TEXAS MATH SOLUTION

Accelerated Grade ɔ

Module 2 Topic 2 Lesson 4

Be Greater Than

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Be Greater Than

4

Solving Inequalities with Inverse Operations

WARM UP

Graph each inequality on a number line.

1. $x > 5$
2. $x \geq 2\frac{1}{2}$
3. $x < 6.2$
4. $x \leq 9$

LEARNING GOALS

- Solve and graph one- and two-step inequalities.
- Solve word problems leading to inequalities of the form $px + q > r$ and $px + q < r$.
- Graph the solution sets of inequalities and interpret the solutions in context.

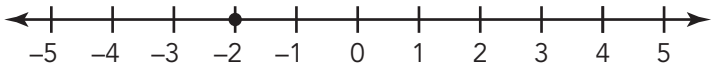
KEY TERMS

- inequality
- solve an inequality
- solution set
- Properties of Inequalities

You have solved a variety of equations. How is solving inequalities similar to or different from solving equations?

Equations Versus Inequalities

Consider the equation $4x + 9 = 1$. The solution is shown on the number line.



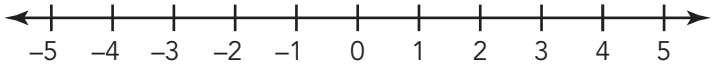
An **inequality** is any mathematical sentence that has an inequality symbol. The solution set of an inequality is all values the make the inequality statement true.

1. Verify the solution is correct. Are there any other solutions to this equation? Explain your reasoning.

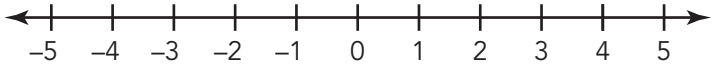
Consider the set of numbers {−5, −4, −3, −2, −1, 0, 1}.

2. Use substitution to determine which values are solutions to each inequality. Plot the solutions for each inequality on the given number line.

a. $4x + 9 > 1$



b. $4x + 9 \leq 1$



3. Use your number lines to make predictions about other solutions to each inequality. Create a number line to illustrate each of your conjectures.

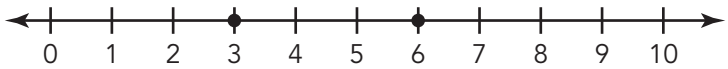
ACTIVITY
4.1 Adding and Subtracting with Inequalities

In this lesson, you will learn to *solve an inequality*. To **solve an inequality** means to determine the values of the variable that make the inequality true. The objective when solving an inequality is similar to the objective when solving an equation. You want to isolate the variable on one side of the inequality symbol by using the operations of addition, subtraction, multiplication, and division.

The values of the variable that make an inequality true are together called the **solution set** of the inequality.

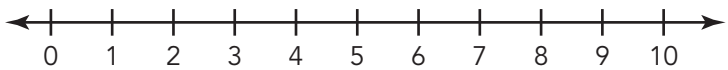
Let's investigate what happens when each side of an inequality is added or subtracted by the same number.

Consider the relationship between the two numbers 3 and 6. Since 3 is to the left of 6, you know that $3 < 6$.



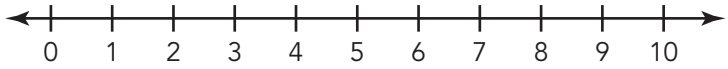
1. Perform each operation to the numbers 3 and 6. Then, plot the new values on the number line. Finally, write a corresponding inequality statement.

a. Add $\frac{1}{2}$ to each number.



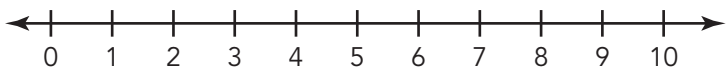
$3 + \frac{1}{2}$ ___ $6 + \frac{1}{2}$

b. Add 2 to each number.



3 + 2 ___ 6 + 2

c. Add 3 to each number.



3 + 3 ___ 6 + 3

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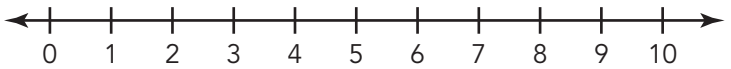
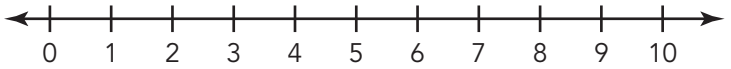
d. Subtract $\frac{1}{2}$ from each number.



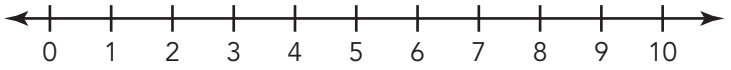
$3 - \frac{1}{2}$ ___ $6 - \frac{1}{2}$

e. Subtract 2 from each number.



$3 - 2$ ___ $6 - 2$

f. Subtract 3 from each number.



$3 - 3$ ___ $6 - 3$

2. When you add the same number to each side of the inequality or subtract the same number from each side of the inequality, what do you notice about the resulting inequality symbol?

3. Explain why Simone is correct.

Simone

No matter what number I add to or subtract from both sides of the inequality, the relationship between the two sides of the inequality stays the same:

$$3 < 6$$

$$3 + a < 6 + a$$

$$3 - a < 6 - a$$

4. Consider the inequality $x - 2 > 6 - 2$.

   a. Write an inequality to describe the possible values of x.

   b. What could you do to both sides of the original inequality to determine your answer to part (a)?

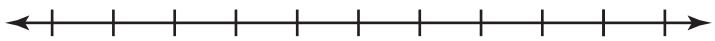
5. Suppose you have the inequality $x - 2 > 6$. Determine the possible values of x and sketch the solution set on a number line. Explain your reasoning.

6. Mike is 5 years older than his brother Jim.
   For each question, write and solve an equation or inequality to describe Jim's possible ages. Then, graph the solution set on the number line.

   a. How old will Jim be when Mike is 29 years old?

   b. How old will Jim be when Mike is at least 25 years old?

   c. How old will Jim be when Mike is younger than 30 years old?



Recall that the solution to any inequality is shown on a number line by a ray whose starting point is an open or closed circle.
A closed circle means that the starting point is part of the solution set of the inequality.
An open circle means that the starting point is not a part of the solution set of the inequality.

7. Solve each inequality and graph the solution set on the number line. Then choose one value from your solution set and one value outside your solution set to check your work.

a. $13 < x + 11$

b. $10 + x \geq 45$

c. $x < 5 + 9$

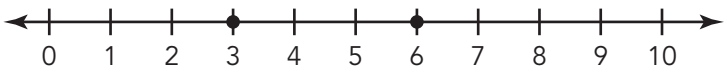
d. $x - 3 \leq 21$

8. Choose one of the inequalities from Question 7 and write a real-world situation that can be modeled by the algebraic statement.

ACTIVITY
4.2 Multiplying and Dividing by Positive Numbers

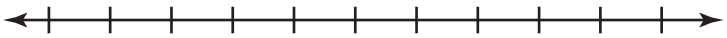
Next, let's investigate what happens when each side of an inequality is multiplied or divided by the same positive number.

Consider the inequality $3 < 6$.



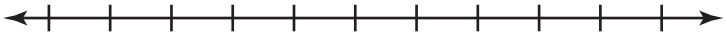
1. Perform each operation to the numbers 3 and 6. Then, plot the new values on the number line. Finally, write a corresponding inequality statement.

a. Multiply each number by $\frac{1}{2}$.



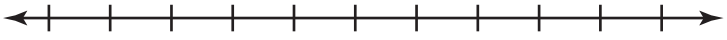
$3\left(\frac{1}{2}\right)$ ___ $6\left(\frac{1}{2}\right)$

b. Multiply each number by 2.



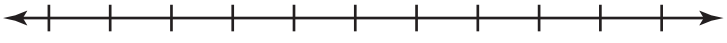
3(2) ___ 6(2)

c. Multiply each number by 3.

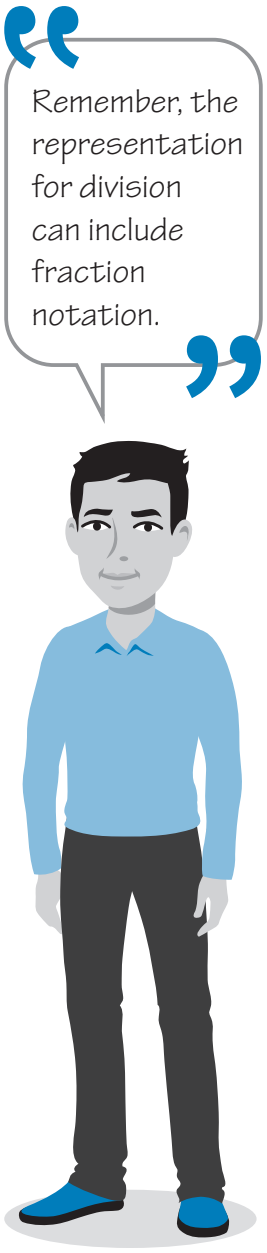


3(3) ___ 6(3)

d. Divide each number by $\frac{1}{2}$.



$3 \div \frac{1}{2}$ ___ $6 \div \frac{1}{2}$ or $\frac{3}{\frac{1}{2}}$ ___ $\frac{6}{\frac{1}{2}}$



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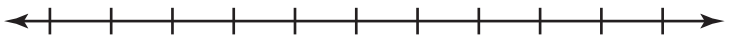
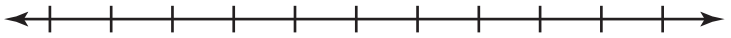
e. Divide each number by 2.



$3 \div 2 \_\_\_ 6 \div 2$ or $\frac{3}{2} \_\_\_ \frac{6}{2}$

f. Divide each number by 3.



$3 \div 3 \_\_\_ 6 \div 3$ or $\frac{3}{3} \_\_\_ \frac{6}{3}$

2. When you multiply the same positive number to each side of the inequality or divide the same positive number from each side of the inequality, what do you notice about the resulting inequality symbol?

3. Identify the constraints of the value *a* that makes Robin's claim correct.

Robin

No matter what positive number I multiply to or divide from both sides of the inequality, the relationship between the two sides of the inequality stays the same:

$$3 < 6$$

$$3(a) < 6(a)$$

$$\frac{3}{a} < \frac{6}{a}$$

4. Consider the inequality $2x < 6(2)$.

a. Write an inequality to describe the possible values of x.

b. What could you do to both sides of the original inequality to determine your answer to part (a)?

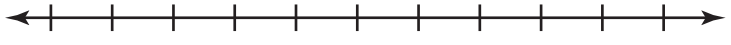
5. Suppose you have the inequality $2x < 6$. Determine the possible values of x. Explain your reasoning.

6. Michelle is 3 times as old as her sister Beth.
For each question, write and solve an equation or inequality to describe Beth's possible ages. Then, graph the solution set on the number line.

   a. How old will Beth be when Michelle is at least 27 years old?

   b. How old will Beth be when Michelle is younger than 30 years old?

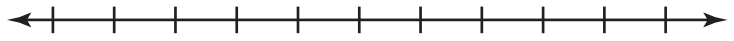
   c. How old will Beth be when Michelle is 42 years old?



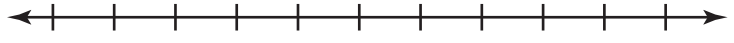
7. **Solve each inequality and graph the solution set on the number line.**

   a. $10x \geq 45$

   b. $\frac{x}{3} \leq 21$



   c. $3 < x \div 8$

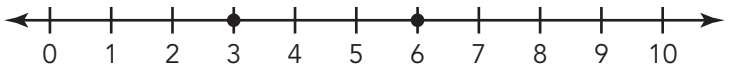


8. **Choose one of the inequalities from Question 7 and write a real-world situation that can be modeled by the algebraic statement.**

ACTIVITY
4.3 Multiplying and Dividing by Negative Numbers

Finally, let's investigate what happens when each side of an inequality is multiplied or divided by the same negative number.

Consider the inequality $3 < 6$.



1. Perform each operation to the numbers 3 and 6. Then, plot the new values on the number line. Finally, write a corresponding inequality statement.

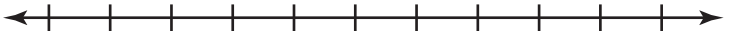
a. Multiply each number by $-\frac{1}{2}$.

$3\left(-\frac{1}{2}\right)$ ___ $6\left(-\frac{1}{2}\right)$

b. Multiply each number by −2.

3(−2) ___ 6(−2)

c. Multiply each number by −3.



3(−3) ___ 6(−3)

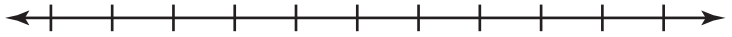
d. Divide each number by $-\frac{1}{2}$.

$3 \div \left(-\frac{1}{2}\right) \_\_ 6 \div \left(-\frac{1}{2}\right)$ or $\frac{3}{\left(-\frac{1}{2}\right)} \_\_ \frac{6}{\left(-\frac{1}{2}\right)}$

e. Divide each number by −2.

$3 \div (-2) \_\_ 6 \div (-2)$ or $\frac{3}{(-2)} \_\_ \frac{6}{(-2)}$

f. Divide each number by −3.



$3 \div (-3) \_\_ 6 \div (-3)$ or $\frac{3}{(-3)} \_\_ \frac{6}{(-3)}$

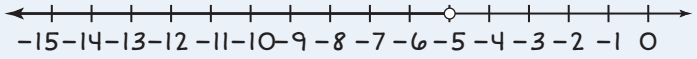
2. When you multiply the same negative number to each side of the inequality or divide the same negative number from each side of the inequality, what do you notice about the resulting inequality symbol?

3. Jenna and Brendan are trying to solve $-4x < 20$. Consider their solutions and explanations.

Brendan's Solution

$-4x < 20$

$x < -5$

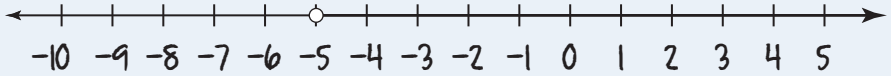


I divided both sides by -4 to solve the inequality.

Jenna's Solution

$-4x < 20$

$x > -5$



I know that when I multiply or divide any given number by a negative number, I have to pay attention to the sign of my answer. So when I divide both sides of the inequality by − 4, the inequality sign should reverse.

a. Determine who is correct. List three values from each person's solution, and verify that those solutions make the original inequality $-4x < 20$ true. What do you notice? Explain your reasoning.

Check for Brendan's solution. Check for Jenna's solution.

b. Circle the correct solution and explanation, and cross out the incorrect solution and explanation from Brendan's and Jenna's work.

4. **Solve each inequality and graph the solution set. Then, list three values from each solution set, and verify that each value makes the original inequality true.**

a. $8x > 16$

b. $\frac{x}{3} \leq -4$

c. $-5x < 35$

d. $\frac{x}{-2} \geq 5$

ACTIVITY
4.4 Solving Two-Step Inequalities

Aaron wants to buy new football pads that cost $55.00 at GoodSportsBuys.com. The online store charges $11 for shipping on orders less than $75. He also wants to buy some packages of eyeblack strips for $4 each, but he does not want to pay more than the $11.00 shipping fee.

1. **Write and solve an inequality that describes the possible number of packages of eyeblack strips Aaron can purchase and still remain in the $11.00 shipping fee category. Let p represent the number of packages of eyeblack strips. Explain your solution in terms of the problem situation.**

You just solved a problem that involved setting up and solving a two-step inequality. Let's compare and contrast the strategies and solutions of an equation and inequality that are similar in structure.

2. **Describe the steps you would take to solve the equation $3x - 2 = 7$. Then, solve the equation.**

3. A set of possible solutions for each inequality is shown. Circle the solutions that make the inequality true. Then, list three additional solutions to the inequality.

   a. $3x - 2 \geq 7$

      {–2, –1, 0, 1, 2, 3, 4, 5, 6, 7}

   b. $3x \geq 9$

      {–2, –1, 0, 1, 2, 3, 4, 5, 6, 7}

   c. $x \geq 3$

      {–2, –1, 0, 1, 2, 3, 4, 5, 6, 7}

4. What do you notice about the solutions you circled in Question 3, parts (a) through (c)?

5. What do you notice about the three additional solutions you wrote for each inequality?

6. Compare the sequence of the three inequalities to the steps you used to solve the equation in Question 2. What do you notice? Explain your reasoning.

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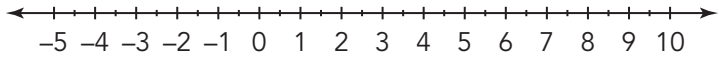
7. Graph the solution set for $3x - 2 \geq 7$.



You can check your solution to an inequality by choosing a value that is in your solution set and substituting it into the original inequality. If that substituted value makes the inequality true, then you have verified a correct solution.

8. Choose a value from the solution set of the inequality $3x - 2 \geq 7$, and verify that it is a solution.

9. Analyze the solution strategy and solution for each inequality.

Ella

$$-\frac{1}{2}x + \frac{3}{4} < 2$$
$$-4\left(-\frac{1}{2}x + \frac{3}{4} < 2\right)$$
$$2x - 3 > -8$$
$$2x > -5$$
$$x > \frac{-5}{2}$$
$$x > -2.5$$

Describe the strategy that Ella used correctly.

Jeff

$$-12x + 20 < 32$$
$$\frac{-12x + 20}{-4} < \frac{32}{-4}$$
$$3x - 5 < -8$$
$$3x < -3$$
$$x < -1$$

Identify the error in Jeff's strategy and determine the correct solution.

10. Solve each inequality or equation, and show your work. Then, graph the solution set on a number line.

a. $2x + 5 < -17$

b. $97 \leq -8x + 1$

c. $6.5x - 1.1 > 6.9$

d. $10 < \frac{2x - 3}{5}$

e. $18 \geq -x + 7$

NOTES

TALK the TALK

Summarizing Inequalities

The **Properties of Inequalities** allow you to solve inequalities involving any numbers.

Properties of Inequalities	For all numbers a, b, and c,...
Addition Property of Inequalities	If $a < b$, then $a + c < b + c$. If $a < b$, then $a + c > b + c$.
Subtraction Property of Inequalities	If $a < b$, then $a - c < b - c$. If $a < b$, then $a - c > b - c$.
Multiplication Property of Inequalities	If $a < b$, then $a \cdot c > b \cdot c$, for $c > 0$. If $a < b$, then $a \cdot c > b \cdot c$, for $c > 0$.
Division Property of Inequalities	If $a < b$, then $\frac{a}{c} < \frac{b}{c}$, for $c > 0$. If $a < b$, then $\frac{a}{c} > \frac{b}{c}$, for $c > 0$.

These properties also hold true for $\leq$ and $\geq$.

1. **Write a paragraph to summarize your understanding of the Properties of Inequalities.**

2. **State the Division Property of Inequalities for $c < 0$.**

3. **Describe how to solve any inequality. How do you check to see if a value is a solution to an inequality?**

NOTES

4. **Write a real-world situation that can be modeled by each inequality. Be sure to define your variables.**

   a. $20 > 5 + 3.25m$

   b. $65 + 20x \geq 150$

   c. $66 - \frac{1}{2}s > 32$

Assignment

LESSON 4: Be Greater Than

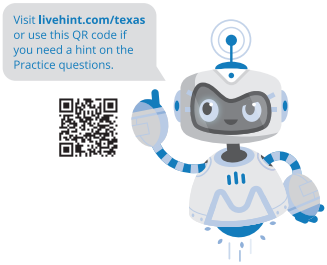
Write

Explain how solving an inequality is similar to and different from solving an equation.

Remember

To solve an inequality means to determine what value or values will replace the variable to make the inequality true.

Practice



1. Match each inequality with the correct solution.

a. $x < -2$	i. $4x + 12 < 20$
b. $x < 2$	ii. $55 < 35 + 10x$
c. $x > -2$	iii. $-\frac{3}{2}x + 12 > 15$
d. $x > 2$	iv. $-8x < 16$

2. Solve each one-step inequality and graph the solution set on a number line.

a. $x + 7 \geq 13$	b. $-4 > x - 3$
c. $\frac{x}{4} \leq \frac{5}{2}$	d. $18.3 > 6.1x$
e. $3 < \frac{x}{-8}$	f. $-10x \geq 45$

3. Solve each two-step inequality and graph the solution set on a number line.

a. $-17 < 3 - 5x$	b. $21 - 9x \geq -6$
c. $-500 \leq 11x - 60$	d. $-x + 38 < 59$

4. Carole has $53.95 and she washes cars for $8 each. Carole wants to attend a musical that costs $145.75.
   a. Write and solve an inequality to determine the minimum number of cars Carole must wash to be able to buy the ticket to the musical.
   b. Is the answer to the question that same as the solution to the inequality? Explain.
5. David has $15 to spend at the gourmet candy store. He wants to buy gummy bears and jelly beans. Gummy bears are $5.25 per pound and jelly beans are $3.90 per pound. If David already has $1\frac{3}{4}$ pounds of jelly beans, how many pounds of gummy bears can he buy? (Weights are measured to the nearest hundredth.) Write and solve an inequality to determine the maximum number of pounds of gummy bears David can buy.

Stretch

Solve each inequality and graph the solution set on a number line.

1. $7(4x + 9) - 13 \geq -87$	2. $0.25(3 - x) < 0.375$
3. $78 < -9x - 3(-56 + 12x)$	4. $0.20x - 0.08(x - 10) \leq 24.80$

Review

Solve each two-step equation.

1. $2(3x + 4) = 19$

2. $-3.2x + 9.1 = 4.62$

Rewrite each linear expression by factoring out the coefficient of the variable.

3. $-2x + 7$

4. $3x - 12$

Use properties to rewrite each expression with the fewest possible terms.

5. $\left(\frac{6}{7}x + 4\frac{1}{3}\right) + \left(-1\frac{1}{2}x - 9\right)$

6. $(10.7x - 19.2) - (81.6x - 33.6)$