TEXAS MATH SOLUTION

Accelerated Grade 6

Module 1 Topic 4 Lesson 1

Depth, Width, and Legnth

Teacher's Implementation Guide

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Depth, Width, and Length

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Deepening Understanding of Volume

MATERIALS

Scissors

Lesson Overview

In this lesson, students are introduced to geometric solids. Students will investigate various figures and sort them based on the definition of a polygon or a polyhedron. The intent of this lesson is for students to determine the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths. In addition, they will review and practice decimal multiplication by calculating volumes of right rectangular prisms.

Grade 6
Expressions, Equations, and Relationships

(8) The student applies mathematical process standards to use geometry to represent relationships and solve problems. The student is expected to:

(C) write equations that represent problems related to the area of rectangles, parallelograms, trapezoids, and triangles and volume of right rectangular prisms where dimensions are positive rational numbers.

(D) determine solutions for problems involving the area of rectangles, parallelograms, trapezoids, and triangles and volume of right rectangular prisms where dimensions are positive rational numbers.

ELPS

1.A, 1.C, 1.D, 1.F, 2.C, 2.D, 2.G, 2.H, 2. I, 3.A, 3.B, 3.C, 3.D, 3.E, 3.J, 4.A, 4.B, 4.C, 4.D, 4.F, 4.K, 5.E, 5.G

Essential Ideas

- A polygon is a closed figure formed by three or more line segments.
- A polyhedron is a three-dimensional figure that has polygons as faces.
- A regular polyhedron is a three-dimensional solid that has congruent regular polygons as faces and has congruent angles between all faces.
- A cube is a regular polyhedron whose six faces are congruent squares.

- A unit cube is a cube that is one unit in length, one unit in width, and one unit in height.
- Volume is the amount of space occupied by an object.
- The formula for the volume of a cube is $V = lwh$, where l is the length, w is the width and h is the height, or $V = Bh$, where B is the area of the base and h is the height.
- When multiplying decimals, the number of decimal places in the product is equal to the sum of the decimal places in the factors.

Lesson Structure and Pacing: 2 Days

Day 1

Engage

Getting Started: Common Figures

Students analyze and sort various figures to develop an understanding of the characteristics of shapes.

Develop

Activity 1.1: Name that Figure

Students investigate cubes and other right rectangular prisms as polyhedra and explore parts of polyhedra, such as faces, edges, and vertices. Students also identify real-world objects that can be represented with right rectangular prisms.

Day 2

Activity 1.2: Volume of Rectangular Prisms

Students calculate the volumes of cubes with fractional edge lengths. They investigate how to pack a rectangular prism with fractional edge lengths using cubes of the appropriate unit fraction edge length. Students explore how changing the dimensions of the interior cubes does not change the volume.

Activity 1.3: Volume Formulas

Students recall the formula for the volume of a right rectangular prism and apply the formula to solve problems involving rectangular prisms with positive rational number dimensions. In this activity students review the rules and strategies for multiplying decimals.

Demonstrate

Talk the Talk: Fractionally Full

Students calculate the volume of a right rectangular prism by packing it with unit cubes of the appropriate unit fraction edge length. They will solve real world problems involving volume with fractional and decimal edge lengths.

ENGAGE

Getting Started: Name that Figure

Facilitation Notes

In this activity, students analyze and sort various figures to develop an understanding of the characteristics of shapes. This activity is designed to get students thinking about the attributes shared among figures.

Have students complete this activity with a partner. Then share the groupings as a class. Allow students to sort the figures any way that makes sense to them as long as they can state their reasoning. Vocabulary will be introduced in the first activity and students will reuse these figures. As students share their groupings listen for the vocabulary they are using.

Possible groupings

- Two-dimensional figures and three-dimensional solids
- Figures that do or do not contain dashed line segments
- Figures that do or do not contain sides
- Figures that are rectangular prisms or cones or pyramids
- Figures that are or are not polygons
- Figures that are open or closed

Differentiation strategies

To scaffold support for students,

- Explain why dashed lines are used in some of the figures.
- Provide 3-dimensional models for the 3-dimensional figures.

Questions to ask

- How can you determine if a figure with no measurement labels is a square or a rectangle?
- Does this figure have a name?
- What is the name of this figure?
- What do the dashed line segments represent in Figures 3 and 7?
- Dashed line segments might have appeared in which other figures?
- How many sides are there in Figures 2, 3, and 8?

Summary

There are many ways to describe figures. A common language is important when studying mathematics and describing geometric figures.

Activity 1.1
Name that Figure

DEVELOP

Facilitation Notes

In this activity, students investigate cubes and other right rectangular prisms as polyhedra and explore parts of polyhedra, such as faces, edges, and vertices. In this activity students will also identify real-world objects that can be represented with right rectangular prisms.

Ask a student to read the definitions and discuss as a class. Allow students to work in pairs or groups to sort the figures from the Getting Started into three groups: Polygons, Polyhedrons, or Neither. Discuss their responses to Question 1 as a class.

Questions to ask

- Explain why each figure is a polygon.
- What is the fewest number of line segments that a polygon can have? What is the greatest number of line segments that a polygon can have?
- Explain why each figure is a polyhedron.
- Describe the faces of each polyhedron.
- How many faces does each polyhedron have? Can you see all the faces from the diagram?
- What is the fewest number of faces that a polyhedron can have? What is the greatest number of faces that a polyhedron can have?
- Explain why some figures do not fit either definition.

Differentiation strategies

- To scaffold support for students, provide 3-dimensional models and use them to help explain the definitions.

As an extension to the activity,

- Provide other examples and non-examples, such as an oblique rectangular prism, a triangular prism, a cylinder and cone. Ask students why they do or do not fit the definitions addressed in this lesson.
- Have students investigate the relationship among vertices, edges and faces of polyhedron, and then check their findings by researching Euler's formula $V - E + F = 2$.

Misconceptions

Limited student experiences may lead students to believe that all prisms look like those that refract light (with two triangles that must be on the sides) and that all pyramids have square bases like those in Egypt. Contradict their misconceptions with discussion and examples.

Ask a student to read the descriptions for each polyhedron shown. Additional vocabulary is introduced. Students will be representing three-dimensional shapes using nets made up of rectangles and triangles later in this topic. Answer Questions 2 through 7 as a class.

Questions to ask

- What are congruent parallel faces?
- Is it possible to draw a cube or rectangular prism such that all faces are visible?
- Is there an alternate method for drawing a right rectangular prism or cube? Sketch one.
- Locate and label the length, width, and height of each prism.
- Are there any times that items are referred to as cubes but aren't really cubes by definition?
- Is an ice cube really a cube by definition?
- Are there any objects in this room that are shaped like right rectangular prisms?
- What other words have you used to describe a right rectangular prism?

Summary

When describing and grouping geometric shapes, figures, and solids, a common mathematical vocabulary is necessary. Distinguishing between polygons and polyhedra introduces the unit cube which sets the stage for determining volume.

Activity 1.2

Volume of Rectangular Prisms

Facilitation Notes

In this activity, students calculate the volumes of cubes with fractional edge lengths. They investigate how to pack a rectangular prism with fractional edge lengths using cubes of the appropriate unit fraction edge length. Students explore how changing the dimensions of the interior cubes does not change the volume.

Ask a student to read the introduction and then discuss the formula for the volume of a cube. Have students work with a partner to answer Questions 1 and 2.

Questions to ask

- What is the unit of measure used to describe the volume of the cube?

- What is the relationship between the unit of measure of a side length and the unit of measure of the volume of the solid?
- What are the rules for multiplying fractions?
- What are the rules for multiplying mixed numbers?
- If you are given the volume of a cube, how are the dimensions determined?

Have students read through the Worked Example on their own and then discuss.

Questions to ask

- What is a unit fraction?
- How do you determine the LCM of two or more numbers?
- What is the rule for dividing fractions by fractions?
- How many $\frac{1}{2}$ inch cubes would fit into the right rectangular prism?
- If the dimensions of the right rectangular prism did not change, would the volume change?

Differentiation strategies

To scaffold support for students,

- Provide a point of reference by physically packing a prism with cubes.
- Draw a rectangular prism on graph paper to help visualize fractional side lengths.

Answer Question 3 as a class. Then have students use the method from the Worked Example to answer Question 4 with their partner.

Questions to ask

- What is the volume of a cube with a $\frac{1}{8}$ inch width?
- How many $\frac{1}{8}$ inch cubes would fit into the right rectangular prism?
- What unit fraction did you use? Why?
- How many cubes of that unit fraction could you pack into the rectangular prism?
- What was the volume of each?
- When you use the volume formula, did you get the same answer?
- If you choose a different unit fraction to use, would that change the volume of the rectangular prism?

Differentiation strategies

To scaffold support for students,

- Have them concentrate on solving only part (a).
- Provide a template of the Worked Example with a third column for students to show their work for the current problem.

To extend the activity, ask:

- How does the volume of a cube change if you double the side length?
- How does the volume of a cube change if you reduce the side length by one-half?

Summary

Unit cubes with fractional side lengths can be used to determine the volume of right rectangular prisms with fractional side lengths.

Activity 1.3
Volume Formulas

Facilitation Notes

In this activity, students recall the formula for the volume of a right rectangular prism and apply the formula to solve problems involving rectangular prisms with positive rational number dimensions. Students review the rules and strategies for multiplying decimals.

Ask a student to read aloud the introduction about the formula used to determine the volume of a rectangular prism. Discuss as a class the worked example for multiplying decimals. Estimating gives students a rough idea of what an answer should be, so it does not have to be precise. This activity provides students with an opportunity to gain fluency in multiplying decimals. Have students complete the questions without a calculator.

Questions to ask about the right rectangular prism

- What unit of measurement describes length, width, and height?
- What unit of measurement describes the area of the base, *B*?
- What unit of measurement describes the volume?

Differentiation strategies

To scaffold support for students,

- Use a stack of paper to demonstrate the formula $V = Bh$.
- Use a milk crate to help students visualize a cubic foot.
- Address the concept of base, and the fact that in a rectangular prism, the base can be any of the 3 parallel faces.

To extend the activity,

- Provide models of prisms other than those with rectangular bases.

- Have students calculate the volume of prisms with triangular or trapezoidal bases.
- Have students extend the volume formula for use with cylinders.

Questions to ask about the Worked Example

- Is there only one correct way to estimate?
- How does multiplying decimals compare to multiplying whole numbers?
- What is the rule used to place the decimal in the appropriate place in the product of two factors?
- Does the rule work for the product of any two factors?
- If estimation is used first before computing the product of two decimals, will you need to use that rule?

Discuss and complete Question 1 as a class.

Have students work in a pair or in groups to complete Questions 2 through 5. Discuss responses as a class.

Questions to ask

- How did you determine where to place the decimal point in the answer?
- What is the rule used to place the decimal in the appropriate place in the product of two factors?
- Does the rule work for the product of any two factors?

Summary

When multiplying decimals, the number of decimal places in the product is equal to the sum of the decimal places in the factors. Right rectangular prisms with equal dimensions have equal volumes. Estimating before calculating reduces mathematical errors.

Talk the Talk: Fractionally Full

DEMONSTRATE

Facilitation Notes

In this activity, students calculate the volume of a right rectangular prism by packing it with unit cubes of the appropriate unit fraction edge length. They solve real world problems involving volume with fractional and decimal edge lengths.

Have students complete Questions 1 through 4 and then discuss as a class.

Questions to ask for Question 1

- Which unit fraction was used to determine the volume?
- How did you determine which unit fraction to use?
- How many $\frac{1}{8}$ inch cubes fit in the right rectangular prism?
- What is the volume of the right rectangular prism?
- Verify the volume of the right rectangular prism by multiplying the length times the width times the height.

Questions to ask for Question 2

- What is the volume of the cubic boxes in which Haley packages her earrings?
- How many inches are equivalent to $\frac{1}{6}$ of a foot? $\frac{1}{3}$ of a foot?
- How did you determine how many cubic boxes will fit into the shipping box?

Questions to ask for Question 3

- What are the dimensions of the floor of the closet?
- Does this situation require you to determine area or volume?
- How was the amount of carpeting determined?
- What unit of measure was used to determine the area of the floor?
- Will the $\frac{1}{2}$ foot cubic boxes fill the storage closet?

Questions to ask for Question 4

- What is the difference between estimating the volume and determining the volume?
- How did you estimate the volume for part (a)?
- How did you estimate the volume for part (b)?

Summary

The formula $V = Bh$ is used to calculate the volume of any prism, where B represents the area of the base of the prism and h represents the height. Volume is estimated before it is calculated.

Depth, Width, and Length

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Deepening Understanding of Volume

WARM UP

Determine the least common multiple of the numbers in each pair.

1. 2, 10
2. 3, 8
3. 6, 14
4. 10, 15

LEARNING GOALS

- Determine the volume of right rectangular prisms with fractional edge lengths using unit cubes with unit fractional dimensions.
- Connect the volume formulas $V = lwh$ and $V = Bh$ with a unit-cube model of volume for rectangular prisms.
- Apply the formulas $V = lwh$ and $V = Bh$ to determine volumes in real-world problems.
- Fluently add, subtract, and multiply multi-digit decimals using the standard algorithms.

KEY TERMS

- point
- line segment
- polygon
- geometric solid
- polyhedron
- face
- edge
- vertex
- right rectangular prism
- cube
- pyramid
- volume

You know about three-dimensional figures such as cubes and other rectangular prisms. You also know how to operate with positive rational numbers. How can you use what you know to calculate measurements of any rectangular prism, even one with fractional edge lengths?

Warm Up Answers

1. 10
2. 24
3. 42
4. 30

Answers

Answers will vary.

Getting Started

Common Figures

Cut out the cards found at the end of the lesson. Sort the figures into two or more groups. Name each category and be prepared to share your reasoning.

2 • TOPIC 4: Decimals and Volume

ACTIVITY **1.1**

Name that Figure

It is important to speak a common language when studying mathematics.

A word you may have used in the past may actually have a more precise definition when dealing with mathematics. For example, the word *point* has many meanings outside of math. However, the mathematical definition of **point** is a location in space. A mathematical point has no size or shape, but it is often represented by using a dot and is named by a capital letter. A **line segment** is a portion of a line that includes two points and all the points between those two points. Knowing these definitions will help you learn the meanings of other geometric words.

Recall, a **polygon** is a closed figure formed by three or more line segments.

A **geometric solid** is a bounded three-dimensional geometric figure.

A **polyhedron** is a three-dimensional solid figure that is made up of polygons. A **face** is one of the polygons that makes up a polyhedron. An **edge** is the intersection of two faces of a three-dimensional figure. The point where multiple edges meet is known as a **vertex** of a three-dimensional figure.

Let's revisit the different figures you sorted.

1. **Sort the figures into one of these three categories and explain your reasoning.**

Polygon | **Polyhedron** | **Neither**

Answers

1. Polygon; Each is a closed figure formed by three or more line segments. Figure 1 and Figure 6

   Polyhedron; Each figure is a geometric solid made of polygons. Figure 2, Figure 3, Figure 8

   Neither
   Figure 4; This figure is not made of line segments, so it is neither a polygon nor a polyhedron.

   Figure 5; This is not a closed figure, so it is neither a polygon nor a polyhedron.

   Figure 7; This geometric solid is not made of line segments or polygons, so it is neither a polygon nor a polyhedron.

ELL Tip

There is a lot of potentially new vocabulary in this lesson. In the activity, students are only going to be asked to sort polyhedrons and polygons. However, this activity also involves the terms *point, line segment, geometric solid, face, edge,* and *vertex*. Create a handout that has all of the terms listed in one column, a blank cell for "definition" in the second, and a blank cell for "drawing" in the third. Have students read this page independently, then ask the class to fill out their charts with the definition and an example drawing. This graphic organizer will help ELL students keep this vocabulary straight.

Answers

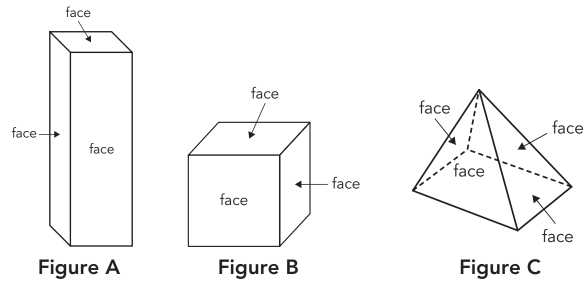
2. Each of the faces are polygons. In Figure A, the faces are rectangles or possibly squares. In Figure B, the faces are squares. In Figure C, the faces are triangles, and the base is a rectangle.
3. The congruent parallel faces are: front-back, left-right, and bottom-top.

4a. The front, top, and right faces of the cube are visible. The back, bottom, and left faces are not visible.

4b. The length, width, and height are all equal.

4c. A cube also has 3 pairs of congruent parallel rectangular faces. Those faces are all squares, which are rectangles.

Three polyhedra are shown.



A unit cube is a cube whose sides are all 1 unit long.

Figure A is a *right rectangular prism.* A **right rectangular prism** is a polyhedron with three pairs of congruent and parallel rectangular faces.

Figure B is an example of a *cube*, which is a special kind of right rectangular prism. A **cube** is a polyhedron that has congruent squares as faces.

Figure C is an example of a *rectangular pyramid*. A **pyramid** is a polyhedron with one base and the same number of triangular faces as there are sides of the base.

2. Describe the different faces of each polyhedron.

3. Study the right rectangular prism. Identify the three pairs of congruent parallel faces.

4. Study the cube.

a. Describe the locations of the cube faces you can see and the locations of the faces you cannot see.

b. What do you know about the length, width, and height of the cube?

c. Describe how the cube is also an example of a right rectangular prism.

5. **Compare the numbers of faces, edges, and vertices of the cube and the other right rectangular prism. Write what you notice.**

6. **Study the rectangular pyramid. How do the faces of the rectangular pyramid differ from the faces of the rectangular prisms?**

7. **List examples in the real-world objects that are shaped like right rectangular prisms or pyramids.**

NOTES

ACTIVITY 1.2 Volume of Rectangular Prisms

Volume is the amount of space occupied by an object. The volume of an object is measured in cubic units.

The volume of a cube is calculated by multiplying the length times the width times the height.

Volume of a cube = $l \times w \times h$

1. **Calculate the volume of each cube with the given side length.**

   a. $\frac{9}{10}$ **centimeter** b. $1\frac{1}{3}$ **centimeters**

2. **Suppose a cube has a volume of 27 cubic meters. What are the dimensions of the cube?**

Answers

5. Both the cube and the other right rectangular prism have 6 faces, 12 edges, and 8 vertices.
6. The triangular pyramid only has 1 rectangular face. The others faces are triangles.
7. Answers will vary.

Answers

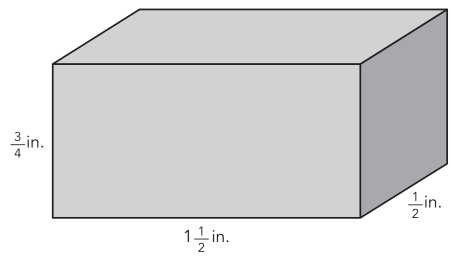
1a. $\frac{729}{1000}$ cubic centimeters

1b. $2\frac{10}{27}$ cubic centimeters

2. The cube is 3 m long, 3 m wide, and 3 m tall.

To determine the volume of a rectangular prism, you can also pack the prism with cubes. You may have done this in elementary school.

Consider the rectangular prism shown. What do you notice about the side lengths? Can you determine its volume by packing it with cubes?



WORKED EXAMPLE

To determine the volume of the right rectangular prism with dimensions $1\frac{1}{2} \times \frac{1}{2} \times \frac{3}{4}$, you can fill the prism with cubes. However, the unit cubes that you may have used in elementary school will not work here. Instead, smaller unit cubes with fractional side lengths are required.

Assign a unit fraction to the dimensions of each cube. Use the least common multiple (LCM) of the fraction denominators to determine the unit fraction.	LCM(2, 4) = 4 So, each cube will measure $\frac{1}{4}$ in. $\times \frac{1}{4}$ in. $\times \frac{1}{4}$ in. The volume of each unit cube is $\frac{1}{64}$ cubic inches.
Determine the number of cubes needed to pack the prism in each dimension.	length: $1\frac{1}{2} \div \frac{1}{4} = 6$ width: $\frac{1}{2} \div \frac{1}{4} = 2$ height: $\frac{3}{4} \div \frac{1}{4} = 3$
Determine the number of cubes that make up the right rectangular prism.	$6 \times 2 \times 3 = 36$
Multiply the number of cubes by the volume of each cube to determine the volume of the right rectangular prism.	$36 \times \frac{1}{64} = \frac{36}{64}$ $= \frac{9}{16}$

The volume of the right rectangular prism is $\frac{9}{16}$ cubic inches.

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NOTES

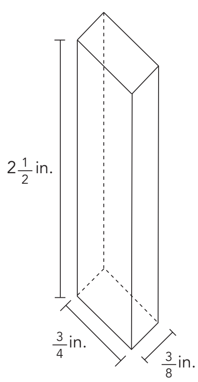
3. **Interpret the Worked Example.**

   a. **How was the number of cubes needed to pack the prism in each dimension determined?**

   b. **Instead of cubes with a width of $\frac{1}{4}$ inch, suppose you used cubes each with a width of $\frac{1}{8}$ inch. How does this change the volume of the rectangular prism?**

4. **Use the method from the worked example to determine the volume of each rectangular prism.**

   a.



   b. **$1\frac{3}{4}$ in. by $2\frac{1}{3}$ in. by $\frac{1}{2}$ in.**

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Answers

3a. The number of cubes needed to pack the prism was determined by dividing each dimension (length, width, and height) by the side length of the unit cube, $\frac{1}{4}$. These numbers were then multiplied.

3b. Volume does not change.Cubes with width $\frac{1}{8}$ in. have a volume of $\frac{1}{512}$ cubic inches.

$\frac{3}{2} \div \frac{1}{8} = 12, \frac{1}{2} \div \frac{1}{8} = 4,$
$\frac{3}{4} \div \frac{1}{8} = 6$
$12 \times 4 \times 6 = 288$
$288 \times \frac{1}{512}$ cubic inches $= \frac{9}{16}$ cubic inches

4a. The length of $\frac{3}{4}$ in. can be packed with six $\frac{1}{8}$-in. cubes. The width of $\frac{3}{8}$ in. can be packed with three $\frac{1}{8}$-in. cubes. The height of $\frac{5}{2}$ in. can be packed with twenty $\frac{1}{8}$-in. cubes. This gives $6 \times 3 \times 20 = 360$ cubes, each with a volume of $\frac{1}{512}$ cubic inches: $360 \times \frac{1}{512}$ cubic inches $= \frac{45}{64}$ cubic inches.

4b. LCM(4, 3, 2) = 12

The unit fraction would be $\frac{1}{12}$. The volume of each unit cube would be $\frac{1}{1728}$ cubic in.

$\frac{7}{4} \div \frac{1}{12} = 21, \frac{7}{3} \div \frac{1}{12} = 28,$
$\frac{1}{2} \div \frac{1}{12} = 6$
$21 \times 28 \times 6 = 3528$
$3528 \times \frac{1}{1728}$ cubic inches
$= \frac{49}{24}$ cubic inches

ACTIVITY 1.3

Volume Formulas

You have calculated the volume of a rectangular prism using the formula $V = lwh$, where V is the volume, l is the length, w is the width, and h is the height. You also know that the area of a rectangle can be calculated using the formula $A = l \cdot w$.

Consider the two formulas:

$$V = l \cdot w \cdot h$$
$$A = l \cdot w$$

If B is used to represent the area of the base of a rectangular prism, then you can rewrite the formula for area: $B = l \cdot w$.

Now consider the two formulas:

$$V = l \cdot w \cdot h$$
$$B = l \cdot w$$

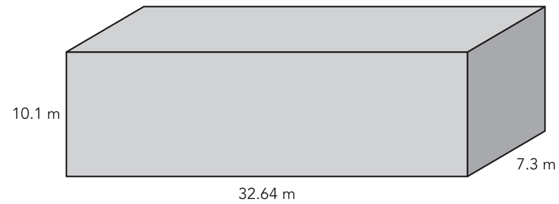
You can use the formula $V = Bh$ to calculate the volume of any prism. However, the formula for calculating the value of B will change depending on the shape of the base.

Using both of these formulas, you can rewrite the formula for the volume of a rectangular prism as $V = B \cdot h$, where V represents the volume, B represents the area of the base, and h represents the height.

In order to calculate the volume of various geometric solids you will need to perform multiplication. In this activity, you will calculate the volume of rectangular prisms with decimal side lengths.

It is good practice to estimate before you actually calculate. If you have an estimate, you can use it to decide whether your answer is correct.

Consider the right rectangular prism shown.



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To calculate the volume of the prism, first calculate the area of the base, *B*, by multiplying 32.64 meters by 7.3 meters.

Kenny said, "I use estimation to help place the decimal point correctly in the product."

WORKED EXAMPLE

The area of the base is 32.64 meters × 7.3 meters.

He estimates his two numbers.

32.64 is close to 30
7.3 is close to 7
$30 \times 7 = 210$

So he knows his product is close to 210, but larger since he rounded down. Next, he calculates the product of 32.64 × 7.3.

$$\begin{array}{r} 32.64 \\ \times \quad 7.3 \\ \hline 9792 \\ \underline{228480} \\ 238.272 \end{array}$$

Kenny knows the product will be close to but greater than 210, so he must place the decimal point after the 8. The area of the base of the rectangular prism is 238.272 square meters.

1. Calculate the volume of the right rectangular prism.

Answers

1. 2406.5472 cubic meters

Answers

2a. 44.488 cubic ft

2b. 4.74 cubic cm

2c. 23.406 cubic m

3a. See below.

3b. The decimal point moves to the right when multiplying by a power of 10 that is greater than 1 (10, 100, 1000, etc.). The decimal point moves to the left when multiplying by a power of 10 less than 1 (0.1, 0.01, 0.001, etc.).

2. Each number sentence represents the base, *B*, times height, *h*, of different rectangular prisms. Complete each number sentence by inserting a decimal point to show the correct volume.

   a. 53.6 sq. ft × 0.83 ft = 44488 cu. ft

   b. 7.9 sq. cm × 0.6 cm = 474 cu. cm

   c. 0.94 sq. m × 24.9 m = 23406 cu. m

3. Casey thought that using a pattern would help her understand how to calculate the product in a decimal multiplication problem.

   a. Complete the table.

Problem	Product	Problem	Product	Problem	Product
32 × 100		3.2 × 100		0.32 × 100	
32 × 10		3.2 × 10		0.32 × 10	
32 × 1		3.2 × 1		0.32 × 1	
32 × 0.1		3.2 × 0.1		0.32 × 0.1	
32 × 0.01		3.2 × 0.01		0.32 × 0.01	
32 × 0.001		3.2 × 0.001		0.32 × 0.001	

   b. Describe any patterns that you notice.

3a.

Problem	Product	Problem	Product	Problem	Product
32 × 100	3200	3.2 × 100	320	0.32 × 100	32
32 × 10	320	3.2 × 10	32	0.32 × 10	3.2
32 × 1	32	3.2 × 1	3.2	0.32 × 1	0.32
32 × 0.1	3.2	3.2 × 0.1	0.32	0.32 × 0.1	0.032
32 × 0.01	0.32	3.2 × 0.01	0.032	0.32 × 0.01	0.0032
32 × 0.001	0.032	3.2 × 0.001	0.0032	0.32 × 0.001	0.00032

4. A rectangular prism with $B = 26$ square centimeters and $h = 31$ centimeters has a volume of 806 cubic centimeters. Use this information to determine the volume of the other rectangular prisms.

a. 2.6 sq. cm × 31 cm

b. 2.6 sq. cm × 3.1 cm

c. 0.26 sq. cm × 3.1 cm

d. 2.6 sq. cm × 0.31 cm

e. 0.26 sq. cm × 31 cm

f. 2.6 sq. cm × 0.031 cm

g. 0.026 sq. cm × 0.31 cm

h. 0.26 sq. cm × 0.31 cm

5. Look at the patterns in Question 4.

a. How can some of the rectangular prisms have the same volume?

b. How can you tell without multiplying which rectangular prisms will have the same volume?

Answers

4a. 80.6 cubic cm

4b. 8.06 cubic cm

4c. 0.806 cubic cm

4d. 0.806 cubic cm

4e. 8.06 cubic cm

4f. 0.0806 cubic cm

4g. 0.00806 cubic cm

4h. 0.0806 cubic cm

5a. The volume of the rectangular prisms can be the same when the relationship between the dimensions is the same, even if the values are different.

5b. The rectangular prisms in Question 4 that have the same total number of decimal places in the base area and height will have the same volume.

Answers

1. Cubes with width $\frac{1}{4}$ in. have a volume of $\frac{1}{64}$ cubic inch.
   $\frac{5}{4} \div \frac{1}{4} = 5$, $1 \div \frac{1}{4} = 4$,
   $\frac{1}{2} \div \frac{1}{4} = 2$
   $5 \times 4 \times 2 = 40$
   40 unit cubes with side length $\frac{1}{4}$ in. could fit in the rectangular prism.
   $40 \times \frac{1}{64}$ cubic inches $= \frac{5}{8}$ cubic inches
2. Cubes with width $\frac{1}{6}$ foot have a volume of $\frac{1}{216}$ cubic ft.
   $\frac{7}{6} \div \frac{1}{6} = 7$, $\frac{1}{3} \div \frac{1}{6} = 2$,
   $\frac{1}{3} \div \frac{1}{6} = 2$
   $7 \times 2 \times 2 = 28$
   Twenty-eight $\frac{1}{6}$-foot cubes would fit in the box.

3a. She will need 12 square feet of carpeting.

3b. The length of $4\frac{1}{2}$ feet can be packed with nine $\frac{1}{2}$-foot cubes. The width of $2\frac{2}{3}$ feet can be packed with five $\frac{1}{2}$-foot cubes. The height of 6 feet can be packed with twelve $\frac{1}{2}$-foot cubes. This gives $9 \times 5 \times 12 = 540$ cubes.

4a. Estimate: (14)(0.1)(2) $= 2.8$, 2.8 cubic feet; 2.679 cubic feet

4b. Estimate: (9)(2.5)(4) $= 90$, 90 cubic inches; 97.65 cubic inches

NOTES

TALK the TALK

Fractionally Full

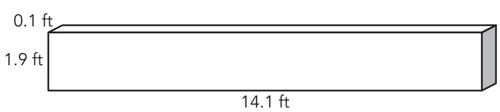
1. Determine the volume of a right rectangular prism with dimensions $1\frac{1}{4}$ feet $\times$ 1 foot $\times$ $\frac{1}{2}$ foot using the unit fraction method you learned in this lesson.

2. Haley makes earrings and packages them into cube boxes that measure $\frac{1}{6}$-foot wide. How many $\frac{1}{6}$-foot cubic boxes can she fit into a shipping box that is $1\frac{1}{6}$ feet by $\frac{1}{3}$ foot by $\frac{1}{3}$ foot?

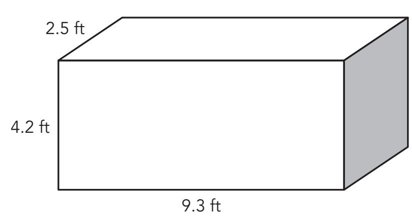
3. The school athletic director has a storage closet that is $4\frac{1}{2}$ feet long, $2\frac{2}{3}$ feet deep, and 6 feet tall.
   a. She wants to put carpet in the closet. How much carpeting will she need?
   b. The athletic director wants to store cube boxes that are $\frac{1}{2}$ foot wide. How many boxes will the storage closet hold?

4. Estimate the volume of each right rectangular prism. Then calculate its volume.

a.



b.



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ELL Tip

Success on this question depends on understanding what a storage closet is and what carpet is. For new ELL students, if they are unaware of what carpet is, they will be unable to identify part a. as an area problem, as opposed to part b., which is a volume problem. Have students draw a picture of each situation in their notebooks, and then check the work of the ELL students before they move on to solving the problems.

Figure 1

Figure 2

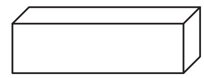


Figure 3

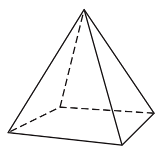


Figure 4

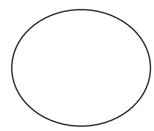


Figure 5

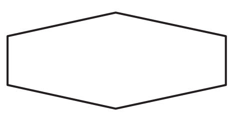


Figure 6

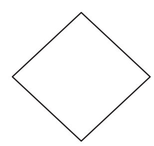


Figure 7

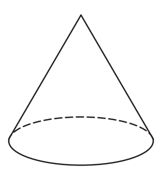
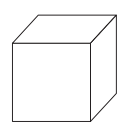


Figure 8



LESSON 1: Depth, Width, and Length • 13

Assignment

LESSON 1: Depth, Width, and Length

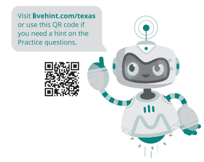
Write

Suppose a rectangular prism has fractional edge lengths. Describe how you can determine the dimensions of cubes that will fill the rectangular prism completely with no overlaps or gaps.

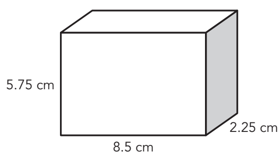
Remember

The volume of a rectangular prism is a product of its length, width, and height:
$V = l \cdot w \cdot h$.

Practice



1. Consider the right rectangular prism shown.

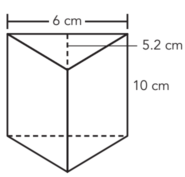


a. List the numbers of faces, edges, and vertices of the rectangular prism.
b. Estimate the volume of the rectangular prism.
c. Calculate the volume of the rectangular prism.

2. Calculate the volume of the rectangular prism with each set of given dimensions.
a. 7 in. × 4 in. × 2 in.
b. 5.2 cm × 5.2 cm × 12 cm
c. 11.3 cm × 3.5 cm × 10.1 cm
d. 4.5 m × 9 m × 6.7 m
e. 2.2 ft × 5.5 ft × 15 ft

Stretch

Calculate the volume for the triangular prism.



Assignment Answers

Write

Possible answer: First, determine the least common multiple of the denominators of the fractional edge lengths. Then, use this as the denominator of a unit fraction for the length, width, and height of each cube. Next, divide the measure of each side by the unit fraction and then calculate the product of the three quotients.

Practice

1a. 6 faces, 12 edges, 8 vertices

1b. Estimates will vary: 108 cubic centimeters

1c. 109.96875 cubic centimeters

2a. 56 cubic inches

2b. 324.48 cubic centimeters

2c. 399.455 cubic centimeters

2d. 271.35 cubic meters

2e. 181.5 cubic feet

Stretch

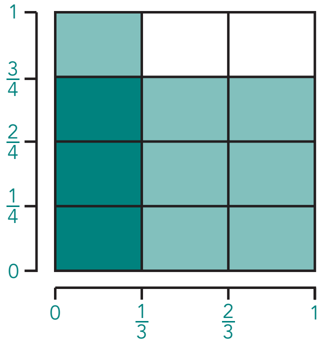
1. 156 cubic centimeters

Assignment Answers

Review

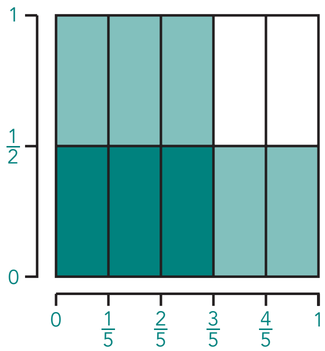
1. about 17 songs
2. 5 campers

3a.



$\frac{3}{4} \times \frac{1}{3} = \frac{3}{12} = \frac{1}{4}$

3b.



$\frac{1}{2} \times \frac{3}{5} = \frac{3}{10}$

4a. 6

4b. 6

5a. 60

5b. 72

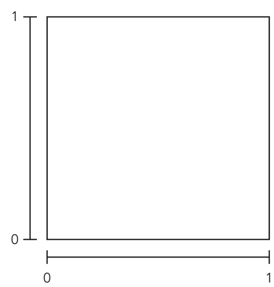
Review

1. Elena wants to put together some of her favorite songs on her computer. She wants to store 60 minutes worth of music. Elena wonders how many songs she will be able to include. She looks online and finds a source that says the average song length is $3\frac{1}{2}$ minutes. If this is true, about how many songs will Elena be able to store? Show your work.
2. Ling is a camp counselor at a local summer camp. She is in charge of the weekly craft activity for 40 campers. Ling plans to make fabric-covered frames that each require $\frac{1}{6}$ yard of fabric. When Ling sets up for her craft activity, she measures the four separate fabric remnants her director gave her. The table shows how much of each fabric she has. How many campers can use plaid fabric? Show your work.

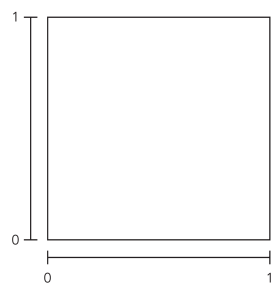
Fabric	Amount (yards)
Plaid	$\frac{11}{12}$
Tie-dyed	$1\frac{7}{9}$
Striped	$2\frac{2}{9}$
Polka-dotted	$1\frac{3}{4}$

3. Represent each product using an area model. Then calculate the product.

a. $\frac{3}{4} \times \frac{1}{3}$



b. $\frac{1}{2} \times \frac{3}{5}$



4. Determine the GCF of each set of numbers.
   a. 72 and 30
   b. 30 and 54

5. Determine the LCM of each set of numbers.
   a. 10 and 12
   b. 8 and 9

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