Module 2: Investigating Proportionality

TOPIC 3: CIRCLES AND VOLUME

Students conclude this module by applying proportional reasoning to solve problems involving circles. Students use translation and dilation to establish the similarity of all circles, which plays a critical role in an informal argument for the circumference formula. Students consider how stacking similar circles and similar squares can build cones and pyramids. They use a limit argument to establish the formula for the volume of a pyramid and then use Cavalieri's Principle to provide an informal argument for the volume formula for a cone.

Where have we been?

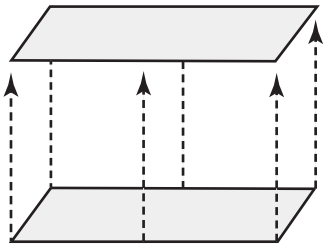
The concept of proportionality was first established for students in middle school. Students developed an understanding of equivalent ratios in grade 6 and began identifying the constant of proportionality in proportional relationships in grade 7. They use this reasoning in this course, recognizing the radian measure of an angle as the constant of proportionality of the arc length to the radius.

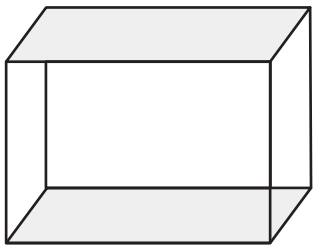
Where are we going?

In future topics, students will use radian measures and the trigonometric ratios to develop their understanding of the unit circle. They will "unroll" the unit circle along the *x*-axis of the coordinate plane to represent the sine, cosine, and tangent functions graphically. Understanding trigonometric functions on a coordinate plane allows students to model periodic behavior and to solve more complex real-world problems.

Using Rigid Motions to Create Solid Figures

When you translate a rectangle through space in a direction that is perpendicular to the plane containing the rectangle, a solid figure is formed, which you know is a rectangular prism.





Building a solid figure using rigid motions can give you insight into the mathematical formula used to describe the volume of the solid figure.

Eureka!

Archimedes of Syracuse, Sicily, who lived from 287 BCE to 212 BCE, was an ancient Greek mathematician, physicist, and engineer. Archimedes discovered formulas for computing volumes of spheres, cylinders, and cones.

Archimedes has been honored in many ways for his contributions. He has appeared on postage stamps in East Germany, Greece, Italy, Nicaragua, San Marino, and Spain. His portrait appears on the Fields Medal for outstanding achievement in mathematics. You can even say that his honors are out of this world. There is a crater on the moon named Archimedes, a mountain range on the moon named the Montes Archimedes, and an asteroid named 3600 Archimedes.

Talking Points

It can be helpful to understand volume for college admissions tests.

Here is an example of a sample question:

The water level in a 4 ft long by 3 ft wide by 2 ft tall fish tank is 1 foot. All of this water is poured into a 3 ft long by 2 ft wide by 4 ft tall fish tank. What is the height of the water in the second tank?

The volume of the water in the first tank is $4 \times 3 \times 1$, or 12 ft^3. This is poured into a fish tank with a base whose area is 6 ft^2. To get a volume of 12 ft^3, you have to multiply 6 ft^2 by 2 ft.

So, the height of the water in the second tank must be 2 ft.

Key Terms

radian

One radian is defined as the measure of a central angle whose arc length is the same as the radius of the circle.

segment of a circle

A segment of a circle is a region of the circle bounded by a chord and the included arc.

Cavalieri's Principle

For two-dimensional figures, if the lengths of one-dimensional slices—just line segments—of two figures are the same, then the figures have the same area.

great circle

A great circle of a sphere is a cross-section of a sphere when a plane passes through the center of the sphere.