TEXAS MATH SOLUTION

Accelerated Grade ɔ

Module 2 Topic 2 Lesson 3

Formally Yours

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Formally Yours

Using Inverse Operations to Solve Equations

WARM UP

Solve each equation.

1. $2.3p = -11.73$

2. $\frac{3}{4}r = 10$

3. $y + 5.92 = 1.63$

4. $7\frac{2}{5} + t = 3\frac{1}{4}$

LEARNING GOALS

- Use properties of equality to solve equations.
- Write two-step equations.
- Solve two-step equations of the form $px + q = r$ and $p(x + q) = r$ with efficiency.
- Check solutions to equations algebraically.
- Solve literal equations for specific variables.

KEY TERMS

- two-step equation
- literal equation

You have solved equations using double number lines. How can you use the Properties of Equality and inverse operations to solve equations?

How Does that Work?

Recall that to solve an equation means to determine the value or values for a variable that make the equation true. In the process of solving equations, you must always maintain equality, using the Properties of Equality.

Properties of Equality	For all numbers a, b, and c, . . .
Addition Property of Equality	If $a = b$, then $a + c = b + c$.
Subtraction Property of Equality	If $a = b$, then $a - c = b - c$.
Multiplication Property of Equality	If $a = b$, then $ac = bc$.
Division Property of Equality	If $a = b$ and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$.

1. Solve $2x + 6 = 13$ using a double number line model.

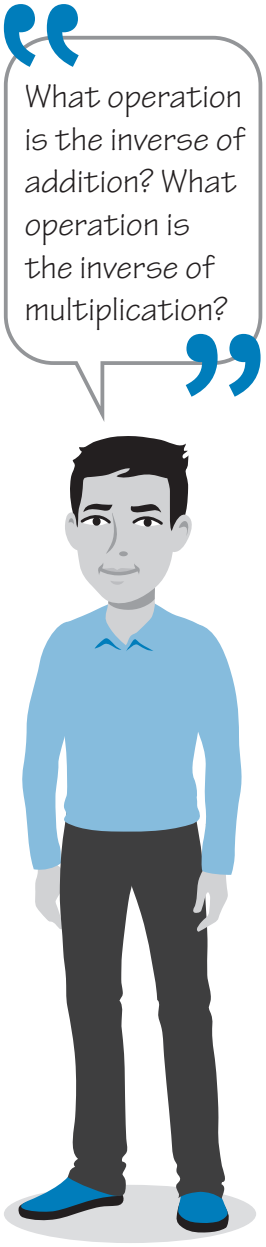
2. Explain which Properties of Equality you used in the process of solving the equation.

ACTIVITY
3.1

Strategies for Applying Inverse Operations

Throughout this topic, you have written and solved *two-step equations*. A **two-step equation** requires two inverse operations, or applying two Properties of Equality, to isolate the variable.

Demaryius, Calvin, and Isaac each solved $2x + 6 = 13$ in a different way. Analyze their solution strategies.



Demaryius

$$2x + 6 = 13$$
$$\frac{2x + 6}{2} = \frac{13}{2}$$
$$\frac{2(x + 3)}{2} = \frac{13}{2}$$
$$x + 3 = 6.5$$
$$-3 = -3$$
$$x = 3.5$$

Calvin

$$2x + 6 = 13$$
$$\frac{2x}{2} + 6 = \frac{13}{2}$$
$$x + 6 = 6.5$$
$$-6 = -6$$
$$x = 0.5$$

Isaac

$$2x + 6 = 13$$
$$-6 = -6$$
$$2x = 7$$
$$\frac{2x}{2} = \frac{7}{2}$$
$$x = 3.5$$

1. Compare the strategies used by Demaryius and Calvin.

2. Compare the strategies used by Demaryius and Isaac.

3. Solve each equation by first applying either the Addition or Subtraction Property of Equality.

a. $56 = -10 + 2x$

b. $6x + 25 = 79$

To make the addition and subtraction simpler, you can leave fractions in improper form. They already have a common denominator.

c. $38 = 4x - 14$

d. $13 + \frac{x}{3} = 35$

4. Solve each equation by first applying either the Multiplication or Division Property of Equality.

a. $56 = -10 + 2x$

b. $6x + 25 = 79$

c. $38 = 4x - 14$

d. $13 + \frac{x}{3} = 35$

Consider the equations and your solutions in Questions 3 and 4.

5. Do you prefer one order over the other? If so, why? If your preference changes depending on the equation, explain why.

ACTIVITY
3.2

Writing and Solving Two-Step Equations

1. Shelly is throwing a graduation party. She is sending invitations to her friends and family. She finds a company that charges $6 for a 10-pack of personalized invitations, plus a $5 shipping fee for the entire order, no matter how many 10-packs are ordered. Shelly wants to calculate the cost of an order, based on the number of packs of invitations she orders.

a. Define variables for the two quantities that are changing in this scenario.

b. Write an equation that represents the total cost of any order based on the number of packs of invitations.

c. Use your equation to determine how many packs of invitations are ordered if the total is $53. What about if the total is $29?

2. **Pete's Garage charges $45 per hour for labor when performing auto repairs. The office manager must have the cost of parts and the hours of each job ticket to complete the bills for the customers.**

   a. **Define variables for the *three* quantities that are changing in this scenario.**

   b. **Write an equation that represents the total cost of the auto repairs.**

   c. **Assume that for a given car, the cost of the parts is $101. Use your equation to determine how many hours the mechanic worked on the car if the total bill was $269.75.**

3. **Felicia's Pet Grooming charges $15 for each dog washed and groomed on the weekend. The cost of the dog shampoo and grooming materials for a weekend's worth of grooming is $23.76. Felicia is interested in her weekend profits.**

   a. **Define variables for the two quantities that are changing in this scenario.**

   b. **Write an equation that represents the total profits based on the number of dogs groomed.**

   c. **Use your equation to determine how many dogs Felicia groomed if her profits were $261.24.**

4. Frankie works as a pet sitter all week long but he is more in demand on some days than others. He posts his rates as $12 per visit plus a surcharge, which depends on the day. On his busiest days, Frankie can serve 8 houses for pet sitting. He is interested in his daily profits.

   a. Define variables for the two quantities that are changing in this scenario.

   b. Write an equation that represents the maximum total profits based on the surcharge for that day. Write your equation in the form $a(x + b) = c$.

   c. Beverly and Sean are trying to determine Frankie's Saturday surcharge per house if he makes $142. Beverly thinks the first step in solving the equation is to divide by the coefficient of the parentheses. Sean thinks the first step is to distribute that value through the parentheses. Who's correct?

   d. Determine the Saturday surcharge by solving the equation you wrote in part (b). What is the total fee Frankie charges for pet sitting on a Saturday?

ACTIVITY
3.3 Solving Equations with Efficiency

A savvy mathematician (you!) can look at an equation, see the structure of the equation, and look for the most efficient solution strategy.

As you have seen, there are multiple ways to solve equations. Sometimes an efficient strategy involves changing the numbers in the equation—in mathematically appropriate ways!

1. Analyze each correct solution strategy to the equation $1.1x + 4.3 = 6.2$.

Sherry

$$1.1x + 4.3 = 6.2$$
$$1.1x + 4.3 - 4.3 = 6.2 - 4.3$$
$$1.1x = 1.9$$
$$x = \frac{1.9}{1.1}$$
$$x = \frac{19}{11}$$

Maya

$$1.1x + 4.3 = 6.2$$
$$11x + 43 = 62$$
$$11x + 43 - 43 = 62 - 43$$
$$11x = 19$$
$$x = \frac{19}{11}$$

Remember, to maintain equality, any operation applied to one side of the equation must be applied to the other side of the equation.

a. Explain how the two solutions strategies are alike and how they are different.

b. What Property of Equality did Maya apply before she started solving the equation?

2. Brian used Maya's strategy to solve the equation $2.6x - 1.4 = 38$. Identify his mistake and then determine the correct solution.

Brian

$$2.6x - 1.4 = 38$$
$$26x - 14 = 38$$
$$26x = 52$$
$$x = 2$$

3. Use Maya's strategy to solve each equation. Then check your solution in the original equation.

a. $-9.6x + 1.8 = -12.3$ **b. $2.99x - 1.4 = 13.55$**

Now let's consider strategies to solve two different equations that contain fractions.

WORKED EXAMPLE

	$\frac{11}{3}x + 5 = \frac{17}{3}$	$\frac{1}{2}x + \frac{3}{4} = 2$
Step 1:	$3\left(\frac{11}{3}x + 5\right) = 3\left(\frac{17}{3}\right)$	$4\left(\frac{1}{2}x + \frac{3}{4}\right) = 4(2)$
Step 2:	$11x + 15 = 17$	$2x + 3 = 8$
Step 3:	$x = \frac{17 - 15}{11}$	$x = \frac{8 - 3}{2}$
	$= \frac{2}{11}$	$= \frac{5}{2}$

You should be fluent in operating with decimals and fractions, but these strategies can ease the difficulty of the calculations when solving equations.

4. Answer each question about the strategies used to solve each equation in the Worked Example.

a. Explain Step 1. Why might this strategy improve your efficiency with solving equations?

b. What property was applied in Step 2?

c. Explain Step 3.

NOTES

5. Louise used the strategy from the Worked Example to solve $3 = \frac{1}{4}x - \frac{1}{4}$. Identify her mistake and determine the correct solution.

Louise

$3 = \frac{1}{4}x - \frac{1}{4}$

$3 = 4\left(\frac{1}{4}x - \frac{1}{4}\right)$

$3 = x - 1$

$4 = x$

6. Use the strategy from the Worked Example to solve $\frac{2}{3}x + \frac{4}{5} = \frac{5}{3}$. Check your solution in the original equation.

Consider the solution strategies used to solve two more equations.

WORKED EXAMPLE

	$-20x + 80 = 230$	$-38 = -6x - 14$
Step 1:	$10(-2x + 8) = 10(23)$	$-2(19) = -2(3x + 7)$
Step 2:	$-2x + 8 = 23$	$19 = 3x + 7$
Step 3:	$x = \frac{23 - 8}{-2} = -\frac{15}{2}$	$\frac{19 - 7}{3} = x$
		$4 = x$

7. Answer each question about the strategies used to solve each equation in the Worked Example.

a. How is the strategy used in this pair of examples different from the strategies used in Questions 1 and 2?

b. When might you want to use this strategy?

c. Use the strategy from the Worked Example to solve $44x - 24 = 216$. Check your solution in the original equation.

ACTIVITY
3.4

Solving Literal Equations

You have already learned a lot of important formulas in mathematics. These formulas are also *literal equations*. **Literal equations** are equations in which the variables represent specific measures. Common literal equations occur in measurement and geometry concepts.

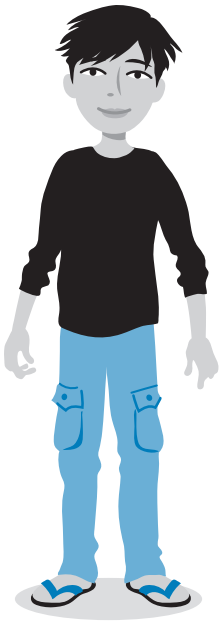
1. The formula to convert from degrees Celsius to degrees Fahrenheit is $F = \frac{9}{5}C + 32$.

a. Calculate the temperature in Celsius, if it is 39°F.

b. Calculate the temperature in Celsius, if it is 25°F.

c. Solve the equation for the temperature in Celsius.

To solve for a variable means to isolate that variable on one side of the equation with a coefficient of 1.



2. The formula for the perimeter of a rectangle can be written as $P = 2l + 2w$, where l and w represent the length and width of the rectangle.

   a. Rewrite the formula by factoring out the coefficient of the variables.

   b. Next, solve the equation for the length.

   c. Solve the equation in part (a) for the width.

   d. How are the equations in parts (b) and (c) alike? Explain why this makes sense.

3. The formula for the area of a trapezoid can be written as $A = \frac{1}{2}(b_1 + b_2)h$, where b_1 and b_2 are the lengths of the bases and h is the length of the height of the trapezoid.

   a. Rewrite the formula as a product of two factors.

   b. Solve the equation for the height of the trapezoid.

   c. Solve the equation in part (a) for one of the bases.

d. When would it be helpful to solve the trapezoid area formula for one of the bases?

4. Solve each equation for the specified variable.

a. $S = 2\pi rh + 2\pi r^2$ **for** h

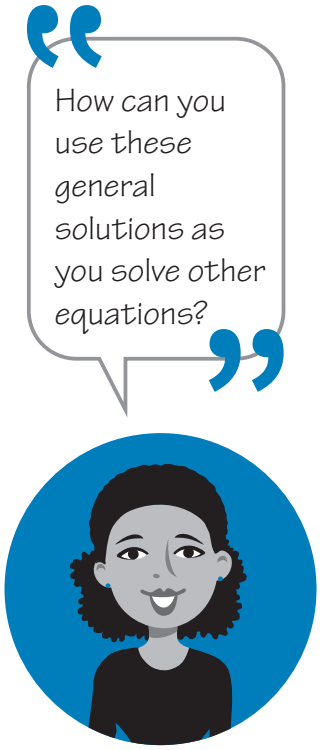
b. $V = \pi r^2 h + \frac{2}{3}\pi r^3$ **for** h.

Throughout this topic, you have solved many linear equations.

5. Analyze each general form.

a. Write a general solution for equations of the form $ax + b = c$ **by solving the equation for** x.

b. Write a general solution of the form $a(x + b) = c$ **by solving the equation for** x.



ACTIVITY
3.5 Solving More Equations

Number riddles are popular types of problems to solve using two-step equations.

1. Solve each number riddle by writing and solving an equation.

a. What is a number that when you multiply it by 3 and subtract 5 from the product, you get 28?

b. What is a number that when you multiply it by 4 and add 15 to the product, you get 79?

c. Make a number riddle for a partner to solve.

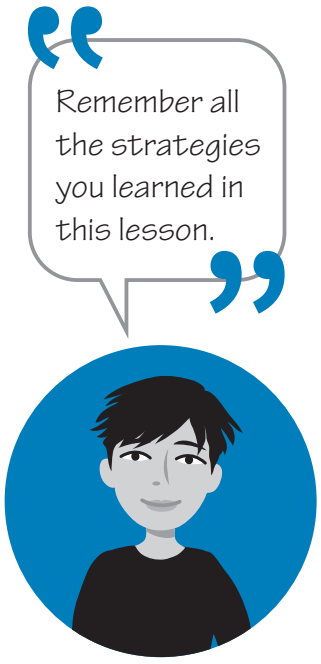
2. Solve each equation. Check your solutions.

a. $2 + 7x = 16$

b. $5 + \frac{x}{2} = 16$

c. $-17 = 2x - 8$

d. $0.5x - 0.3 = 0.2$



e. $-\frac{1}{4} - \frac{1}{2}x = -\frac{19}{4}$

f. $-\frac{2}{5}x + 4 = 18$

NOTES

g. $-5 = -3(x + 11)$

h. $8(x + 6) = 18$

i. $\frac{1}{2}(5 - x) = \frac{1}{4}$

j. $6.4 = 1.2(4 + 2x)$

TALK the TALK

Get Creative

1. Any equation in the form $ax + b = c$ can be solved in two steps, but do you need to write out both steps to solve the equation?

   a. Isolate the variable x, so that it has a coefficient of 1.

   b. Use your answer from part (a) to solve $4x + 5 = 61$.

NOTES

2. Similarly, any equation in the form $a(x + b) = c$ can be solved without writing out both steps of the two-step solution process.

   a. Isolate the variable x, so that it has a coefficient of 1.

   b. Use your answer to part (a) to solve $4(x - 7) = 20$.

3. Write a real-world situation that can be modeled by each equation.

   a. $3b - 5 = 22$

   b. $19 = 2.5 + 4.5n$

   c. $\frac{1}{2}t + 2 = 16$

Assignment

LESSON 3: Formally Yours

Write

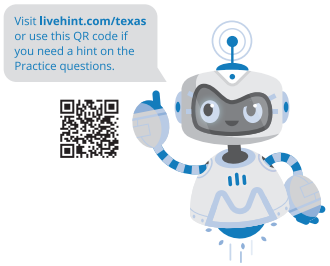
Explain the process of solving a two-step linear equation.

Remember

You can use the Properties of Equality to rewrite equations and increase your efficiency with solving equations.

- If the equation contains fractions, you can multiply both sides of the equation by the least common denominator.
- If the equation contains decimals, you can multiply both sides of the equation by a multiple of 10.
- If the equation contains large values, you can divide both sides of the equation by a common factor.

Practice



1. Madison Middle School has a Math and Science Club that holds meetings after school. The club has decided to enter a two-day competition that involves different math and science challenges. The first day of competition involves solving multi-step math problems. Teams will receive two points for every problem they get correct in the morning session and three points for every question they get correct in the afternoon session.
   a. Write an equation to represent the situation. Remember to define your variable(s).
   b. The team scores four points in the morning session, but finishes the day with 28 points. Solve the equation and interpret the solution in the context of the problem.
   c. The second day of the competition was the science portion, involving hands-on science problems. Each correct science problem is worth 5 points. If the team started the day with 28 points and ended with 53 points, how many science problems did they get correct? Write and solve an equation to answer the question.
2. Employees at Driscoll's Electronics earn a base salary plus a 20% commission on their total sales for the year. Suppose the base salary is $40,000.
   a. Write an equation to represent the total earnings of an employee. Remember to define your variable(s).
   b. Stewart wants to make $65,000 this year. How much must he make in sales to achieve this salary? Write and solve an equation to answer this question.
   c. Describe the equation $52{,}000 + 0.3s = 82{,}000$ in terms of the problem situation.
3. The manager of a home store is buying lawn chairs to sell at his store. Each pack of chairs contains 10 chairs. The manager will sell each chair at a markup of 20% of the wholesale cost, plus a $2.50 stocking fee.
   a. Write an equation that represents the retail price of a chair, r, in terms of the wholesale price, w.
   b. Use your equation to calculate the retail price of the chair if the wholesale price is $8.40.
   c. Use your equation to calculate the wholesale price if the retail price is $13.30.

4. What is a number that when you multiply it by 0.9 and subtract 6.3 from the product, you get 4.5? Write and solve an equation to solve the riddle.
5. Craig and four of his friends had a car wash to earn some extra money. They split the profits and Craig got an extra $18 to repay his parents for the car wash supplies. If Craig got $32, how much total money did they split among themselves? Write and solve an equation to answer the question.
6. Susana bought a laptop for $500. It was marked $50 off because it was out of the box and slightly scratched. She also got a 20% student discount, which was taken off the original price. What was the original price of the laptop? Write and solve an equation to answer the question.
7. Solve each equation. Check your solution.

   a. $1 = 3x - 11$
   b. $7x + 2 = -12$
   c. $9 = \frac{y}{4} - 2$
   d. $13 - \frac{a}{7} = 6$
   e. $-5b - 12 = 18$
   f. $-8 = 2h - 14$
   g. $-3(2x + 7) = 18$
   h. $-14 = -2(5 - x)$
   i. $45.99c - 50 = 133.96$
   j. $1.1x + 2.35 = -8.1$

8. Solve each equation for the indicated variable.

   a. $ax + by = c$, for y
   b. $h = \frac{1}{2}gt^2 + 160t$, for g

Stretch

Solve each equation. Check your solution.

1. $1.95(6.2 - 3x) - 4.81 = -18.46$
2. $\frac{2}{3}\left(x - \frac{5}{2}\right) - \frac{7}{6} = -\frac{13}{3}$

Review

Solve each equation using a double number line model.

1. $4x - 5 = 7$
2. $\frac{1}{3}x + 2 = 5$

Evaluate each expression for the indicated value.

3. $-\frac{1}{2}a^2 + \frac{5}{6}a$, for $a = \frac{6}{7}$
4. $-5.3r - 7.6 + 0.4r$, for $r = -2.4$

Determine each quotient.

5. $2\frac{3}{8} \div -2\frac{1}{2}$
6. $-14.8 \div -1.2$