TEXAS MATH SOLUTION

Accelerated Grade ɔ

Module 2 Topic 2 Lesson 3

Formally Yours

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Formally Yours

3

Using Inverse Operations to Solve Equations

MATERIALS

None

Lesson Overview

Students learn the formal strategies for solving two-step equations and formalize the language of solving equations. They review the Properties of Equality and use the properties to justify applying inverse operations to solve equations. Because of the Properties of Equality, if an operation is applied to both sides of an equation, the transformed equation has the same solution as the original equation. Students then use inverse operations to solve equations.

Students learn strategies for developing efficiency in solving two-step equations. They learn that, because of the Properties of Equality, they can multiply or divide all terms of an equation by the same rational number to ease computations. They apply the strategies learned throughout the lesson to solve two-step linear equations, including literal equations and number riddles. As they solve equations, they also check their solutions. Finally, students summarize solving two-step equations and write real-world scenarios that model situations involving equations.

Grade 7
Expressions, Equations, and Relationships

(10) The student applies mathematical process standards to use one-variable equations and inequalities to represent situations. The student is expected to:

(A) write one-variable, two-step equations and inequalities to represent constraints or conditions within problems.

(C) write a corresponding real-world problem given a one-variable, two-step equation or inequality.

(11) The student applies mathematical process standards to solve one-variable equations and inequalities. The student is expected to:

(A) model and solve one-variable, two-step equations and inequalities.

(B) determine if the given value(s) make(s) one-variable, two-step equations and inequalities true.

(C) write and solve equations using geometry concepts, including the sum of the angles in a triangle, and angle relationships.

ELPS

1.A, 1.C, 1.D, 1.E, 1.F, 2.C, 2.D, 2.G, 2.H, 2.I, 3.A, 3.B, 3.C, 3.D, 3.E, 4.A, 4.B, 4.C, 4.D, 4.K, 5.E

Essential Ideas

- A solution to an equation is any variable value that makes that equation true.
- The Properties of Equality state that if an operation is performed on both sides of the equation, to all terms of the equation, the equation maintains its equality.
- When the Properties of Equality are applied to an equation, the transformed equation will have the same solution as the original equation.
- Strategies to improve equation-solving efficiency include terms of an equation with fractions by the least common denominator, multiplying the terms of an equation with decimals by the appropriate multiple of 10, and dividing out a common factor of the terms of an equation.
- To determine if a solution to an equation is correct, substitute the value of the variable back into the original equation and if the equation remains equivalent, the solution is correct.

Lesson Structure and Pacing: 3 Days

Day 1

Engage

Getting Started: How Does That Work?

Students review the Properties of Equality. They solve an equation using the double number line model and explain which properties of equality were used in the solving process.

Develop

Activity 3.1: Strategies for Applying Inverse Operations

Students use inverse operations to solve two-step equations. They learn that the order in which they apply the inverse operations does not matter as long as the operation is applied to all the terms in the equation. Students solve four problems applying inverse operations in different orders and discuss when they prefer each order of applying the inverse operations.

Day 2

Activity 3.2: Writing and Solving Two-Step Equations

Students work with four contexts that require a two-step equation. They define the variables in the scenario and write an equation to represent the scenario. They then use their two-step equation to solve for the independent variable given values of the dependent variable.

Activity 3.3: Solving Equations with Efficiency

Students analyze provided student work focused on developing efficiency in solving two-step equations. They learn that, because of the Multiplication Property of Equality, they can multiply all terms of an equation containing decimals by a multiple of 10 to form an equation of integers. Similarly, they can multiply all terms by the least common denominator when the equation contains fractions. In other cases, because of the Division Property of Equality, students can divide all terms of the equation by the greatest common factor to make the numbers smaller and computations easier.

Day 3

Activity 3.4: Solving Literal Equations

Students solve a variety of literal equations. They solve problems about temperature conversions, the perimeter of a rectangle, the area of a trapezoid, and surface area and volume. They also generalize the solutions to two-step equations of the form $ax + b = c$ and $a(x + b) = c$.

Activity 3.5: Solving More Equations

Students apply the strategies learned throughout the lesson to solve two-step linear equations. They write and solve several two-step equations in the form of number riddles (word equations). Then they solve a wide variety of linear equations and check their solutions.

Demonstrate

Talk the Talk: Get Creative

Students summarize solving two-step equations. They then write real-world scenarios that model situations involving equations. Consider prompting students by discussing areas of interest such as technology, music, and finance.

ENGAGE

Getting Started: How Does that Work?

Facilitation Notes

In this activity, students review the Properties of Equality. They solve an equation using the double number line model and explain which properties of equality were used in the solving process.

Ask a student to read the introductory paragraph aloud. Discuss the properties and examples as a class.

Differentiation strategy

Divide the class into 6 groups. Use the 4 properties listed, as well as the Reflexive Property and the Commutative Property. (Students may want to use the two additional properties to rewrite the equations into a more familiar form throughout the lesson.) Assign each group one property to explain in detail to the class. The group members should use the variables *a*, *b*, and *c*, substitute values for the variables, and create a one-step equation to explain the meaning of the property.

Questions to ask

- Are there any number constraints when you use any of the Properties of Equality?
- When using the Addition Property, could you write $c + a = c + b$? Why or why not?
- When using the Addition Property, could you write $c + a = b + c$? Why or why not?
- When using the Subtraction Property, could you write $c - a = c - b$? Why or why not?
- When using the Multiplicative Property, could you write $ca = cb$? Why or why not?
- When using the Multiplicative Property, could you write $ca = bc$? Why or why not?
- When using the Division Property, could you write $\frac{c}{a} = \frac{b}{c}$? Why or why not?

Misconceptions

- Students may be confused by the expressions *ac* and *ab*. This may be a good time to explain that multiplication can be written in various ways, by using
  - A "×" sign, for example 4 × 3.
  - Parentheses, for example: (4)(3) or (4)3 or 4(3).
  - A "·", for example 4 · 3.
  - Juxtaposition of a number and a variable or two variables, for example: 4*a*, *ab*.

- Encourage students to use a fraction to denote division rather than the "÷" sign. When using the "÷" sign with more than one term in the dividend, parentheses are required.

Summary

When solving equations, equality must be maintained by using the Addition, Subtraction, Multiplication and Division Properties of Equality.

Activity 3.1

Strategies for Applying Inverse Operations

DEVELOP

Facilitation Notes

In this activity, students use inverse operations to solve two-step equations. They learn that the order in which they apply the inverse operations does not matter as long as the operation is applied to all the terms in the equation. Students solve four problems applying inverse operations in different orders and discuss when they prefer each order of applying the inverse operations.

Have students work with a partner or in groups to complete Questions 1 and 2. Share responses as a class.

Questions to ask

- What does it mean to isolate the variable?
- What properties did this student use to solve the equation?
- What property did Demaryius use when he rewrote $2x + 6$ as $2(x + 3)$?
- Why did Demaryius only write "-3" two times? Why isn't -3 written under the term, x?
- What was Calvin's mistake?
- Why did Isaac only write "-6" two times? Why isn't -6 written under the term, $2x$?
- Why do you think the subtraction of 6 was written vertically instead of horizontally? Would it be correct if you wrote the subtraction horizontally?
- What student used the same method as you?

Differentiation strategy

To extend the activity, have students redo Demaryius and Isaac's work by multiplying by $\frac{1}{2}$ rather than dividing by 2. Explain the necessity for parentheses in Demaryius' strategy.

Have students work with a partner or in groups to complete Questions 3 through 5. Share responses as a class.

Differentiation strategies

- To scaffold support with the fact that all equations are not written in the format, $ax + b = c$, suggest students use the Reflexive and Commutative Properties to rewrite each equation so that it looks more familiar to them before solving it.
- Some students have difficulty seeing the two sides of the equation, and therefore have difficulty maintaining the properties of equality. Suggest to these students to make a vertical line underneath the equals sign to visualize their actions on both sides of the equation.
- Some students may be following the examples of Demaryius or Isaac line-by-line to solve the equations. Explain where lines may be condensed. For example, in Isaac's work: $2x = 7$ can be written as $\frac{2x}{2} = \frac{7}{2}$.

Questions to ask

- How did you know what constant to add or subtract?
- Why is the constant being added or subtracted written twice each time?
- Why is the constant being added or subtracted never written under the term with a variable?
- How do you know what value to divide by?
- How do you know when to multiply and when to divide?
- How can you tell that your solution is correct?
- What does it mean to solve an equation?
- How is this process related to the double number line process?

Summary

When solving two-step equations, two inverse operations are required; the operations can be applied in either order.

Activity 3.2
Writing and Solving Two-Step Equations

Facilitation Notes

In this activity, students work with four contexts that require a two-step equation. They define the variables in the scenario and write an equation to represent the scenario. They then use their two-step equation to solve for the independent variable given values of the dependent variable.

Differentiation strategy

To scaffold support with writing the equations, have students write the equation in words first, then substitute algebraic expressions for each component. For example, for Question 1: \$6 × number of packs + shipping = cost. Then, $6p + 5 = c$.

Have students work with a partner or in groups to complete Questions 1 and 2. Share responses as a class.

Questions to ask for Question 1

- What is the cost of one invitation? Can you buy just one invitation?
- How many packs of invitations would Shelly have to order if she was sending out 31 invitations?
- What is the shipping fee if Shelly orders 500 invitations?
- What values remain constant in this context?
- What values vary in this context?
- How are the values that remain constant and the values that vary represented in your equation?
- Why does it make sense to subtract the shipping fee from the total cost before dividing by the cost of a 10-pack of invitations?
- How did you use your equation to calculate the number of 10-packs of invitations ordered for a total cost of \$53.
- Did you need to know that invitations were sold in 10-packs in order to solve this problem?

Questions to ask for Question 2

- How is the structure of this question the same as that of Question 1?
- What are the two types of charges that determine the total cost of a repair bill?
- How are the values that remain constant and the values that vary represented in your equation?
- Why is the equation you wrote considered a two-step equation?
- What were the steps you used to solve your equation?
- Can a mechanic work part of an hour?

Have students work with a partner or in groups to complete Questions 3 and 4. Share responses as a class.

Questions to ask

- How is this question different from the previous two questions?
- What does the term "profit" mean?

- How is profit determined?
- What many times does Felicia have to pay \$23.76 each weekend?
- How is the money Felicia is making from shampooing dogs represented algebraically?
- How is the money Felica must spend to shampoo the dogs represented?
- Why is subtraction involved in your equation?
- Does it matter what term is written first in your equation? Why or why not?
- Why is the equation you wrote considered a two-step equation?
- How do you know that your answer is correct?
- Is the equation $p = 8\ (12 + s)$ in the correct form? Explain.

Summary

Two-step equations can be used to model some real-world problems.

Activity 3.3

Solving Equations with Efficiency

Facilitation Notes

In this activity, students analyze provided student work focused on developing efficiency in solving two-step equations. They learn that, because of the Multiplication Property of Equality, they can multiply all terms of an equation containing decimals by a multiple of 10 to form an equation of integers. Similarly, they can multiply all terms by the least common denominator when the equation contains fractions. In other cases, because of the Division Property of Equality, students can divide all terms of the equation by the greatest common factor to make the numbers smaller and computations easier.

Differentiation strategy

Some students may need to see the subtraction, multiplication and division steps explicitly in order for the processes to make sense to them. If that is the case, have the students enter those steps into the text.

Ask a student to read the introductory paragraph aloud. Complete Question 1 as a class.

Misconception

Some students may say that the decimal points were moved. Explain that moving the decimal point is not a Property of Equality. Have students look at the change in values to see that multiplication by 10 occurred.

Questions to ask

- How did Sherry get the value 1.9 in her equation?
- What operation did Sherry complete to get $x = \frac{1.9}{1.1}$?
- What operation did Sherry complete to get $x = \frac{19}{11}$?
- What operation did Maya complete to rewrite the equation with whole numbers.
- How would you know when to use Maya's strategy?
- What strategy did you prefer? Why?

Have students work with a partner or in groups to complete Question 2. Share responses as a class.

Questions to ask

- Is it acceptable to put a decimal point in a number that does not have one? If so, where does the decimal point go?
- Why does Brian have to change the right side of the equation, too?
- How do you determine what number to multiply by if all values do not have the same number of decimal places?

Have students solve the equations in Question 3 on their own. Keep in mind, the strategies introduced in the peer analysis examples were done to foster efficiency in equation solving. If students are using a calculator, encourage them to still use Maya's strategy.

Have students work with a partner or in groups to read through the Worked Example and complete Question 4. Share responses as a class.

Questions to ask

- How were the fractions rewritten as whole numbers in each example?
- Why do you think the value 3 was chosen to multiply both sides by?
- Why might it be helpful to place a 1 underneath the 3's on each side of the equation?
- Why do you think the value 4 was chosen to multiply both sides by?
- What would happen if you multiplied both sides by 2 instead?
- What would happen if you multiplied both sides by 8 instead?

Differentiation strategies

- When simplifying by distribution, some students may choose to divide out common factors immediately, resulting in Step 2. Others may see the benefit in distributing, then simplifying. For example, $\frac{33}{3}x + 15 = \frac{51}{3}$, then dividing to get to Step 2.
- Some students may be overwhelmed by this process and not see its benefit. If that is the case, have students work through the steps to solve the equations the customary way by performing the calculations using fractions, then make a comparison about workload.

Have students work with a partner or in groups to complete Questions 5 through 7. Share responses as a class.

Questions to ask

- What property did Louise not consider when solving this equation?
- How did you determine what number to multiply both sides of the equation by?
- Why did you need parentheses to solve this equation?
- What Property of Equality was used in the Worked Examples?
- Why should your answer work in both the original equation and the equation using the new strategy?

Summary

Equations can be rewritten to eliminate decimals, fractions or large numbers using the Properties of Equality in order to solve them more efficiently.

Activity 3.4
Solving Literal Equations

Facilitation Notes

In this activity, students solve a variety of literal equations. They solve problems about temperature conversions, the perimeter of a rectangle, the area of a trapezoid, and surface area and volume. They also generalize the solutions to two-step equations of the form $ax + b = c$ and $a(x + b) = c$.

Ask a student to read the information about literal equations. Discuss the definition as a class.

Have students work with a partner or in groups to complete Question 1. Share responses as a class.

Questions to ask

- What does it mean to "solve the equation for the temperature in Celsius"?
- Do you get a numeric answer when solving a literal equation?
- How is solving a literal equation different from solving a regular equation?
- How did solving Questions 1a and 1b assist you in solving the literal equation?

Differentiation strategy

Have students solve part (a) and (b) a second time, this time using the formula developed in part (c). Discuss the benefit of solving the literal equation first, then substituting a value in for F.

Have students work with a partner or in groups to complete Questions 2 through 4. Share responses as a class.

Differentiation strategy

To scaffold support with literal equations, try this strategy.

- Have students write the literal equation twice, side-by-side.
- Provide or help students select a set of variables that work for the equation.
- Have the students substitute the values for all variables in the first equation except the one for which they are solving. This way, the equation will look more familiar.
- Have students solve the first equation they created.
- Have students use their equation-solving process as a guide to solve the second literal equation.
- Have students check that their new equation is correct by substitution.

Questions to ask

- How did you determine what inverse operation to complete first?
- How did you use the steps from Question 3 to assist you in solving Question 4?
- How did writing the formula as a pair of factors assist you in isolating h?
- Is there another way you could have solved this literal equation?
- How can you determine if both solutions for the literal equation are correct?

Have students work with a partner to complete Question 5.

Questions to ask

- What does the variable *a* represent in part (a)? in part (b)?
- What does the variable *b* represent in part (a)? in part (b)?
- What does the variable *c* represent in part (a)? in part (b)?
- How would you describe the equation solved for *x* in words?

Differentiation strategy

To extend the activity, have students concentrate on the formulas for two-step equations in Question 5. Have them generate an equation similar to how they did in *Keeping It Together* in Lesson 2. Then have students solve the equation using the formal steps learned in this lesson and then using the literal equation developed in part (a). Duplicate the process using part (b). Discuss advantages and disadvantages of using literal equations.

Summary

Literal equations are solved using the same Properties of Equality as solving customary equations.

Activity 3.5

Solving More Equations

Facilitation Notes

In this activity, students apply the strategies learned throughout the lesson to solve two-step linear equations. They write and solve several two-step equations in the form of number riddles (word equations). They then solve a wide variety of linear equations and check their solutions.

Have students work with a partner or in groups to complete Question 1. Share responses as a class.

Questions to ask

- What did you do first to solve the riddles?
- Did you use the same strategy for parts (a) and (b)?
- How does solving the riddles compare to solving a two-step equation?

Have students work with a partner or in groups to complete Question 2. Share responses as a class.

Questions to ask

- Did you rewrite any of the equations in a different order before you started solving it? If so, what properties did you use?

- Did you use either of the two literal equations you developed in the last activity?
- How did you determine what the inverse operations were?
- How did you know when to add or subtract?
- How did you know when to multiply or divide?
- How did you determine the order in which to perform the inverse operations?
- Is there more than one correct way to solve this equation?
- Why must each inverse operation be performed on both sides of the equation?
- Did you use any methods to eliminate decimals or fractions before you started using inverse operations? If so, explain your process.
- Is there more than one solution?
- How do you know if the solution is correct?
- How did you verify your solution?

Summary

There are two specific structures of two-step equations, $ax + b = c$ and $a(x + b) = c$. There is flexibility in the order of the steps in solving them as long as the Properties of Equality are followed.

DEMONSTRATE

Talk the Talk: Get Creative

Facilitation Notes

In this activity, students summarize solving two-step equations. They then write real-world scenarios that model situations involving equations. Consider prompting students by discussing areas of interest such as technology, music, and finance.

Have students complete Questions 1 through 3 with a partner. Then share the responses as a class.

Questions to ask

- Do you prefer to solve a two-step equation as it is written or by solving the literal equation first?
- What does the variable represent in your equation?
- What does the coefficient of the variable represent in your equation?
- What does the constant represent in your equation?
- Why does this situation require subtraction?
- Why does this situation require addition?

- Was it easier to make sense of the coefficient of $\frac{1}{2}$ by using it as multiplication by $\frac{1}{2}$ or division by 2?

Summary

There are two specific structures of two-step equations, $ax + b = c$ and $a(x + b) = c$. The general equations can be rewritten to solve for x and can be used to model real-world situations.

Formally Yours

3

Using Inverse Operations to Solve Equations

WARM UP

Solve each equation.

1. $2.3p = -11.73$
2. $\frac{3}{4}r = 10$
3. $y + 5.92 = 1.63$
4. $7\frac{2}{5} + t = 3\frac{1}{4}$

LEARNING GOALS

- Use properties of equality to solve equations.
- Write two-step equations.
- Solve two-step equations of the form $px + q = r$ and $p(x + q) = r$ with efficiency.
- Check solutions to equations algebraically.
- Solve literal equations for specific variables.

KEY TERMS

- two-step equation
- literal equation

You have solved equations using double number lines. How can you use the Properties of Equality and inverse operations to solve equations?

Warm Up Answers

1. $p = -5.1$
2. $r = \frac{40}{3} = 13\frac{1}{3}$
3. $y = -4.29$
4. $t = -\frac{83}{20} = -4\frac{3}{20}$

Answers

1. Answers may vary. Students should draw their model. There are two common approaches: (1) Subtract 6 and then divide by 2 and (2) Divide by 2 and then subtract 3.
2. The order listed by students will vary based on their approach in Question 1, but the two properties most likely used were the Subtraction Property of Equality and the Division Property of Equality. Multiplication Property of Equality could also be used if division by 2 is interpreted as multiplication by $\frac{1}{2}$.

Getting Started

How Does that Work?

Recall that to solve an equation means to determine the value or values for a variable that make the equation true. In the process of solving equations, you must always maintain equality, using the Properties of Equality.

Properties of Equality	For all numbers *a*, *b*, and *c*, . . .
Addition Property of Equality	If $a = b$, then $a + c = b + c$.
Subtraction Property of Equality	If $a = b$, then $a - c = b - c$.
Multiplication Property of Equality	If $a = b$, then $ac = bc$.
Division Property of Equality	If $a = b$ and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$.

1. Solve $2x + 6 = 13$ using a double number line model.

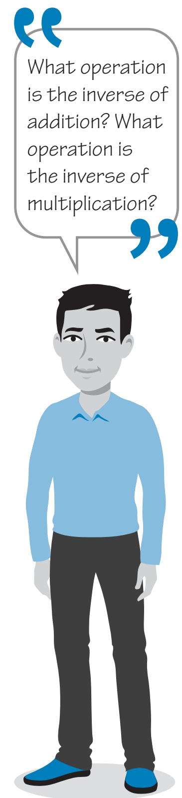
2. Explain which Properties of Equality you used in the process of solving the equation.

ACTIVITY 3.1

Strategies for Applying Inverse Operations

Throughout this topic, you have written and solved *two-step equations*. A **two-step equation** requires two inverse operations, or applying two Properties of Equality, to isolate the variable.

Demaryius, Calvin, and Isaac each solved $2x + 6 = 13$ in a different way. Analyze their solution strategies.



Demaryius

$$2x + 6 = 13$$
$$\frac{2x + 6}{2} = \frac{13}{2}$$
$$\frac{2(x + 3)}{2} = \frac{13}{2}$$
$$x + 3 = 6.5$$
$$-3 = -3$$
$$x = 3.5$$

Calvin

$$2x + 6 = 13$$
$$\frac{2x}{2} + 6 = \frac{13}{2}$$
$$x + 6 = 6.5$$
$$-6 = -6$$
$$x = 0.5$$

Isaac

$$2x + 6 = 13$$
$$-6 = -6$$
$$2x = 7$$
$$\frac{2x}{2} = \frac{7}{2}$$
$$x = 3.5$$

1. **Compare the strategies used by Demaryius and Calvin.**

2. **Compare the strategies used by Demaryius and Isaac.**

Answers

1. Demaryius and Calvin both used the Division and Subtraction Properties of Equality, but Calvin did not divide the entire left-hand side of the equation by 2. To correctly apply the Division Property of Equality, all terms must be divided by the same number.
2. Demaryius and Isaac both used the Division and Subtraction Properties of Equality, but they used them in opposite orders.

Answers

3a. $x = 33$
3b. $x = 9$
3c. $x = 13$
3d. $x = 66$
4a. $x = 33$
4b. $x = 9$
4c. $x = 13$
4d. $x = 66$

3. Solve each equation by first applying either the Addition or Subtraction Property of Equality.

To make the addition and subtraction simpler, you can leave fractions in improper form. They already have a common denominator.

a. $56 = -10 + 2x$

b. $6x + 25 = 79$

c. $38 = 4x - 14$

d. $13 + \frac{x}{3} = 35$

4. Solve each equation by first applying either the Multiplication or Division Property of Equality.

a. $56 = -10 + 2x$

b. $6x + 25 = 79$

c. $38 = 4x - 14$

d. $13 + \frac{x}{3} = 35$

Consider the equations and your solutions in Questions 3 and 4.

5. Do you prefer one order over the other? If so, why? If your preference changes depending on the equation, explain why.

ACTIVITY 3.2 Writing and Solving Two-Step Equations

1. Shelly is throwing a graduation party. She is sending invitations to her friends and family. She finds a company that charges \$6 for a 10-pack of personalized invitations, plus a \$5 shipping fee for the entire order, no matter how many 10-packs are ordered. Shelly wants to calculate the cost of an order, based on the number of packs of invitations she orders.

   a. Define variables for the two quantities that are changing in this scenario.

   b. Write an equation that represents the total cost of any order based on the number of packs of invitations.

   c. Use your equation to determine how many packs of invitations are ordered if the total is \$53. What about if the total is \$29?

Answers

5. Answers will vary.

Answers

1a. Let p represent the number of 10-packs of invitations ordered and c represent the total cost of the order.

1b. $c = 5 + 6p$

1c. If the total cost was \$53, Shelly ordered 8 10-packs of invitations ($p = 8$). If the total cost was \$29, Shelly ordered 4 10-packs of invitations ($p = 4$).

Answers

2a. Let p represent the cost of parts, h represent the number of hours worked, and c represent the total cost of the repairs.

2b. $p + 45h = c$

2c. $h = 3.75$ The mechanic worked on the car for 3.75 hours, or 3 hours and 45 minutes.

3a. Let p represent the total profit and d represent the number of dogs groomed.

3b. $p = 15d - 23.76$

3c. $d = 19$. Felicia groomed 19 dogs if her profits were $261.24.

2. Pete's Garage charges $45 per hour for labor when performing auto repairs. The office manager must have the cost of parts and the hours of each job ticket to complete the bills for the customers.

a. Define variables for the *three* quantities that are changing in this scenario.

b. Write an equation that represents the total cost of the auto repairs.

c. Assume that for a given car, the cost of the parts is $101. Use your equation to determine how many hours the mechanic worked on the car if the total bill was $269.75.

3. Felicia's Pet Grooming charges $15 for each dog washed and groomed on the weekend. The cost of the dog shampoo and grooming materials for a weekend's worth of grooming is $23.76. Felicia is interested in her weekend profits.

a. Define variables for the two quantities that are changing in this scenario.

b. Write an equation that represents the total profits based on the number of dogs groomed.

c. Use your equation to determine how many dogs Felicia groomed if her profits were $261.24.

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4. Frankie works as a pet sitter all week long but he is more in demand on some days than others. He posts his rates as $12 per visit plus a surcharge, which depends on the day. On his busiest days, Frankie can serve 8 houses for pet sitting. He is interested in his daily profits.

a. Define variables for the two quantities that are changing in this scenario.

b. Write an equation that represents the maximum total profits based on the surcharge for that day. Write your equation in the form $a(x + b) = c$.

c. Beverly and Sean are trying to determine Frankie's Saturday surcharge per house if he makes $142. Beverly thinks the first step in solving the equation is to divide by the coefficient of the parentheses. Sean thinks the first step is to distribute that value through the parentheses. Who's correct?

d. Determine the Saturday surcharge by solving the equation you wrote in part (b). What is the total fee Frankie charges for pet sitting on a Saturday?

Answers

4a. Let s represent the surcharge for a particular day and p represent Frankie's profit.

4b. $p = 8(12 + s)$

4c. Both Beverly and Sean are correct. If the equation is solved using Beverly's strategy, it can be solved in two steps. If the equation is solved using Sean's strategy, it can be solved in three steps, but still only 2 inverse operations.

4d. $h = 5.75$. The Saturday surcharge is $5.75. Frankie charges a total of $17.75.

Answers

1a. Both used the inverse operations of subtraction and then division. The solutions are the same.

Sherry solved the equation using the decimals. Maya converted her equation to whole numbers before she started applying the inverse operations.

1b. Maya applied the Multiplication Property of Equality by multiplying each term of the equation by 10.

2. Brian did not multiply 38 by 10. The approximate value of x is 15.15.

ACTIVITY 3.3

Solving Equations with Efficiency

A savvy mathematician (you!) can look at an equation, see the structure of the equation, and look for the most efficient solution strategy.

As you have seen, there are multiple ways to solve equations. Sometimes an efficient strategy involves changing the numbers in the equation—in mathematically appropriate ways!

1. Analyze each correct solution strategy to the equation $1.1x + 4.3 = 6.2$.

Sherry

$$1.1x + 4.3 = 6.2$$
$$1.1x + 4.3 - 4.3 = 6.2 - 4.3$$
$$1.1x = 1.9$$
$$x = \frac{1.9}{1.1}$$
$$x = \frac{19}{11}$$

Maya

$$1.1x + 4.3 = 6.2$$
$$11x + 43 = 62$$
$$11x + 43 - 43 = 62 - 43$$
$$11x = 19$$
$$x = \frac{19}{11}$$

a. Explain how the two solutions strategies are alike and how they are different.

b. What Property of Equality did Maya apply before she started solving the equation?

Remember, to maintain equality, any operation applied to one side of the equation must be applied to the other side of the equation.

2. Brian used Maya's strategy to solve the equation $2.6x - 1.4 = 38$. Identify his mistake and then determine the correct solution.

Brian

$$2.6x - 1.4 = 38$$
$$26x - 14 = 38$$
$$26x = 52$$
$$x = 2$$

3. **Use Maya's strategy to solve each equation. Then check your solution in the original equation.**

   a. $-9.6x + 1.8 = -12.3$

   b. $2.99x - 1.4 = 13.55$

Now let's consider strategies to solve two different equations that contain fractions.

WORKED EXAMPLE

	$\frac{11}{3}x + 5 = \frac{17}{3}$	$\frac{1}{2}x + \frac{3}{4} = 2$
Step 1:	$3\left(\frac{11}{3}x + 5\right) = 3\left(\frac{17}{3}\right)$	$4\left(\frac{1}{2}x + \frac{3}{4}\right) = 4(2)$
Step 2:	$11x + 15 = 17$	$2x + 3 = 8$
Step 3:	$x = \frac{17 - 15}{11}$	$x = \frac{8 - 3}{2}$
	$= \frac{2}{11}$	$= \frac{5}{2}$

You should be fluent in operating with decimals and fractions, but these strategies can ease the difficulty of the calculations when solving equations.

4. **Answer each question about the strategies used to solve each equation in the Worked Example.**

   a. **Explain Step 1. Why might this strategy improve your efficiency with solving equations?**

   b. **What property was applied in Step 2?**

   c. **Explain Step 3.**

Answers

3a. $x = -\frac{47}{32} = -1\frac{15}{32}$

3b. $x = 5$

4a. In Step 1, the terms of the equations were multiplied by the least common denominator of the fractions in the equation. This strategy eliminates the need to later get a common denominator during the solving process.

4b. The Distributive Property was applied to each equation.

4c. In Step 3, both inverse operations are shown in one step and then the final answer is given.

Answers

5. Louise did not multiply the left side of the equation by 4. $x = 13$
6. Multiply all terms of the equation by the least common denominator, 15. $x = \frac{13}{10}$

7a. In this strategy, the Distributive Property was used to factor the left side of the equation and then the Division Property of Equality was applied before using the inverse operations.

NOTES

5. Louise used the strategy from the Worked Example to solve $3 = \frac{1}{4}x - \frac{1}{4}$. Identify her mistake and determine the correct solution.

Louise

$3 = \frac{1}{4}x - \frac{1}{4}$

$3 = 4\left(\frac{1}{4}x - \frac{1}{4}\right)$

$3 = x - 1$

$4 = x$

6. Use the strategy from the Worked Example to solve $\frac{2}{3}x + \frac{4}{5} = \frac{5}{3}$. Check your solution in the original equation.

Consider the solution strategies used to solve two more equations.

WORKED EXAMPLE

	$-20x + 80 = 230$	$-38 = -6x - 14$
Step 1:	$10(-2x + 8) = 10(23)$	$-2(19) = -2(3x + 7)$
Step 2:	$-2x + 8 = 23$	$19 = 3x + 7$
Step 3:	$x = \frac{23 - 8}{-2} = -\frac{15}{2}$	$\frac{19 - 7}{3} = x$
		$4 = x$

7. Answer each question about the strategies used to solve each equation in the Worked Example.

a. How is the strategy used in this pair of examples different from the strategies used in Questions 1 and 2?

b. When might you want to use this strategy?

c. Use the strategy from the Worked Example to solve $44x - 24 = 216$. Check your solution in the original equation.

ACTIVITY 3.4

Solving Literal Equations

You have already learned a lot of important formulas in mathematics. These formulas are also *literal equations*. **Literal equations** are equations in which the variables represent specific measures. Common literal equations occur in measurement and geometry concepts.

1. The formula to convert from degrees Celsius to degrees Fahrenheit is $F = \frac{9}{5}C + 32$.

   a. Calculate the temperature in Celsius, if it is 39°F.

   b. Calculate the temperature in Celsius, if it is 25°F.

   c. Solve the equation for the temperature in Celsius.

Answers

7b. Answers will vary. Students may mention when the numbers in the equation are large or when the coefficient of the variable is negative as key opportunities to use this strategy.

7c. $x = \frac{60}{11}$

Answers

1a. The temperature is approximately 3.9°C.

1b. The temperature is approximately −3.9°C.

1c. $C = \frac{5}{9}(F - 32)$

Answers

2a. $P = 2(l + w)$

2b. $\frac{P}{2} - w = l$

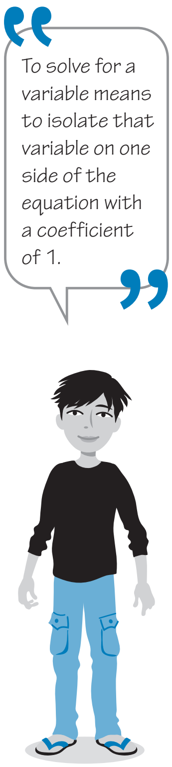
2c. $\frac{P}{2} - l = w$

2d. The equations look the same except length and width are in opposite places. This makes sense beside either distance of a rectangle can be assigned the name *length* and the other distance is assigned the name *width*.

3a. $A = \frac{h}{2}(b_1 + b_2)$

3b. $\frac{2A}{b_1 + b_2} = h$

3c. $\frac{2A}{h} - b_2 = b_1$



2. The formula for the perimeter of a rectangle can be written as $P = 2l + 2w$, where l and w represent the length and width of the rectangle.

a. Rewrite the formula by factoring out the coefficient of the variables.

b. Next, solve the equation for the length.

c. Solve the equation in part (a) for the width.

d. How are the equations in parts (b) and (c) alike? Explain why this makes sense.

3. The formula for the area of a trapezoid can be written as $A = \frac{1}{2}(b_1 + b_2)h$, where b_1 and b_2 are the lengths of the bases and h is the length of the height of the trapezoid.

a. Rewrite the formula as a product of two factors.

b. Solve the equation for the height of the trapezoid.

c. Solve the equation in part (a) for one of the bases.

d. When would it be helpful to solve the trapezoid area formula for one of the bases?

4. Solve each equation for the specified variable.

a. $S = 2\pi rh + 2\pi r^2$ for h

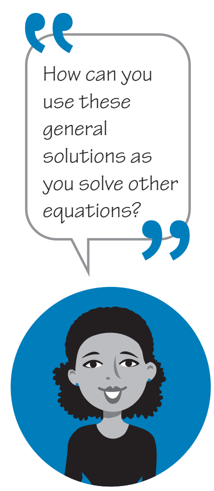
b. $V = \pi r^2 h + \frac{2}{3}\pi r^3$ for h.

Throughout this topic, you have solved many linear equations.

5. Analyze each general form.

a. Write a general solution for equations of the form $ax + b = c$ by solving the equation for x.

b. Write a general solution of the form $a(x + b) = c$ by solving the equation for x.



Answers

3d. Answers will vary.

4a. $\frac{S - 2\pi r^2}{2\pi r} = h$ or $\frac{S}{2\pi r} - r = h$

4b. $\frac{V - \frac{2}{3}\pi r^3}{\pi r^2} = h$ or $\frac{V}{\pi r^2} - \frac{2}{3}r = h$

5a. $x = \frac{c - b}{a}$

5b. $x = \frac{c}{a} - b$

Answers

1a. $3x - 5 = 28$, $x = 11$.
The number is 11.

1b. $4x + 15 = 79$, $x = 16$.
The number is 16.

1c. Answers will vary.

2a. $x = 2$

2b. $x = 22$

2c. $x = \frac{-9}{2}$

2d. $x = 1$

ACTIVITY **3.5**

Solving More Equations

Number riddles are popular types of problems to solve using two-step equations.

1. Solve each number riddle by writing and solving an equation.

a. What is a number that when you multiply it by 3 and subtract 5 from the product, you get 28?

b. What is a number that when you multiply it by 4 and add 15 to the product, you get 79?

c. Make a number riddle for a partner to solve.

2. Solve each equation. Check your solutions.

a. $2 + 7x = 16$

b. $5 + \frac{x}{2} = 16$

c. $-17 = 2x - 8$

d. $0.5x - 0.3 = 0.2$

"Remember all the strategies you learned in this lesson."

14 • TOPIC 2: Two-Step Equations and Inequalities

e. $-\frac{1}{4} - \frac{1}{2}x = -\frac{19}{4}$

f. $-\frac{2}{5}x + 4 = 18$

NOTES

g. $-5 = -3(x + 11)$

h. $8(x + 6) = 18$

i. $\frac{1}{2}(5 - x) = \frac{1}{4}$

j. $6.4 = 1.2(4 + 2x)$

TALK the TALK

Get Creative

1. Any equation in the form $ax + b = c$ can be solved in two steps, but do you need to write out both steps to solve the equation?

   a. Isolate the variable x, so that it has a coefficient of 1.

   b. Use your answer from part (a) to solve $4x + 5 = 61$.

Answers

2e. $x = 9$

2f. $x = -35$

2g. $x = \frac{-28}{3}$

2h. $x = \frac{-15}{4}$

2i. $x = \frac{9}{2}$

2j. $x = \frac{2}{3}$

Answers

1a. $x = \frac{(c - b)}{a}$

1b. $x = \frac{61 - 5}{4}$

Answers

2a $x = \frac{c}{a} - b$

2b. $x = \frac{20}{4} + 7$

3a. Sample answer. Marshall is taking his two little brothers to the movies. It costs *b* dollars per admission ticket, and Marshall has a $5 off coupon. The total amount he spends for the three of them is $22.

3b. Sample answer. Madison spent a total of $19 at the grocery store. She purchased *n* boxes of cereal for $4.50 each and a bag of oranges for $2.50.

3c. Sample answer. Lezlee gets on an escalator on the ground floor, which is 2 feet above street level. The escalator travels at a rate of $\frac{1}{2}$ vertical foot per second for *t* seconds until it reaches the top, which is 16 feet above street level.

NOTES

2. Similarly, any equation in the form $a(x + b) = c$ can be solved without writing out both steps of the two-step solution process.

a. Isolate the variable *x*, so that it has a coefficient of 1.

b. Use your answer to part (a) to solve $4(x - 7) = 20$.

3. Write a real-world situation that can be modeled by each equation.

a. $3b - 5 = 22$

b. $19 = 2.5 + 4.5n$

c. $\frac{1}{2}t + 2 = 16$

Assignment

LESSON 3: Formally Yours

Write

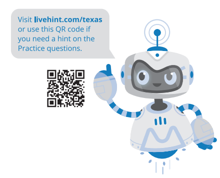
Explain the process of solving a two-step linear equation.

Remember

You can use the Properties of Equality to rewrite equations and increase your efficiency with solving equations.

- If the equation contains fractions, you can multiply both sides of the equation by the least common denominator.
- If the equation contains decimals, you can multiply both sides of the equation by a multiple of 10.
- If the equation contains large values, you can divide both sides of the equation by a common factor.

Practice



1. Madison Middle School has a Math and Science Club that holds meetings after school. The club has decided to enter a two-day competition that involves different math and science challenges. The first day of competition involves solving multi-step math problems. Teams will receive two points for every problem they get correct in the morning session and three points for every question they get correct in the afternoon session.
   a. Write an equation to represent the situation. Remember to define your variable(s).
   b. The team scores four points in the morning session, but finishes the day with 28 points. Solve the equation and interpret the solution in the context of the problem.
   c. The second day of the competition was the science portion, involving hands-on science problems. Each correct science problem is worth 5 points. If the team started the day with 28 points and ended with 53 points, how many science problems did they get correct? Write and solve an equation to answer the question.
2. Employees at Driscoll's Electronics earn a base salary plus a 20% commission on their total sales for the year. Suppose the base salary is $40,000.
   a. Write an equation to represent the total earnings of an employee. Remember to define your variable(s).
   b. Stewart wants to make $65,000 this year. How much must he make in sales to achieve this salary? Write and solve an equation to answer this question.
   c. Describe the equation $52{,}000 + 0.3s = 82{,}000$ in terms of the problem situation.
3. The manager of a home store is buying lawn chairs to sell at his store. Each pack of chairs contains 10 chairs. The manager will sell each chair at a markup of 20% of the wholesale cost, plus a $2.50 stocking fee.
   a. Write an equation that represents the retail price of a chair, r, in terms of the wholesale price, w.
   b. Use your equation to calculate the retail price of the chair if the wholesale price is $8.40.
   c. Use your equation to calculate the wholesale price if the retail price is $13.30.

Assignment Answers

Write

To solve an equation, isolate the variable so that the variable has a coefficient of 1. To isolate the variable, use the inverse operation of each operation contained within the equation. Verify the solution algebraically by substituting the value of the variable into the original equation to see if it makes the equation true.

Practice

1a. Let m be the number of problems correct in the morning session, a be the number of problems correct in the afternoon session, and t be the total score. $2m + 3a = t$

1b. $4 + 3a = 28$, $a = 8$. The team correctly solved 8 problems in the afternoon session.

1c. Let y be the number of science problems correct. $28 + 5y = 53$, $y = 5$. The team correctly solved 5 science problems.

2a. Let s represent the amount of sales and let t represent the total earnings. $40{,}000 + 0.20s = t$

2b. $40{,}000 + 0.20s = 65{,}000$, $s = 125{,}000$. Stewart must sell $125,000 worth of electronics to make this salary.

2c. Answers may vary. The sales manager's earnings can be represented using this equation. He makes a base salary of $52,000 plus 30% on his sales. His total earnings are $82,000.

3a. $r = 1.20w + 2.50$

3b. $r = 12.58$. The retail price of the chair will be $12.58.

3c. $w = 9$. The wholesale price of the chair is $9.00.

Assignment Answers

4. $0.9x - 6.3 = 4.5$, $x = 12$. The number is 12.
5. $32 = \frac{x}{5} + 18$, where x represents the total amount of money Craig and his friends split among themselves. $x = 70$. The friends split $70 among themselves.
6. $500 = p - 50 - 0.20p$, where p represents the original price of the laptop. $p = 687.50$. The original price of the laptop was $687.50

7a. $x = 4$

7b. $x = -2$

7c. $y = 44$

7d. $a = 49$

7e. $b = -6$

7f. $h = 3$

7g. $x = \frac{-13}{2}$

7h. $x = -2$

7i. $c = 4$

7j. $x = -9.5$

8a. $y = \frac{c - ax}{b}$

8b. $g = \frac{2(h - 160t)}{t^2}$

Stretch

1. $x = 4.4$
2. $x = \frac{-9}{4}$

Review

1. $x = 3$
2. $x = 9$
3. $\frac{17}{49}$
4. 4.16
5. $\frac{-19}{20}$
6. $12.\overline{3}$

4. What is a number that when you multiply it by 0.9 and subtract 6.3 from the product, you get 4.5? Write and solve an equation to solve the riddle.
5. Craig and four of his friends had a car wash to earn some extra money. They split the profits and Craig got an extra $18 to repay his parents for the car wash supplies. If Craig got $32, how much total money did they split among themselves? Write and solve an equation to answer the question.
6. Susana bought a laptop for $500. It was marked $50 off because it was out of the box and slightly scratched. She also got a 20% student discount, which was taken off the original price. What was the original price of the laptop? Write and solve an equation to answer the question.
7. Solve each equation. Check your solution.
   a. $1 = 3x - 11$
   b. $7x + 2 = -12$
   c. $9 = \frac{y}{4} - 2$
   d. $13 - \frac{a}{7} = 6$
   e. $-5b - 12 = 18$
   f. $-8 = 2h - 14$
   g. $-3(2x + 7) = 18$
   h. $-14 = -2(5 - x)$
   i. $45.99c - 50 = 133.96$
   j. $1.1x + 2.35 = -8.1$
8. Solve each equation for the indicated variable.
   a. $ax + by = c$, for y
   b. $h = \frac{1}{2}gt^2 + 160t$, for g

Stretch

Solve each equation. Check your solution.

1. $1.95(6.2 - 3x) - 4.81 = -18.46$
2. $\frac{2}{3}\left(x - \frac{5}{2}\right) - \frac{7}{6} = -\frac{13}{3}$

Review

Solve each equation using a double number line model.

1. $4x - 5 = 7$
2. $\frac{1}{3}x + 2 = 5$

Evaluate each expression for the indicated value.

3. $-\frac{1}{2}a^2 + \frac{5}{6}a$, for $a = \frac{6}{7}$
4. $-5.3r - 7.6 + 0.4r$, for $r = -2.4$

Determine each quotient.

5. $2\frac{3}{8} \div -2\frac{1}{2}$
6. $-14.8 \div -1.2$