Module 3: Investigating Growth and Decay

TOPIC 1: INTRODUCTION TO EXPONENTIAL FUNCTIONS

In this topic, students build upon their previous understanding of sequences and common ratio to recognize that some geometric sequences are exponential functions and others are not. Next, students examine the structure of exponential functions, connecting the common ratio of a geometric sequence with the base of the power in an exponential function. Students explore the constant ratio between intervals of $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ for the function $f(x) = 2^x$. Through this exploration, they conclude that $2^{\frac{1}{2}} = \sqrt{2}$ and $2^{\frac{1}{3}} = \sqrt[3]{2}$. By graphing these values, the misconception that $f\left(\frac{1}{2}\right)$ is halfway between $f(1)$ and $f(2)$ is addressed. Finally, students transform exponential functions and generalize the effect of these transformations on (x, y).

Where have we been?

In middle school, students learned the rules of exponents and used those rules to rewrite expressions in equivalent forms. They transformed geometric objects in the coordinate plane and noted the effect of each transformation on the ordered pairs of the image. This topic begins where students left with geometric sequences in a previous topic. Students can already write a recursive and an explicit formula for a given geometric sequence.

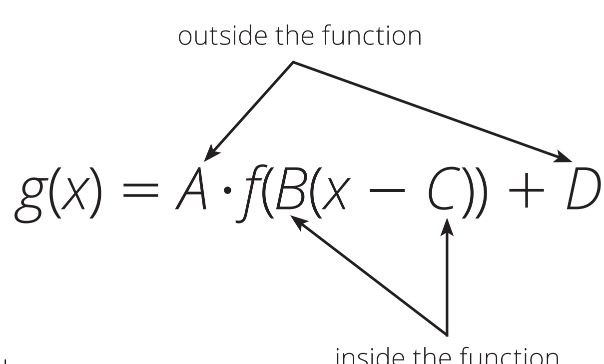
Where are we going?

Throughout this topic, students apply what they know about the key characteristics of functions (e.g., intercepts, intervals of increase or decrease, domain and range, the rules of transformations) to include exponential functions. This prepares them for the work they will do with quadratic functions, both in this course and with more complex functions in subsequent courses.

Inside and Outside the Function

Different transformations of a function—such as a vertical stretching or compressing or horizontal stretching or compressing, vertical or horizontal translations, and reflections—can be specified when writing a function in transformation form.

Values inside the function affect the horizontal transformations of the graph, and values outside the function affect the vertical transformations of the graph.



The Matthew Effect

It might seem unfair, but some banks will charge you money for not having money. And they'll pay you money if you have a lot of it.

In 2015, the big three banks in the United States made about $6 billion in overdraft fees. By contrast, personal interest income in 2017 was over $1.4 trillion!

When you deposit money in a bank account that accrues interest, your money doesn't just sit there, waiting for you to withdraw it. The bank lends this money to people who want to buy cars, houses, and pay for college. Banks collect interest on these loans and reward you for your contribution. The more money you have in an interest-earning account, the more you are rewarded!

Talking Points

Exponential functions is an important topic to know about for college admissions tests.

Here is a sample question:

> **A biology class predicted that a population of animals will double in size every year. The population at the start of 2018 was about 500 animals. If *P* represents the population *n* years after 2018, what equation represents the model of the population over time?**

To solve this, students should know that this represents an exponential function, because the population doubles each year. This can be written as:

$$\textit{initial value} \cdot (\textit{growth rate})^{\textit{time}}$$

The initial value is 500 animals, and the growth rate is 2, for doubling. So, the function $P(n) = 500 \cdot 2^n$ models the population over time in years, n.

Key Terms

exponential function

An exponential function is a function of the form $f(x) = a \cdot b^x$, where a and b are real numbers, and b is greater than 0 but is not equal to 1.

horizontal asymptote

A horizontal asymptote is a horizontal line that a function gets closer and closer to, but never intersects.