Module 4: Seeing Structure

TOPIC 1: SOLVING QUADRATIC EQUATIONS

Students use what they know about square roots and graphs of quadratic equations to solve equations of the form $x^2 = n$ and $ax^2 - c = n$. They see in the graphs that the solutions are both equidistant from the axis of symmetry. Students then learn to factor or complete the square to solve quadratic equations and real-world problems. Finally, they derive the Quadratic Formula. Students see the structure of solutions to quadratic equations in the Quadratic Formula: the axis of symmetry plus or minus the distance to the parabola.

Where have we been?

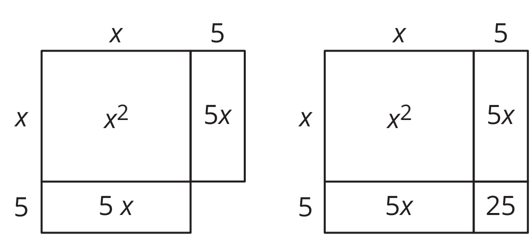
Students know the characteristics that define a quadratic function. They have explored zeros of functions and have interpreted their meaning in contextual situations. Students know that the factored form of a quadratic equation gives the zeros of the function. They can sketch quadratic equations using key characteristics from an equation written in general form, factored form, or vertex form. Importantly, students have extensive experience with locating solutions to equations using a graphical representation.

Where are we going?

The techniques for solving quadratics will be applicable as students solve higher-order polynomials in future courses. Understanding the structure and symmetry of a quadratic equation allows students to solve quadratics with complex roots as well as higher-order polynomials.

Completing the Square

The quadratic expression $x^2 + 10x$ can be represented in a square shape as $x^2 + 5x + 5x$. To complete the square, add $5 \cdot 5$, or 25. The expression $x^2 + 10 + 25$ can then be written in factored form as $(x + 5)(x + 5)$, or $(x + 5)^2$.



The Roots of Zero

The word *zero* has had a long and interesting history. The word comes from the Hindu word *sunya*, which meant "void" or "emptiness." In Arabic, this word became *sifr*, which is also where the word *cipher* comes from. In Latin, it was changed to *cephirum*, and finally, in Italian it became *zevero* or *zefiro*, which was shortened to *zero*.

The ancient Greeks, who were responsible for creating much of modern formal mathematics, did not even believe zero was a number!

Talking Points

It can be helpful to understand quadratic functions for college admissions tests.

Here is an example of a sample question:

Solve: $x^2 + 6x + 9 = 0$

To solve this problem, you can first factor the quadratic expression on the left-hand side of the equation:

$$(x + 3)(x + 3)$$

The Zero Product Property tells us that when factors have a product of 0, one or more of the factors is equal to 0. So, set each factor equal to 0 and solve:

$$x + 3 = 0$$
$$x = -3$$

Key Terms

degree of a polynomial

The degree of a term in a polynomial is the exponent of the term. The greatest exponent in a polynomial determines the degree of the polynomial.

double root

The quadratic function $y = x^2$ has two solutions at $y = 0$. Therefore, it has two zeros: $x = +\sqrt{0}$ and $x = -\sqrt{0}$. These two zeros of the function, or roots of the equation, are the same number, 0, so $y = x^2$ is said to have a double root.

completing the square

Completing the square is a process for writing a quadratic expression in vertex form which then allows you to solve for the zeros.

Quadratic Formula

The Quadratic Formula is $x = \frac{-b \pm \sqrt{b - 4ac}}{2a}$.