TEXAS MATH SOLUTION

Accelerated Grade 6

Module 2 Topic 3 Lesson 1

Many Ways to Measure

Student Edition

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1 Many Ways to Measure

Using Ratio Reasoning to Convert Units

WARM UP

Answer each question about a common measurement conversion.

1. How many inches are in 1 foot?
2. How many feet are in 1 yard?
3. How many grams are in 1 kilogram?
4. How many milliliters are in 1 liter?
5. How many centimeters are in 1 meter?
6. How many fluid ounces are in 1 cup?
7. How many quarts are in 1 gallon?
8. Which of the previous questions include units that are part of the U.S. customary system of measurement, and which include units that are part of the metric system?

LEARNING GOALS

- Use ratio reasoning with double number lines to convert measurement units.
- Use ratio reasoning with ratio tables to convert measurement units.
- Use scaling up or scaling down to convert and transform measurement units appropriately.
- Use unit analysis to convert and transform measurement units appropriately.

KEY TERM

- convert

In previous grades, you have worked with the U.S. customary system and the metric system of measurement. This year, you have also learned about ratios. How can you use ratio reasoning to convert from one measurement unit to another in order to solve problems?

Customary to Whom?

In the U.S., customary units are primarily used for business, personal, and social purposes. Sciences, including the medical field, use the metric system.

You've learned about the relationships between inches and feet, feet and yards, quarts and gallons, meters and millimeters—to name a few.

1. Name a U.S. customary system unit and a metric system unit that would be an appropriate size to measure each object or quantity.

Object/Quantity	U.S. Customary System	Metric System
Your height		
Length of your pencil		
Distance from your school to the beach		
Weight of your math book		
Amount of water in a bottle		
Amount of water in a swimming pool		

2. Circle the most appropriate measurement for each item.

a. The weight of a dog
- 15 pounds
- 18 ounces
- 1 ton
- 25 fluid ounces

b. The amount of gas in a car's tank
- 50 milliliters
- 2 kiloliters
- 55 liters
- 12 kiloliters

c. The height of your classroom
- 90 inches
- 1 mile
- 2 yards
- 12 feet

d. The height of a basketball hoop
- 3 meters
- 70 centimeters
- 500 millimeters
- 1 kilometer

ACTIVITY
1.1 Reasoning About Unit Conversions

You can use more than one measurement to describe the same length, weight, or capacity. For example, you may say that a football field is 100 yards long or 300 feet long. You could also say that the football field is about 90 meters long. In each case, the lengths are the same—you just say them in different ways.

There are many situations in which you need to *convert* measurements to different units. To **convert** a measurement means to change it to an equivalent measurement in different units.

When you convert a measurement to a different unit, the size of the object does not change; only the units and the number of those units change.

1. Name a situation in which converting one measurement to another would be necessary or useful.

Before you start converting units, it is useful to estimate the number of units to expect in a conversion. A few estimates comparing common metric and U.S. customary measures are given.

- One meter is about the same length as one yard.
- One inch is about 2.5 centimeters.
- One kilometer is a little more than half of a mile.
- One foot is about 30 centimeters.
- One liter is about the same as one quart.
- One kilogram is a little more than 2 pounds.

Use the estimates given and your knowledge of metric and U.S. customary measures to answer each question.

2. The numeric value of which measurement will be greater?

a. The length of a table in inches or in feet

b. The length of a table in meters or in centimeters

c. The length of a table in meters or in yards

d. The distance from school to your house in miles or in kilometers

e. The weight of your math book in kilograms or in pounds

3. How did you decide which value would be greater in Question 2?

4. Estimate each measurement conversion.

a. The distance to Toronto is 548 km. About how many miles is that?

b. You order 5 kilograms of food pellets for your guinea pig. About how many pounds are you ordering?

5. Describe the strategies you used to estimate each measurement conversion in Question 4.

Because most conversions compare two quantities using multiplicative strategies, the conversion estimates provided and the conversions within systems that you already know can be written using ratio language. They can also be written symbolically in terms of equality.

Ratio Language	Symbolically
For every inch, there are approximately 2.5 centimeters.	1 in. ≈ 2.5 cm
For every meter, there is approximately 1 yard.	1 m ≈ 1 yd
For every foot, there are approximately 30 centimeters.	1 ft ≈ 30 cm
For every 12 inches, there is exactly 1 foot.	12 in. = 1 ft
For every 1 kilometer, there are exactly 1000 meters.	1 km = 1000 m

The ≈ symbol means that the two values are approximately equal.

When a conversion ratio is presented for use in converting between units of measure, it is often written as an equation: 12 in. = 1 ft. However, it can also be written as a ratio in fractional form: $\frac{12 \text{ in.}}{1 \text{ ft}}$.

Because conversions compare two quantities that are measured in different units, conversion ratios can also be called conversion rates.

6. Rewrite each common conversion using ratio language and as a ratio in fractional form.

a. 3 ft = 1 yd

b. 5280 ft = 1 mi

c. 1 lb ≈ 0.45 kg

d. 4 qt = 1 gal

e. 1 m = 100 cm

f. $\frac{1}{1000}$ m = 1 mm

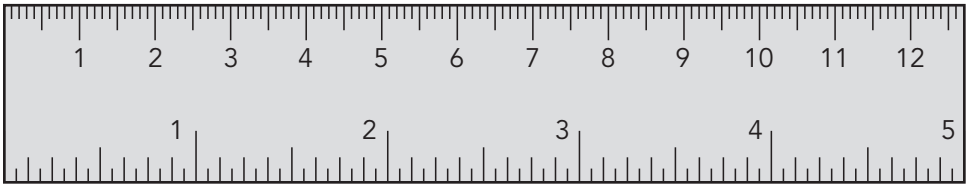
Because these measurement conversions are ratios, you can use ratio reasoning to convert between units. For example, you can determine the number of miles in a 10-kilometer race or the number of fluid ounces in 500 milliliters of a solution.

ACTIVITY
1.2

Using Double Number Lines to Convert Units

When you learned about ratios, you learned how to use double number lines to determine equivalent ratios. You can also use double number lines to convert from one unit to another.

Although you may not have realized it before, many rulers are set up as double number lines and can be used to convert between inches and centimeters.



1. Determine which scale represents inches and which represents centimeters. How did you decide? Label the scales on the ruler.

2. Use the ruler as a double number line to determine each approximate conversion.

a. 1 cm ≈ _____ in.

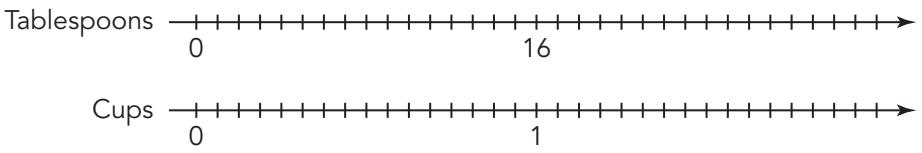
b. 1 in. ≈ _____ cm

c. 5 cm ≈ _____ in.

d. 3 in. ≈ _____ cm

You are baking cookies at your friend's house. After searching the cupboards and drawers, you cannot find the measuring cups, but you can find the tablespoon.

3. Use the double number line to determine how many tablespoons you need of each ingredient in the recipe.



a. 2 cups of sugar

b. $1\frac{3}{4}$ cups of flour

c. $\frac{1}{2}$ cup of raisins

4. Suppose you had found the cup but not the tablespoon. Use the double number line to determine how many cups you need if the recipe calls for 2 tablespoons of vanilla extract.

You want to redecorate your bedroom and need to measure the room for new carpeting, paint, and a border on the walls. You realize that you have only a meter stick. You measure the room, but you need to know the dimensions in feet to purchase the materials. You record these measurements:

- The length of the room is 5 meters.
- The width of the room is 4 meters.
- The height of the room is 2.5 meters.

1 meter ≈ 3.28 feet

5. Use a double number line to determine the measurement of each dimension in feet.

a. length **b. width** **c. height**

ACTIVITY
1.3
Using Ratio Tables and Scaling to Convert Units

You can use ratio tables, as you did when determining equivalent ratios, as another strategy for converting units.

1. Complete the ratio table by converting between pounds and ounces.

Pounds	1	2		$1\frac{1}{4}$	$\frac{1}{2}$		
Ounces	16		4			6	40

2. What strategies did you use to determine the unknown values?

3. Complete the ratio table by converting between milliliters and liters.

Milliliters	1000	100		50	1		575
Liters	1		0.5			0.01	

4. What strategies did you use to determine the unknown values?

Ratio tables are helpful tools for converting within a given system of measurement. Scaling up or down is a similar strategy for determining equivalent ratios that can be more easily used to convert from one unit of measurement to another.

You will use the common conversions shown in the table to convert between customary and metric systems.

Length	Mass	Capacity
1 in. = 2.54 cm	1 oz = 28.35 g	1 pt = 0.47 L
1 cm = 0.39 in.	1 g = 0.035 oz	1 L = 2.11 pt
1 ft = 30.48 cm	1 lb = 0.45 kg	1 qt = 0.95 L
1 m = 3.28 ft	1 kg = 2.2 lb	1 L = 1.06 qt
1 mi = 1.61 km		1 gal = 3.79 L
1 km = 0.62 mi		1 L = 0.26 gal
1 m = 39.37 in.		
1 in. = 0.0254 m		
1 m = 1.09 yd		

Most conversions that require moving between the U.S. customary and metric systems are approximations, so, in general, you will use conversion rates rounded to the nearest hundredth in your calculations.

NOTES

Scaling up or down is another strategy that you already know that can be used to convert between units.

WORKED EXAMPLE

You can use scaling up to determine how many kilograms are in 2.5 pounds. Because you want to determine the number of kilograms for a specific number of pounds, use the conversion rate 1 lb = 0.45 kg or $\frac{1 \text{ lb}}{0.45 \text{ kg}}$.

$$\frac{1 \text{ lb}}{0.45 \text{ kg}} = \frac{2.5 \text{ lb}}{? \text{ kg}} \longrightarrow \frac{1 \text{ lb}}{0.45 \text{ kg}} \overset{\times 2.5}{=} \frac{2.5 \text{ lb}}{1.125 \text{ kg}}$$

5. Why was the conversion rate $\frac{1 \text{ lb}}{0.45 \text{ kg}}$ used rather than the rate $\frac{2.2 \text{ lb}}{1 \text{ kg}}$?

Use scaling up or down to answer each question.

6. The school cafeteria has eight very large cans of tomato sauce for making pizza. Each can contains 2 gallons of sauce. Is there more or less than 50 L of sauce in these 8 cans?

7. Tyrone, the quarterback for the Tigers Football team, can throw a football 40 meters. Jason, the quarterback for the Spartans, can throw a football 45 yards. Who can throw farther? How do you know?

8. Molly says that she is 1.5 meters tall. Shawna is 5 feet tall. Molly says that she is taller, but Shawna disagrees. Who is correct? Explain your reasoning.

9. Larry weighs 110 pounds, Casey weighs 98 pounds, Shaun weighs 42 kg, and Jamal weighs 52 kg. Place the boys in order from the least weight to the greatest weight using pounds and kilograms.

10. Karen has a gold bracelet that weighs 24 grams. She wants to sell the bracelet, but she needs a minimum of one ounce of gold to sell it. Can Karen sell her bracelet? Why or why not?

ACTIVITY
1.4 Using Unit Analysis to Convert Units

In unit analysis, note how how the units are carried through all calculations. Units are divided out in the same way that factors can be divided out.

$$\cancel{\text{Given unit}} \times \frac{\text{desired unit}}{\cancel{\text{given unit}}} = \text{desired unit}$$

To use scaling up or down to convert one unit to another, you set up a proportion and use the conversion rate based on the given measurement that you are converting. In another strategy, *unit analysis,* you are multiplying by a form of 1 to rewrite the given measurement in a different unit.

WORKED EXAMPLE

Determine the quantity in pounds that is equivalent to 4.5 kilograms.

Scaling Up

$$\frac{1 \text{ kg}}{2.2 \text{ lb}} \overset{\times 4.5}{=} \frac{4.5 \text{ kg}}{? \text{ lb}}$$

Unit Analysis

$$4.5 \text{ kg} \left(\frac{2.2 \text{ lb}}{1 \text{ kg}}\right)$$

$$\frac{4.5 \cancel{\text{kg}}}{1} \left(\frac{2.2 \text{ lb}}{1 \cancel{\text{kg}}}\right) = 9.9 \text{ lb}$$

$$\frac{1 \text{ kg}}{2.2 \text{ lb}} = \frac{4.5 \text{ kg}}{9.9 \text{ lb}}$$

$$4.5 \text{ kg} = 9.9 \text{ lb}$$

1. Analyze the Worked Example.

a. Both strategies used a form of 1 to determine the equivalent number of pounds in 4.5 kilograms. How is the form of 1 used in scaling up different from the form of 1 used in unit analysis?

b. Why are the labels for kilograms crossed out in the unit analysis strategy?

Christopher and Max want to determine the number of miles in 31,680 feet using unit analysis.

NOTES

Christopher

$31{,}680\ \text{ft}\left(\frac{5280\ \text{ft}}{1\ \text{mi}}\right) = 167{,}270{,}400\ \text{mi}$

Max

$31{,}680\ \text{ft} \cdot \frac{1\ \text{mi}}{5280\ \text{ft}} = 6\ \text{mi}$

2. Explain why Christopher's answer is not reasonable.

3. Explain what is different in how Christopher and Max set up their multiplication problem. What is important about how the units are arranged in the conversion rates?

Use unit analysis to convert each unit of measurement. Check to make sure your answer is reasonable.

4. A giraffe is 18 feet tall. How tall is the giraffe in inches?

5. **A giraffe is 174 inches tall. How tall is the giraffe in feet?**

6. **The length of the school playground is 32 yards. How many feet long is the playground?**

A marathon is a long-distance foot race with an official distance of 42.195 kilometers (26 miles and 385 yards) that is usually run as a road race. Larger marathons can have tens of thousands of runners. Most of these marathon runners are not professional marathoners but run to raise funds for various charities.

7. **Although a marathon is a popular distance for a race, there are many other distances in which runners can train to race. Complete the table shown by writing the unknown measurements.**

Race	Kilometers	Miles
Short Distance	5	
Medium Distance	10	
Medium Distance	20	
Half Marathon		13.1
Ultramarathon	100	
Ironman Triathlon Swim		2.4
Ironman Triathlon Bike		112

Conversion rates are also common in other contexts, like currency. During the 2016 Summer Olympics, the currency exchange rate between the U.S. dollar and the Brazilian real (pronounced "ray-all") was $1 US for every 3.17 BRL.

8. Alejandra's family went to the Rio de Janeiro Olympics and she budgeted $500 to spend while she was gone.

a. Write the conversion rate: _____ US = _____ BRL.

b. Did Alejandra budget more or less than 500 BRL? Explain.

c. How many BRL could she spend in Rio de Janeiro?

d. After Rio de Janeiro, Alejandra's family traveled to Mexico, where 1 BRL was equal to 5.92 pesos. If Alejandra had 295 BRL remaining, how many pesos did she have?

9. Emma is preparing to re-carpet her room. She measured the room to be 6 yards long and 8 yards wide. When she got to the carpet store, all of the measurements were in square feet.

a. Determine how many square yards of carpet Emma needs to buy to re-carpet her room.

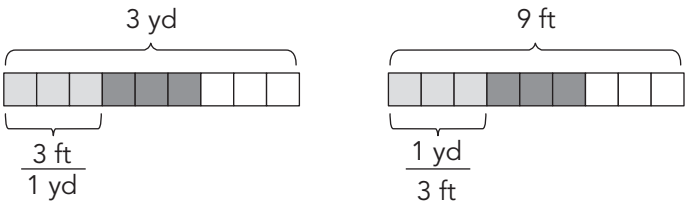
b. Determine how many square feet of carpet Emma needs to buy to re-carpet her room. How can you check your answer?

NOTES

TALK the TALK

Larger or Smaller?

1. Compare the two conversions. How are they similar? How are they different?



2. When you convert a measurement with smaller units to a measurement with larger units, does the number of units increase or decrease?

3. When you convert a measurement with larger units to a measurement with smaller units, does the number of units increase or decrease?

NOTES

4. What information is always needed to convert between measurement units?

For each conversion, explain which strategy you prefer to use and then convert the units.

5. 12 gal = _____ L

6. 240 oz = _____ lb

7. 0.380 km = _____ m

8. 324 in. = _____ yd

Assignment

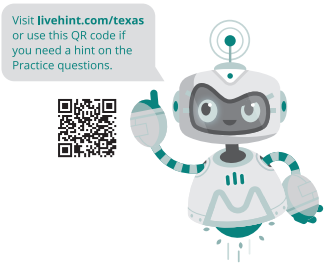
Write

Explain how to convert from one unit to another using ratio reasoning.

Remember

More than one unit can be used to describe the same length, weight, or capacity. To convert units means to change a measurement to an equivalent measurement in different units. You can use models, ratio reasoning, and unit analysis to convert units using conversion rates.

Practice



Use any strategy to convert between the specified units.

1. Janine will be traveling to Botswana, where the unit of currency is called the *pula*, which means "rain" in the local language. Suppose that $1 is equivalent to 7 pula.
   a. If Janine has $500 to spend in Botswana, how many pula will she have to spend?
   b. The safari lodge where she is staying in Chobe National Park costs 434 pula each night. What is the cost per night in dollars?
   c. When she goes to dinner at the safari lodge, the bill comes to 91 pula. How many dollars did Janine spend on dinner?
2. Jonah is going to the hardware store for his Uncle Frederick. He needs to buy 4 yards of electrical wire and 14 quarts of liquid nails.
   a. The store only sells wire by the foot. How many feet does Jonah need?
   b. The store only sells liquid nails by the gallon. How many gallons does Jonah need?
3. Jin Lee is volunteering at a zoo and is helping weigh a penguin's egg. The egg weighs 0.15 kilogram.
   a. Is this more or less than the average weight of 145 grams? Explain.
   b. If Jin Lee expands the penguin area to be about 500 meters wider than it is now, how many more kilometers wide is the area?
4. Harold is buying a new car. Some of the cars he has researched provide measurements in the U.S. customary system and some provide measurements in the metric system.
   a. One car manufacturer reports the mass of the car to be 3307 lb. How many kilograms is this?
   b. Another manufacturer recommends that the owner change the oil every 12,075 kilometers. After how many miles should the owner change the oil?
   c. Harold is a tall man and prefers cars with high ceilings. One car lists 43.3 inches of headroom and another car lists 99.3 centimeters of headroom. Which car has more headroom?
   d. He is concerned about the fuel tank capacity of the new car he wants to buy. He commutes a long distance to work every day and does not want to constantly be filling the tank. He finds 3 cars that he likes online. The Skyte has a fuel capacity of 19 gallons. The Madrid has a fuel capacity of 64.4 liters, and the Cougar has a fuel capacity of 63.6 quarts. Compare the fuel tank capacities of the cars using both gallons and liters. Order the cars from least to greatest fuel tank capacity.

5. A group of 4 campers must navigate through the forest using compasses, topographic maps, and other devices. They scatter and each of them travels to a different location. Using the clues below, determine how far it is from the start to each point on the map.
   - The distance to point A is 1.5 kilometers.
   - It is 0.5 more miles to get to point *B* from the start than to point *A*.
   - The total distance to points *A* and *D* from the start is 3.1 miles.
   - The distance from the start to point *C* is twice the distance from the start to point *B*.

   a. How many kilometers is it from the start to each location?

   b. How many miles is it from the start to each location?
6. A zip line activity is part of an obstacle course that a group of students must get through together. There are several zip lines on the course, the longest of which is about 72 meters long. How can this be stated using the most appropriate unit in the customary system? Show your work.

Stretch

Anthony measured the dimensions of a rectangular box to be 45 cm by 35 cm by 2 m.

1. Determine the volume of the box in cubic meters.
2. Convert the volume of the box to cubic centimeters.

Review

1. At Union Middle School, 99 girls, or 33% of the girls, play basketball. How many girls attend Union Middle School?
2. Kasey gets a 35% employee discount on anything she buys at The Foot Parade. If Kasey got a $5.25 discount on her new flip-flops, how much did they cost originally?
3. Mr. Hawkins manages a small store called Action Sporting Goods. He wants to make sure that his store is stocked with enough equipment for all of the community sports. He surveys 240 of his customers and asks them to choose the one sport that they're most likely to buy sports equipment for this season.

Sport	Percent of Responses
Basketball	30%
Baseball	20%
Football	35%
Wrestling	15%

   a. How many of the surveyed customers will need baseball equipment?

   b. How many of the surveyed customers will need wrestling equipment?

4. Estimate each quotient to the nearest whole number. Then calculate the quotient.

   a. $0.796 \div 9.95$

   b. $23.84 \div 6.4$