Module 4: Seeing Structure

TOPIC 2: APPLICATIONS OF QUADRATIC EQUATIONS

This topic provides students with an opportunity to use what they have learned to model and solve problems in situations involving quadratics. Students start by exploring the complex plane and imaginary numbers. They then investigate a real-world problem that can be modeled by a quadratic inequality. From there, students are given a scenario that can be modeled by a system comprised of a quadratic function and a linear function. They use technology to graph the system and determine the solutions. Students are presented with a real-world situation and use familiar strategies to fit a quadratic regression.

Where have we been?

Students have graphed and solved linear inequalities in one variable and in two variables, as well as systems of linear inequalities. They will use these skills to graph and solve quadratic inequalities, interpreting the solution set in the same way. Students know that a graph represents the solutions to the function it models and that the intersection point(s) of two graphs represent the solution(s) shared by both functions.

Where are we going?

This topic provides students with an opportunity to bring together the advanced techniques that they have learned throughout this course and the previous course. When students move into the next course, their knowledge of first- and second-degree polynomials will be used and expanded upon as they encounter cubics, quartics, rational functions, and logarithmic functions. Modeling advanced scenarios will be heavily used by students who continue on to calculus and post-secondary mathematics.

Quadratic Inequalities

To solve a quadratic inequality, such as $x^2 - 4x + 3 < 0$, you can first solve the corresponding quadratic equation: $x^2 - 4x + 3 = 0$, which will give you the roots $x = 1$ or $x = 3$. Plot the roots to create intervals on the x-axis.

Interval 2 satisfies the original inequality, so the solution includes all numbers between 1 and 3.

Interval 1	Interval 2	Interval 3

× × × × × ×   ✓ ✓ ✓ ✓ ✓   × × × × × ×

Interval 1	Interval 2	Interval 3

Try $x = 0$	Try $x = 2$	Try $x = 4$
$0^2 - 4(0) + 3 < 0$	$2^2 - 4(2) + 3 < 0$	$4^2 - 4(4) + 3 < 0$
$3 < 0$ ✗	$4 - 8 + 3 < 0$	$16 - 16 + 3 < 0$
	$-1 < 0$ ✓	$3 < 0$ ✗

Baby, You're a Firework

Many historians believe fireworks were first created and used in China during the 7th century. To these ancient people, fireworks were a way to scare spirits away—a tradition that is still used for Chinese New Year festivities. Also in this modern time, fireworks are often used during celebrations, and they come in many different shapes and colors. Many aerial fireworks burst at different heights and can travel anywhere from 300 to about 1300 feet in the air!

Talking Points

It can be helpful to understand imaginary and complex numbers for college admissions tests.

Here is an example of a sample question:

Simplify: $\sqrt{-3} + \sqrt{-9} + \sqrt{-16}$.

To solve this problem, you need to understand imaginary numbers and square roots. The imaginary number i is equal to $\sqrt{-1}$. You can rewrite expressions with i just as with any variable.

So, $\sqrt{-3} = \sqrt{(3)(-1)}$, or $\sqrt{3}i$. The sum can be written as

$$\sqrt{3}i + 3i + 4i, \text{ or } (7 + \sqrt{3})\,i$$

Key Terms

Fundamental Theorem of Algebra

The Fundamental Theorem of Algebra states that any polynomial equation of degree n must have, with multiplicity, exactly n complex roots or solutions.

one-to-one function

A function is a one-to-one function if both the function and its inverse are functions.