can be rewritten in equivalent forms.

   a. Rewrite each equation in the form $y = ax + b$.

   b. Explain what the numbers in the equations mean in terms of the problem context.

   c. Refer back to the graph. Explain how the numbers in these equations and your graphs are related.

8 • TOPIC 3: Multiple Representations of Equations

7c. The constants in each graph are the *y*-intercepts, the points at which the lines that model the situations intersect the *y*-axis. The coefficients of the variables are height (cm) increase for each 1 container.

NOTES

8. Use your equations of the form $y = ax + b$ to calculate the stack height of:

   a. two dozen round containers.

   b. two dozen square containers.

9. What height should Storage Pros make its boxes to accommodate the height of a stack of two dozen of either type of container?

10. Storage Pros had extra boxes that were 45 centimeters tall.

   a. How many round containers can be in each stack inside the box?

   b. How many square containers can be in each stack inside the box?

Answers

8a. $h = 27.4$ cm

8b. $h = 24.2$ cm

9. Approximately 28 cm, minimum of 27.4 cm

10a. $c = 46$

10b. $c = 76$

Answers

1. See below.
2. The c term represents the amount of the dependent variable for a quantity of one for the independent variable.

   The d term represents how much *more* is added for all quantities of the independent variable except the first one.
3. The a term represents the amount for the dependent variable if the first quantity of the independent variable had the same rate as all of the others.

   The b term represents the additional amount that needs to be added to the dependent variable to make up for the difference between the rate applied to the initial quantity and the others.

ACTIVITY 2.3

Interpreting Forms of Equations

In the limousine and container scenarios, you represented the situations with two different equations.

1. Complete the table to summarize the different forms of the equations. Use the variables x and y for the independent and dependent variables.

	$y = ax + b$	$y = c + d(x - 1)$
Limousines by Lilly		$y = 99.99 + 23.75(x - 1)$
Transportation with Class		
Round Containers	$y = 0.8x + 8.2$	
Square Containers		

2. Use your equations to explain the meaning of the c and d terms in $y = c + d(x - 1)$.

3. Use your equations to explain the meaning of the a and b terms in $y = ax + b$.

Any letter can be used as a variable. It is common to use a and b in forms of equations, but the different variables were used to reduce the possibility of confusing the equations.

1.

	$y = ax + b$	$y = c + d(x - 1)$
Limousines by Lilly	$y = 23.75x + 76.24$	$y = 99.99 + 23.75(x - 1)$
Transportation with Class	$y = 36.55x + 53.44$	$y = 89.99 + 36.55(x - 1)$
Round Containers	$y = 0.8x + 8.2$	$y = 9 + 0.8(x - 1)$
Square Containers	$y = 0.4x + 14.6$	$y = 15 + 0.4(x - 1)$

4. Refer back to the graphs of the plastic containers and the related equations. Explain if and how the two equations of the form $y = ax + b$ can be visualized on the graph.

5. Which form of the linear equations do you prefer? Explain your reasoning.

Answers

4. The coefficients of the variable term, d in $y = c + d(x - 1)$ and a in $y = ax + b$, are the same value and are equal to the increase in the vertical distance for each one unit increase in the horizontal distance on the graph. The constant term from $y = ax + b$ is the *y-intercept* of graph.

5. Answers will vary. Students may prefer $y = ax + b$ when they are graphing or solving equations, but they may prefer one of the other two forms when representing a problem situation that has a different rate for the first hour or object and the same rate for each subsequent hour or object.

Answers

1. Answers will vary. Sample answer: I chose to use the equations in the form of $y = ax + b$. Limousines by Lilly has a constant term, or y-intercept, of 76.24. Transportation with Class has a constant term, or y-intercept of 53.44. So the graph of Limousines by Lilly would cross the axis at a higher point than Transportation with Class. So graph m represents Transportation with Class and graph n represents Limousines by Lilly.

2. $c = 124.99 + 49.99(h - 3)$

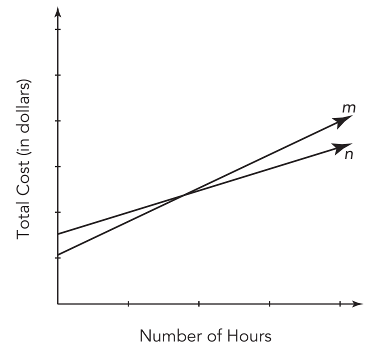
NOTES

TALK the TALK

Back to the Limos!

At the beginning of the lesson, you wrote equations for the fee schedule of Limousines by Lilly and Transportation with Class.

1. Determine which graph represents each equation. Use your equations to explain your reasoning.



2. Suppose Katie decides to charge \$124.99 for the first *three* hours and then \$49.99 for each additional hour. Write an equation to represent Katie's fee schedule.

Assignment

LESSON 2: Stretches, Stacks, and Structure

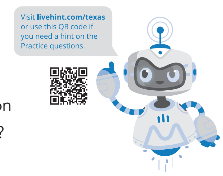
Write

Write a problem situation that could be modeled by a linear equation in x and y that includes the expression $x - c$, where c is a positive integer.

Remember

Different forms of an equation reveal different information about a problem situation and about other representations of the problem situation.

Practice



Write an equation to represent each situation. Define your variables and solve the equation.

1. At the Namaste Yoga Studio, the first two yoga classes are free with a registration fee of \$15. Each class after that is \$45. How many classes can you take for \$1185?
2. Clara has a coupon for \$10 off at her favorite clothing store. The coupon is applied before any discounts are taken. The store is having a sale, and offering 15% off everything. If Clara has \$50 to spend, how much can her purchases total before applying the discount and her coupon? Round to the nearest cent.
3. A dog kennel charges \$40 to board a dog for one night and \$35 per night each night after that. Henry paid a total of \$215 for dog boarding. For how many nights did Henry board his dog?
4. Drake's Drugstore is getting ready for the upcoming summer season. The manager of the store wants to add lawn chairs to the stock. He asks the buyer to determine the two lowest priced wholesalers of lawn chairs. The table shows the data that the buyer collects from two wholesalers.

Packs of Chairs	Price from Wholesaler A (dollars)	Price from Wholesaler B (dollars)
1	\$90.99	\$98.99
2	\$173.98	\$179.98
3	\$256.97	\$260.97
4	\$339.96	\$341.96

a. Let p represent the total number of packs of chairs bought from Wholesaler A and let c represent the total cost. Write an equation to calculate the total cost of any number of packs of chairs.

b. Let p represent the total number of packs of chairs bought from Wholesaler B and let c represent the total cost. Write an equation to calculate the total cost of any number of packs of chairs.

c. Write the equations from parts (a) and (b) in the form $y = ax + b$.

d. Calculate the cost of eight packs of chairs from each wholesaler.

e. The manager wants to buy at least seven packs of chairs. Which wholesaler should the drugstore use this year? Explain your reasoning.

Assignment Answers

Write

Answers will vary.

Practice

1. $1185 = 15 + 45(x - 2)$. Let x represent the number of yoga classes you can take. You can take 28 yoga classes for \$1185.
2. $50 = 0.85(x - 10)$. Let x represent Clara's total before the discounts. Clara's purchases can total \$68.82 before her discounts.
3. $215 = 40 + 35(x - 1)$. Let x represent the number of nights a dog is boarded. Henry boarded his dog for 6 nights.

4a. $c = 90.99 + 82.99(p - 1)$

4b. $c = 98.99 + 80.99(p - 1)$

4c. Wholesaler A: $c = 8 + 82.99p$; Wholesaler B: $18 + 80.99p$

4d. Wholesaler A: \$671.92; Wholesaler B: \$665.92

4e. The drugstore should use Wholesaler B. Wholesaler A costs less than Wholesaler B only for fewer than five packs of chairs, at which the cost is the same. For more than five packs of chairs, Wholesaler B costs less.

Assignment Answers

5a. Six boards from Build It would earn 22.5 reward points, seven boards would earn 25. Six boards from All Things Home would earn 20 reward points, and seven boards would earn 23.

5b. Variable quantities: the number of boards purchased and the number of reward points earned. The number of points earned depends on the number of boards purchased at one time.

5c. See below.

5d. The difference in the number of reward points is 2.5. Buying one additional board during each visit adds 2.5 reward points.

5e. The difference in the number of reward points is 3. Buying one additional board during each visit adds 3 reward points.

5f. Build It: $p = 10 + 2.5(b - 1)$; All Things Home: $p = 5 + 3(b - 1)$

5g. Build It: $p = 7.5 + 2.5b$; All Things Home: $p = 2 + 3b$

5h. Geoffrey will earn 37.5 reward points at Built It and 38 reward points at All Things Home.

5i. At Build It, Geoffrey would have bought 23 boards. At All Things Home, Geoffrey would have bought 21 boards.

5. Geoffrey owns the Super Backyard Shed Company. He makes custom built sheds for residential homeowners, and he buys the majority of his building materials from two large home stores in the area. Both stores, Build It and All Things Home, offer reward cards for the purchase of lumber. The more boards that Geoffrey buys at one time, the more points he will earn. The points can then be used for future purchases. The table shows the number of reward points that he will earn.

Number of Boards Purchased	Store	
	Build It	All Things Home
1	10	5
2	12.5	8
3	15	11
4	17.5	14
5	20	17
6		
7		

a. Complete the table to show the number of reward points earned for the purchase of 6 and 7 boards. Use the table and scenario to answer each question.

b. What are the variable quantities in this problem situation? State which quantity depends on the other.

c. Create graphs for each store's reward points in terms of the number of boards purchased. Identify the bounds and intervals. Be sure to label your graph clearly.

d. How does the number of reward points change when the number of boards bought at Build It is increased by 1? Explain your reasoning.

e. How does the number of reward points change when the number of boards bought at All Things Home is increased by 1? Explain your reasoning.

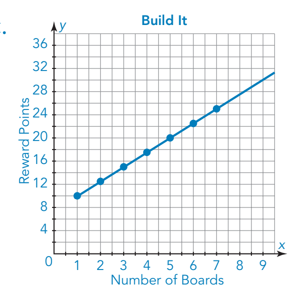
f. Let *p* represent the number of reward points and *b* represent the number of boards purchased at one time. Write equations to represent the number of reward points that Geoffrey will earn in terms of the number of boards purchased from each store.

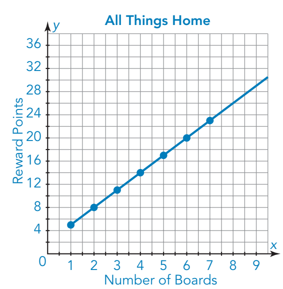
g. Rewrite each equation in the form $y = ax + b$.

h. Determine the number of points that would be earned if Geoffrey buys 12 boards at a time from each store.

i. If Geoffrey earned 65 reward points, how many boards could he have bought at each store?

5c.





Variable Quantity	Lower Bound	Upper Bound	Interval
Number of Boards	0	10	0.5
Reward Points	0	40	2

Stretch

Greg needs to hire someone to clear his driveway of snow this winter season. A neighbor has a plow attached to his truck and charges $30 for each time he plows the driveway. Mel's Landscaping runs a snow-clearing business and charges $50 for the first time they plow and $25 for each additional time they plow. Write and solve an equation to determine when the costs of each option are the same. Under what conditions would Greg choose his neighbor? Mel's Landscaping?

Review

1. The winner of the 95th annual hotdog eating contest consumed 207 hotdogs (and buns!) in 10 minutes. You are determined to break this record!
   a. What would you have to do to break this record?
   b. How many hotdogs would you have to eat every minute?
2. The 96th annual contest begins at noon. Your best friend got caught in traffic and arrives halfway through the event.
   a. How many hotdogs have you consumed?
   b. Assuming you eat at the average rate needed, after the arrival of your best friend, how many total hotdogs will you consume in one minute? two minutes? three minutes?
   c. Identify and define the independent and dependent variables with their units of measure for this situation.
   d. Create a table of values for the in minutes after 12:05 PM and the number of hotdogs consumed.
   e. Write an equation for calculating the value of the dependent variable when the value of the independent variable is given.
   f. Use your equation to determine how long after 12:05 PM it will take you to consume 187 hotdogs.
   g. Use your equation to determine when you would have consumed a total of 83 hotdogs.
   h. What does the answer to part (g.) mean in this problem situation?

3. Solve each equation and check your solution.
   a. $42 = \frac{3}{5}x + 12$
   b. $\frac{-7}{3}x - 11 = -25$

2d.

Time (Minutes)	Number of Hotdogs and Buns consumed
t	h
0	104
1	124.8
2	145.6
3	166.4

Assignment Answers

Stretch

$30x = 50 + 25(x - 1)$

. The options cost the same when the number of snow removals is 5. If Greg needs his driveway cleared no more than 5 times, he would choose his neighbor. If Greg needs his driveway cleared at least 5 times, he would choose Mel's Landscaping.

Review

1a. You would have to eat 208 hotdogs and buns in 10 minutes.

1b. You would have to eat 20.8 hotdogs and buns every minute.

2a. I would have consumed 104 hotdogs and buns.

2b. I would have consumed 124.8 hotdogs and buns after one minute, 145.6 hotdogs and buns after two minutes, and 166.4 hotdogs and buns after three minutes.

2c. Independent variable: t = time in minutes since 12:05 P.M.

Dependent variable: h = hotdogs and buns consumed

2d. See below.

2e. $h = 104 + 20.8t$

2f. $t = 4$. It will take 4 minutes after 12:05 P.M to have consumed a total of 187.2 hotdogs and buns.

2g. $t = -1$

2h. Negative 1 minute means 1 minute before 12:05, or 12:04 P.M.

3a. $x = 50$

3b. $x = 6$