

Composing and Decomposing Shapes Summary

KEY TERMS

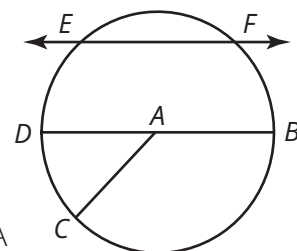
- secant
- major arc
- minor arc
- conjecture
- inscribed angle
- intercepted arc
- tangent
- circumscribed angle
- theorem
- postulate
- diagonal
- coincident
- interior angle of a polygon
- kite
- isosceles trapezoid
- midsegment
- cyclic quadrilateral
- converse
- hypothesis
- conclusion
- biconditional statement
- base angles
- exterior angle
- remote interior angles
- concurrent
- point of concurrency
- circumcenter
- circumscribed circle
- incenter
- median
- centroid
- altitude
- orthocenter

LESSON

1

Running Circles Around Geometry

A diameter (\overline{DB}) of a circle is a line segment passing through the center of the circle with endpoints on the circle. The radius (\overline{AC} , \overline{AD} , \overline{AB}) is the distance from the center of a circle to a point on the circle. A central angle is an angle with its vertex at the center of a circle, like $\angle DAC$. An arc is a part of a circle, like \widehat{DC} . The measure of \widehat{DC} is the measure of the central angle formed by the endpoints. A chord (\overline{EF}) is a segment whose endpoints are points on a circle. A chord is formed by the intersection of the circle and a **secant** line, like \overline{EF} .

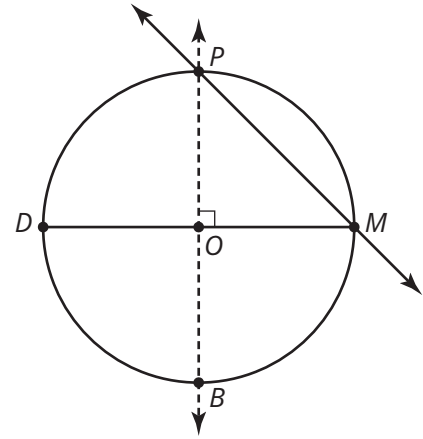


Two points on a circle determine a major arc and a minor arc. The arc with the greater measure is the **major arc**, like \widehat{DBC} . The other arc is the **minor arc**, like \widehat{DC} .

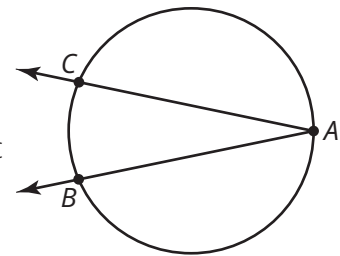
When you **conjecture**, you use what you know through experience and reasoning to presume that something is true. Later, you will prove these things to be true.

Angles POM and DOB are vertical angles. Angles BOM and POD are also vertical angles. Vertical angles are angles that are nonadjacent angles formed by intersecting lines. Vertical angles are also congruent.

Angles POM and POD are a linear pair of angles. Angles DOB and MOB are also a linear pair. A linear pair of angles are two adjacent angles that have noncommon sides that are opposite rays. The angles of a linear pair have measures whose sum is 180° .



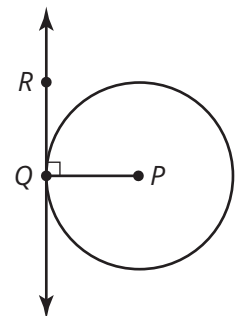
An **inscribed angle** is an angle whose vertex is on a circle and whose sides contain chords of the circle. In this circle, $\angle CAB$ is the inscribed angle. We can also say that \widehat{CB} is an **intercepted arc** because the rays of the angle cross the circle at points C and B . The measure of the intercepted arc is twice that of the inscribed angle.



For example, if $m\angle CAB = 35^\circ$, then $m\widehat{CB} = 70^\circ$.

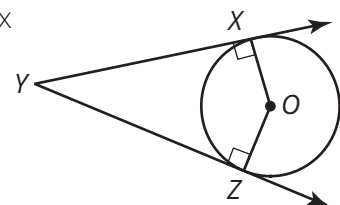
A **tangent** to a circle is a line that intersects the circle in exactly one point, called the point of tangency.

For example, the tangent line \overleftrightarrow{QR} forms a right angle with the radius \overline{PQ} where the two intersect at the point of tangency, point Q .



A **circumscribed angle** has its two sides tangent to the circle with a vertex that is outside of the circle.

For example, $\angle XYZ$ is a circumscribed angle.



In mathematics, you often have to prove a solution is correct. In geometry, theorems are used to verify statements. A **theorem** is a statement that can be proven true using definitions, postulates, or other theorems. A **postulate** is a mathematical statement that is not proved but is considered true.

LESSON

2

The Quad Squad

A quadrilateral may be convex or concave. The quadrilaterals you are most familiar with—trapezoids, parallelograms, rectangles, rhombi, and squares—are convex. A concave quadrilateral has one angle that is greater than 180° .

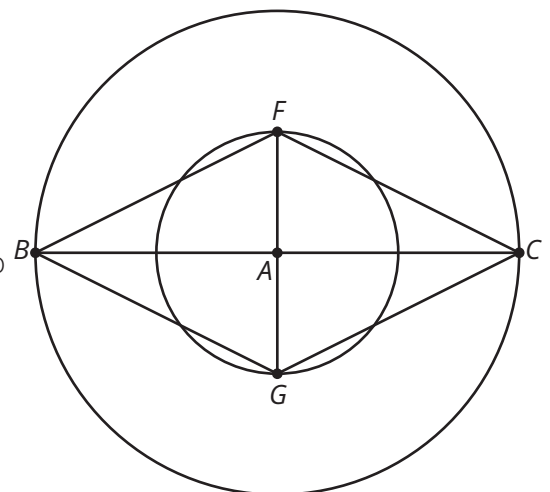
A quadrilateral has exactly two diagonals. A **diagonal** is a line segment that connects non-adjacent vertices of a polygon. The diagonals of any convex quadrilateral create two pairs of vertical angles and four linear pairs of angles. You can use the diameters of circles to draw the diagonals of different quadrilaterals as long as the two segments are not coincident. Two line segments are **coincident** if they lie exactly on top of each other.

An **interior angle of a polygon** is an angle inside the polygon between two adjacent sides. The interior angles of special quadrilaterals have different relationships.

For example, the four interior angles of a rectangle are congruent while only opposite interior angles of a parallelogram are congruent.

A **kite** is a quadrilateral with two pairs of equal adjacent sides. If the diagonals of a quadrilateral are perpendicular, non-congruent, and only one bisects the other, it can only be classified as a kite.

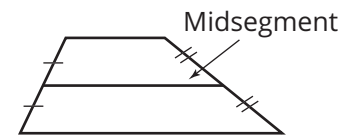
For example, consider the quadrilateral formed using two concentric circles. The inner circle and outer circle have two diameters that bisect each other. Connecting points B , F , C , and G will form a kite that is also a rhombus.



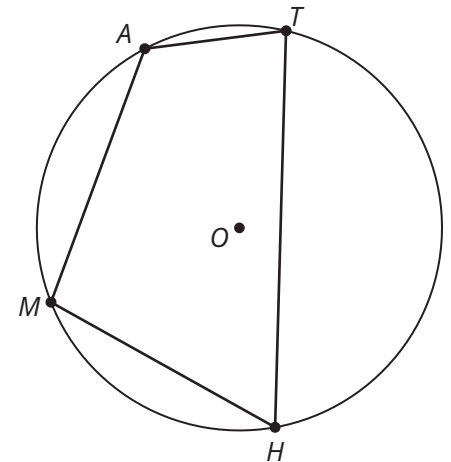
A trapezoid is a quadrilateral with at least one pair of parallel sides, known as the bases. If the other pair of sides are not parallel, they are known as the legs. A special type of trapezoid is an isosceles trapezoid. An **isosceles trapezoid** is a trapezoid with congruent legs.

Property	Parallelogram	Rectangle	Rhombus	Square	Isosceles Trapezoid	Kite
Opposite sides are parallel.	X	X	X	X		
Only one pair of opposite sides is parallel.					X	
Opposite sides are congruent	X	X	X	X		
Only one pair of opposite sides is congruent					X	
Opposite angles are congruent.	X	X	X	X		
At least one pair of opposite angles is congruent.						X
Diagonals bisect each other.	X	X	X	X		
Diagonals are perpendicular.			X	X		X
Diagonals are congruent.		X		X		
All angles are right angles.		X		X		
All sides are congruent.			X	X		
2 pairs of consecutive sides are congruent.			X	X		X

A **midsegment** of a polygon is any line segment that connects two midpoints of the sides of the polygon. The midsegment of a trapezoid that connects the midpoints of the two non-parallel sides of the trapezoid is parallel to each base. The length of this midsegment is equal to half the sum of the lengths of its bases



A **cyclic quadrilateral** is a quadrilateral whose vertices all lie on a single circle. The opposite angles of a cyclic quadrilateral are supplementary. The sum of either pair of opposite angles is 180° .



LESSON

3

Tri- Tri- Tri- and Separate Them

When you divide a quadrilateral along any one of its diagonals, you create two triangles.

The **converse** of an if-then statement is the statement that results from interchanging the **hypothesis** (the “if” part) and the **conclusion** (the “then” part) of the original statement.

For example, consider the conditional statement: “If a polygon has three sides, then the polygon is a triangle.” The hypothesis is, “A polygon has three sides.” The conclusion is, “The polygon is a triangle.” The converse is: “If a polygon is a triangle, then the polygon has three sides.”

When a conditional statement and its converse are both true, this is called a biconditional statement. A **biconditional statement** is a statement written in the form “if and only if p , then q .”

For example, consider the previous conditional statement and its converse. Both statements are true. The biconditional statement is: “If and only if a polygon three sides, then the polygon is a triangle.”

An isosceles triangle has a vertex angle that is formed by the two congruent sides. The congruent sides of an isosceles triangle are called the legs. The other side is called the base and the angles between the base and the congruent sides are called **base angles**.

An equilateral triangle is a triangle in which all three sides are equal in length. All three angle measures of an equilateral triangle are also congruent.

The sum of the measures of the interior angles of a triangle is 180° .

An **exterior angle** of a polygon is an angle that forms a linear pair with an interior angle of the polygon.

For example, $\angle TRB$ is an exterior angle of $\triangle RST$.

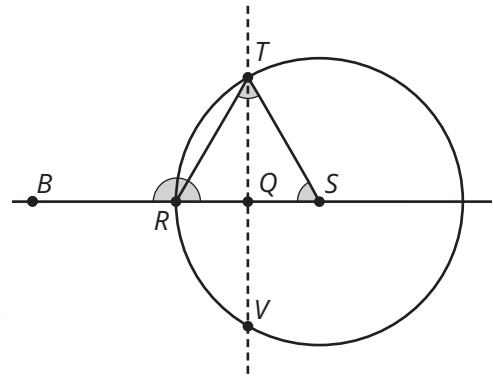
The **remote interior angles** of a polygon are the two angles that are not adjacent to the specified exterior angles.

For example, $\angle RST$ and $\angle STR$ are remote interior angles of $\triangle RST$.

The measure of the exterior angle of a triangle is equal to the sum of the measures of the two remote interior angles.

The sum of the lengths of two sides of a triangle is always greater than the length of the third side.

The midsegment of a triangle is one-half the measure and parallel to the third side.



LESSON

4

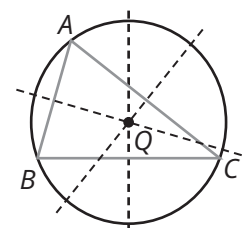
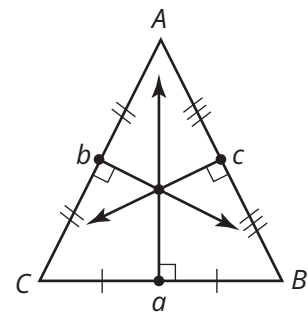
What's the Point?

Concurrent lines, rays, and line segments are three or more lines, rays, or line segments that intersect at a single point. The **point of concurrency** is the point at which concurrent lines, rays, or segments intersect.

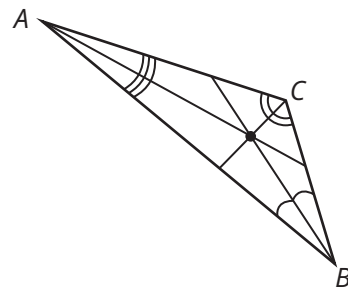
The **circumcenter** is the point of concurrency of the three perpendicular bisectors of a triangle. It is equidistant from the vertices.

A **circumscribed circle** is a circle that passes through all the vertices of a polygon.

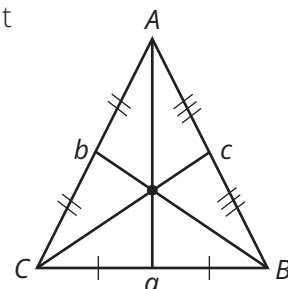
For example, circle Q is circumscribed around $\triangle ABC$.



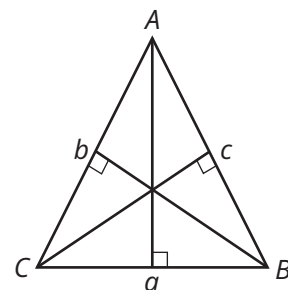
The **incenter** is the point of concurrency of the three angle bisectors of a triangle. It is equidistant from the sides.



A **median** of a triangle is a line segment that connects a vertex to the midpoint of the opposite side. The **centroid** is the point of concurrency of the three medians of a triangle. The centroid of a triangle divides each median into two parts so that the distance from the centroid to the vertex is twice the distance from the centroid to the midpoint of the opposite side.



An **altitude** of a triangle is a line segment that is perpendicular to a side of the triangle and has one endpoint at the opposite vertex. The **orthocenter** is the point of concurrency of the three altitudes of a triangle.



Points of concurrency are a valuable tool in constructing geometric figures. When circumscribing a triangle you can use the circumcenter to identify the center of the circle. The circumcenter is equidistant from each of the vertices, and when the circle passes through each of them, the perpendicular bisectors will become the radii. You can also use the incenter of a triangle to inscribe a circle.

Points of concurrency can lie on the interior of a triangle, the exterior of a triangle, or on the triangle.

	Acute Triangle	Obtuse Triangle	Right Triangle
Incenter	Interior	Interior	Interior
Circumcenter	Interior	Exterior	On Hypotenuse
Centroid	Interior	Interior	Interior
Orthocenter	Interior	Exterior	At Vertex of Right Angle

The incenter, circumcenter, centroid, and orthocenter for an equilateral triangle are the same point.

