TEXAS MATH SOLUTION

Accelerated Grade

Module 2 Topic 3 Lesson 2

Stretches, Stacks, and Structure

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Structure of Linear Equations

WARM UP

Use properties to rewrite.

1. $3(x - 1)$

2. $-9(-2 + x)$

3. $\frac{1}{2}(x - 6)$

4. $6 + 3(x + 4)$

LEARNING GOALS

- Write and solve two-step equations.
- Compare two linear problem situations.
- Rewrite expressions in different forms in problem contexts in order to interpret how quantities are related.
- Compare graphs of linear problem situations.
- Compare and interpret forms of linear equations.

All of the linear equations you have written for problem situations have been in the form $y = ax + b$. Are there other common forms of equations used to express linear problem situations?

Learning the Limo Business

Katie is starting her own limousine rental company. She wisely decides to check her competitors' pricing plans before setting her own plan. The table shows the fees from two rival limousine rental companies.

Examine the fee schedule for the two limousine companies provided in the table.

Number of Hours Rented	Limousines by Lilly Fees (in dollars)	Transportation with Class Fees (in dollars)
1	99.99	89.99
2	123.74	126.54
3	147.49	163.09
4	171.24	199.64
5	194.99	236.19

1. **Which company would you choose if you were renting a limousine? Support your answer with information from the table.**

Katie starts by analyzing the cost structure of Limousines by Lilly.

1. **Consider the cost of renting a limousine from Limousines by Lilly.**

   a. **What does the first hour of a rental from Limousines by Lilly cost?**

   b. **What does each additional rental hour cost from Limousines by Lilly after the first hour?**

   c. **What would it cost to rent a limo from Limousines by Lilly for 10 hours? Explain your reasoning.**

   Do you have an estimate for the cost to rent from Lilly for 10 hours?

   d. **What would it cost to rent a limo from Limousines by Lilly for 13 hours? Explain your reasoning.**

   e. **Explain how you calculated each cost.**

2. **Write an equation for the total cost, _t_, of renting from Limousines by Lilly for any given number of rental hours, _h_.**

You can rewrite your equation for Limousines by Lilly before using it to solve problems. Previously, you have learned to simplify algebraic expressions using a variety of strategies.

3. **Rewrite your equation in the form _ax_ + _b_ = _c_.**

   a. **Name the strategies necessary to rewrite the equation you wrote.**

We used properties to rewrite expressions before. Now, which properties . . . ?

   b. **Rewrite the equation you wrote for Limousines by Lilly. Explain why the resulting equation is a two-step equation.**

   c. **Compare the two equations you wrote for this company. What is the same? What is different?**

   d. **Write a possible fee scenario for Limousines by Lilly to match the rewritten equation.**

4. **Use your equation to calculate how many hours you rented from Limousines by Lilly if the total cost is $266.24.**
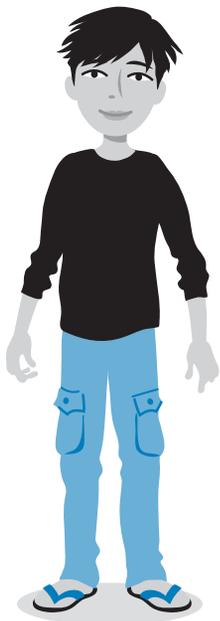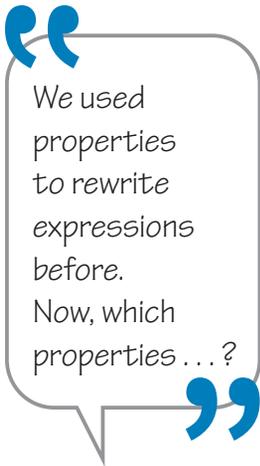
5. Consider the cost of renting a limousine from Transportation with Class.

   a. What does the first hour of a rental from Transportation with Class cost?

   b. What does each additional rental hour cost from Transportation with Class after the first hour?

   c. Write an equation for the total cost, t, of renting from Transportation with Class for any given number of rental hours, h.

   d. Rewrite your equation in the form $ax + b = c$.

   e. Write a possible fee scenario for Transportation with Class to match the rewritten equation.

   f. Use your equation to determine the number of hours that cost $309.29 from Transportation with Class.

6. What suggestions would you provide to Katie on the fees she should charge for her limo rental business?
   Explain your reasoning.

Comparing Graphs of Linear Equations

Your job at Storage Pros is to create new boxes to ship the company's plastic containers. Storage Pros makes all different shapes and sizes of plastic containers. To ship the containers, the lids are removed, allowing the containers to be stacked. Storage Pros wants to design its shipping boxes so that they will hold two dozen stacks of the plastic containers without lids in stacks of two dozen, regardless of the size or shape of the container.

The table shows the data gathered from measuring the heights of different-sized stacks of the various plastic containers.

Number of Containers	Stack Height (centimeters)	
	Round	Square
1	9	15
2	9.8	15.4
3	10.6	15.8
4	11.4	16.2
5	12.2	16.6
6		
7		
13		

1. **What are the variable quantities in this problem situation?**

2. **What quantity depends on the other?**

3. Create a graph for each container shape's stack height in terms of the number of containers used. Determine the bounds and intervals, complete the table, and label your graph clearly. Use the symbols in the legend shown when graphing.

Variable Quantity	Lower Bound	Upper Bound	Interval
Number of Containers			
Stack Height			
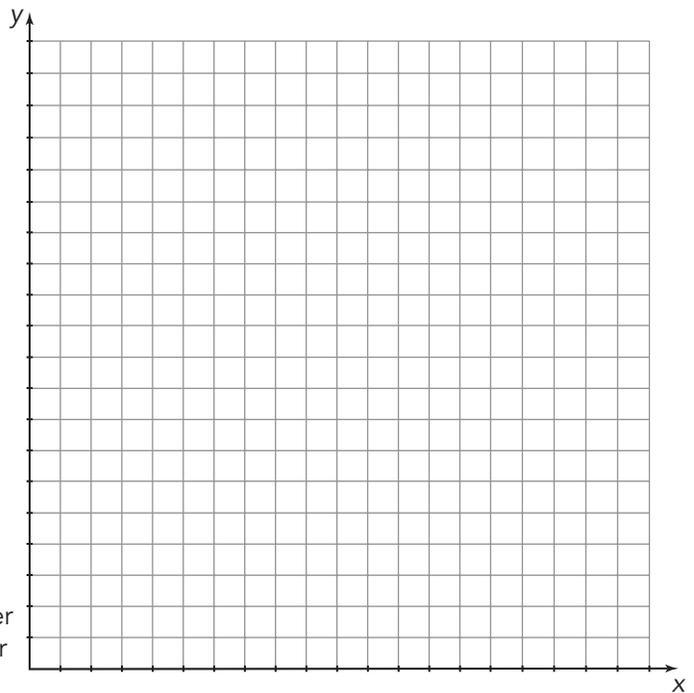
Legend:
■ Square container
● Round container

4. Consider the stack of round containers.

   a. What is the height of the first round container?

   b. How does the height change when one round container is added to a stack of round containers?

   > Draw a line to model the relationship. Do all points on the line make sense?

   c. Let h represent the stack height. Write an equation that represents the stack height of the round containers in terms of the number of round containers, c, in the stack.

   d. Use your table, graph, or equation to determine the stack height of 6, 7, and 13 round containers. Add these values to your table and graph.

5. Consider the stack of square containers.

   a. Let *c* represent the number of containers in a stack of square containers, and let *h* represent the stack height. Write an equation that gives the stack height in terms of the number of containers in the stack.

   b. Use your table, graph, or equation to determine the stack height of 6, 7, and 13 square containers. Add these values to your table and graph.

6. Analyze the equations you wrote for round and square containers.

   a. How are the two equations you wrote similar? Why are these equations similar? Explain your reasoning.

   b. How are the two equations you wrote different? Why are these equations different? Explain your reasoning.

7. The equations you wrote for the heights of the containers can be rewritten in equivalent forms.

   a. Rewrite each equation in the form $y = ax + b$.

   b. Explain what the numbers in the equations mean in terms of the problem context.

   c. Refer back to the graph. Explain how the numbers in these equations and your graphs are related.

8. Use your equations of the form $y = ax + b$ to calculate the stack height of:

   a. two dozen round containers.

   b. two dozen square containers.

9. What height should Storage Pros make its boxes to accommodate the height of a stack of two dozen of either type of container?

10. Storage Pros had extra boxes that were 45 centimeters tall.

   a. How many round containers can be in each stack inside the box?

   b. How many square containers can be in each stack inside the box?

Interpreting Forms of Equations

In the limousine and container scenarios, you represented the situations with two different equations.

1. **Complete the table to summarize the different forms of the equations. Use the variables *x* and *y* for the independent and dependent variables.**

	$y = ax + b$	$y = c + d(x - 1)$
Limousines by Lilly		$y = 99.99 + 23.75(x - 1)$
Transportation with Class		
Round Containers	$y = 0.8x + 8.2$	
Square Containers		

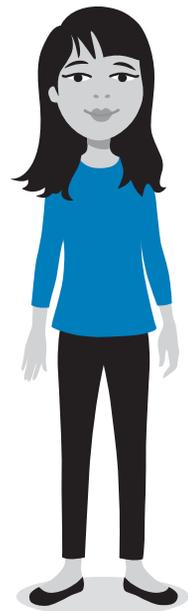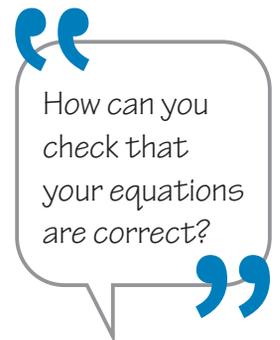2. **Use your equations to explain the meaning of the *c* and *d* terms in $y = c + d(x - 1)$.**

3. **Use your equations to explain the meaning of the *a* and *b* terms in $y = ax + b$.**

Any letter can be used as a variable. It is common to use *a* and *b* in forms of equations, but the different variables were used to reduce the possibility of confusing the equations.

4. Refer back to the graphs of the plastic containers and the related equations. Explain if and how the two equations of the form $y = ax + b$ can be visualized on the graph.

5. Which form of the linear equations do you prefer? Explain your reasoning.

How can you check that your equations are correct?

TALK the TALK 💬

Back to the Limos!

At the beginning of the lesson, you wrote equations for the fee schedule of Limousines by Lilly and Transportation with Class.

1. Determine which graph represents each equation. Use your equations to explain your reasoning.
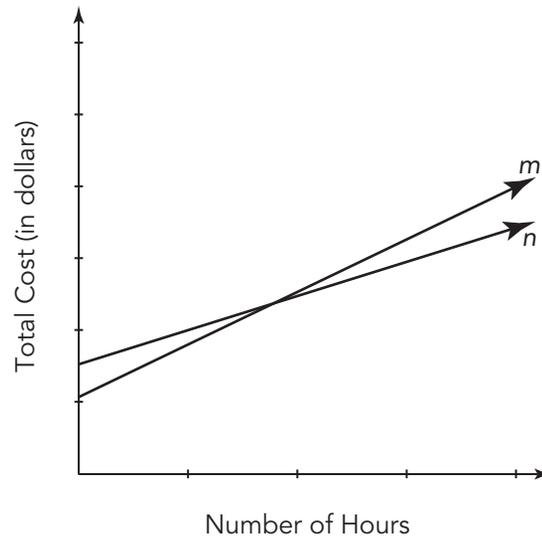
2. Suppose Katie decides to charge $124.99 for the first *three* hours and then $49.99 for each additional hour. Write an equation to represent Katie's fee schedule.

Write

Write a problem situation that could be modeled by a linear equation in x and y that includes the expression $x - c$, where c is a positive integer.

Remember

Different forms of an equation reveal different information about a problem situation and about other representations of the problem situation.

Practice

Write an equation to represent each situation. Define your variables and solve the equation.

1. At the Namaste Yoga Studio, the first two yoga classes are free with a registration fee of $15. Each class after that is $45. How many classes can you take for $1185?

2. Clara has a coupon for $10 off at her favorite clothing store. The coupon is applied before any discounts are taken. The store is having a sale, and offering 15% off everything. If Clara has $50 to spend, how much can her purchases total before applying the discount and her coupon? Round to the nearest cent.

3. A dog kennel charges $40 to board a dog for one night and $35 per night each night after that. Henry paid a total of $215 for dog boarding. For how many nights did Henry board his dog?

4. Drake's Drugstore is getting ready for the upcoming summer season. The manager of the store wants to add lawn chairs to the stock. He asks the buyer to determine the two lowest priced wholesalers of lawn chairs. The table shows the data that the buyer collects from two wholesalers.
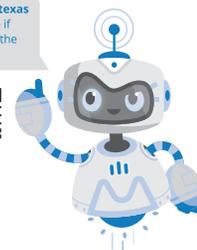
Packs of Chairs	Price from Wholesaler A (dollars)	Price from Wholesaler B (dollars)
1	$90.99	$98.99
2	$173.98	$179.98
3	$256.97	$260.97
4	$339.96	$341.96

a. Let p represent the total number of packs of chairs bought from Wholesaler A and let c represent the total cost. Write an equation to calculate the total cost of any number of packs of chairs.

b. Let p represent the total number of packs of chairs bought from Wholesaler B and let c represent the total cost. Write an equation to calculate the total cost of any number of packs of chairs.

c. Write the equations from parts (a) and (b) in the form $y = ax + b$.

d. Calculate the cost of eight packs of chairs from each wholesaler.

e. The manager wants to buy at least seven packs of chairs. Which wholesaler should the drugstore use this year? Explain your reasoning.

5. Geoffrey owns the Super Backyard Shed Company. He makes custom built sheds for residential homeowners, and he buys the majority of his building materials from two large home stores in the area. Both stores, Build It and All Things Home, offer reward cards for the purchase of lumber. The more boards that Geoffrey buys at one time, the more points he will earn. The points can then be used for future purchases. The table shows the number of reward points that he will earn.

Number of Boards Purchased	Store	
	Build It	All Things Home
1	10	5
2	12.5	8
3	15	11
4	17.5	14
5	20	17
6		
7		

a. Complete the table to show the number of reward points earned for the purchase of 6 and 7 boards. Use the table and scenario to answer each question.

b. What are the variable quantities in this problem situation? State which quantity depends on the other.

c. Create graphs for each store's reward points in terms of the number of boards purchased. Identify the bounds and intervals. Be sure to label your graph clearly.

d. How does the number of reward points change when the number of boards bought at Build It is increased by 1? Explain your reasoning.

e. How does the number of reward points change when the number of boards bought at All Things Home is increased by 1? Explain your reasoning.

f. Let p represent the number of reward points and b represent the number of boards purchased at one time. Write equations to represent the number of reward points that Geoffrey will earn in terms of the number of boards purchased from each store.

g. Rewrite each equation in the form $y = ax + b$.

h. Determine the number of points that would be earned if Geoffrey buys 12 boards at a time from each store.

i. If Geoffrey earned 65 reward points, how many boards could he have bought at each store?

Stretch

Greg needs to hire someone to clear his driveway of snow this winter season. A neighbor has a plow attached to his truck and charges $30 for each time he plows the driveway. Mel's Landscaping runs a snow-clearing business and charges $50 for the first time they plow and $25 for each additional time they plow. Write and solve an equation to determine when the costs of each option are the same. Under what conditions would Greg choose his neighbor? Mel's Landscaping?

Review

1. The winner of the 95th annual hotdog eating contest consumed 207 hotdogs (and buns!) in 10 minutes. You are determined to break this record!
   a. What would you have to do to break this record?
   b. How many hotdogs would you have to eat every minute?
2. The 96th annual contest begins at noon. Your best friend got caught in traffic and arrives halfway through the event.
   a. How many hotdogs have you consumed?
   b. Assuming you eat at the average rate needed, after the arrival of your best friend, how many total hotdogs will you consume in one minute? two minutes? three minutes?
   c. Identify and define the independent and dependent variables with their units of measure for this situation.
   d. Create a table of values for the in minutes after 12:05 PM and the number of hotdogs consumed.
   e. Write an equation for calculating the value of the dependent variable when the value of the independent variable is given.
   f. Use your equation to determine how long after 12:05 PM it will take you to consume 187 hotdogs.
   g. Use your equation to determine when you would have consumed a total of 83 hotdogs.
   h. What does the answer to part (g.) mean in this problem situation?

3. Solve each equation and check your solution.
   a. $42 = \frac{3}{5} x + 12$

   b. $\frac{-7}{3} x - 11 = -25$