Module 3: Exploring Functions

TOPIC 2: EXPONENTIALS

Students begin this topic by examining the structure of exponential functions. Next, students compare the value of a simple interest account and a compound interest account. They graph and write equations for these two scenarios and compare the average rate of change of each for a given interval. Students graph and analyze horizontal reflections and dilations of exponential functions. Considering the transformed graph of $f(Bx)$, they generalize the effect of the B-value transformation on the graph of the function and specifically on the coordinates (x, y). Throughout the rest of the topic, students solve real-world problems that can be modeled by exponential functions, including one that requires students to combine function types to best model the scenario.

Where have we been?

In middle school, students learned the rules of exponents and used those rules to rewrite expressions in equivalent forms. In a previous course, students extensively studied the properties of both linear and exponential functions. They have connected geometric sequences and exponential functions, and they understand that exponential functions increase or decrease by a common multiplier.

Where are we going?

The work in *Exponentials* serves as a bridge between the work students did with exponential functions in the previous course and the deep dive they will take into exponential and logarithmic functions in the next course. Learning the features of exponential functions and contrasting them with linear functions allows students to model situations involving constant growth or decay. As students gain proficiency in solving increasingly complex equations, they are able to model more interesting and complex real-life phenomena.

The *B*-Value

The B-value of a function in transformation form affects the horizontal shape of the graph. The B-value of an exponential function is written in the exponent:

$$f(x) = a \cdot (b)^{Bx}$$

The B-value compresses the graph of an exponential function horizontally. When it is negative, it can also reflect the graph of an exponential function across the y-axis.

I Feel the Earth. Move.

How do scientists measure the intensity of earthquakes? You may know that scientists who study earthquakes—seismologists—refer to a scale known as a Richter scale when reporting the strength of an earthquake. The Richter scale is a kind of exponential scale.

The scale generally goes from 1 to 9 (though it doesn't really have an upper limit), but an earthquake which has an intensity of 6 on the Richter scale is 10 times more powerful than an earthquake which measures 5.

One of the strongest earthquakes in history occurred in Chile on May 22, 1960. This earthquake measured an amazing 9.5 on the Richter scale—over 30,000 times stronger than a magnitude 5 earthquake!

Talking Points

It can be helpful to understand exponential functions for college admissions tests.

Here is an example of a sample question:

You deposit $500 into a bank account that pays 3% interest compounded annually. What equation could you use to compute the amount in the account (in dollars), *A*, at the end of two years?

To solve this problem, you can identify the starting amount, $500, the constant multiplier, which is 1 + the interest rate, or 1.03, and the time in years, 2.

The equation that models the situation is $A = 500 \cdot 1.03^2$.

Check to see if the answer is reasonable: $500 \cdot 1.03^2 = 530.45$.

Key Terms

simple interest

In a simple interest account, a percent of the starting balance is added to the account at each interval. The formula for simple interest is $I = Prt$, where P represents the starting amount, or principal, r represents the interest rate, t represents time, and I represents the interest earned.

compound interest

In a compound interest account, the balance is multiplied by the same amount at each interval.

extracting square roots

The process of removing perfect square numbers from under a radical symbol is called extracting square roots.