

# Module 2: Exploring Constant Change

## TOPIC 2: SHAPES ON A COORDINATE PLANE

In this topic, students investigate strategies for determining the perimeters and areas of rectangles, triangles, parallelograms, and composite plane figures on the coordinate plane. Students also explore the effects of proportional and non-proportional changes to the dimensions of a plane figure on its perimeter and area. Students apply the Distance Formula and slope formula to calculate the area and perimeter of composite figures in real-world scenarios. Students extend this knowledge to include scenarios comparing speed and time

## Where have we been?

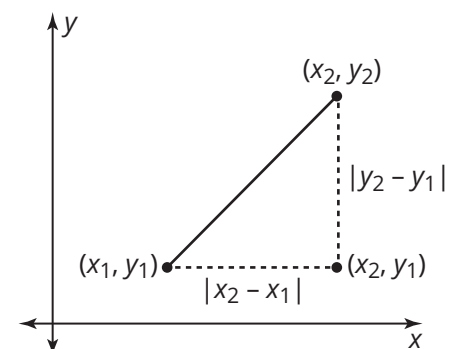
Students have been calculating the area and perimeter of triangles, quadrilaterals, and other figures given side lengths in many of their previous years of school. This topic adds another step to the process by providing only vertices of the figure and requiring students to calculate side lengths before applying area or perimeter formulas. It builds upon students' understanding of the area formulas for squares, rectangles, and triangles; the Pythagorean Theorem; the slope criteria for parallel and perpendicular lines; and methods of solving a system of linear equations.

## Where are we going?

Proving relationships and performing geometric measurements algebraically integrates geometry and algebra. As with much of high school geometry, students formalize concepts that they investigated informally in elementary and middle school. And by connecting algebraic reasoning with geometric properties, students can better make sense of formal Euclidean proofs for these properties.

## Distance Formula

The Distance Formula is derived from the formula for the Pythagorean Theorem:  $a^2 + b^2 = c^2$ . If  $c$  is the side length we want to know, and  $a$  and  $b$  are the leg lengths, then  $c$  is equal to  $\sqrt{a^2 + b^2}$ . Substituting in the horizontal and vertical parts of the right triangle,  $x_2 - x_1$  and  $y_2 - y_1$ , gives us  $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ .



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## The Bermuda Triangle

One of the most famous stretches of ocean in the Atlantic is an area that stretches between the United States, Puerto Rico, and Bermuda known as the Bermuda Triangle.

A heavily traveled area by planes and ships, it has become famous because of the many stories about ships and planes lost or destroyed as they moved through the Triangle.

For years, the Bermuda Triangle was suspected of having mysterious, supernatural powers that fatally affected all who traveled through it. Others believe natural phenomena, such as human error and dangerous weather, are to blame for the incidents. In 2016, a group of scientists announced that hexagon-shaped clouds in the area, which create 170 mph “air bombs” are responsible for most, if not all, of the lost craft in the Triangle.

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### Talking Points

Coordinate geometry can be an important topic to know about for college admissions tests. Here is an example of a sample question:

**In the  $xy$ -plane, a triangle has vertices at  $(5, 0)$ ,  $(\sqrt{2}, 0)$ , and  $(2, \sqrt{10})$ . What is the approximate area of the triangle?**

You can think of the base as the horizontal line segment. Its length is  $5 - \sqrt{2}$ , and the height is  $\sqrt{10}$ . So, the area is

$$\frac{1}{2}(\sqrt{10})(5 - \sqrt{2}) \approx 5.67$$

So, the area of the triangle is approximately 5.67 square units.

### Key Terms

#### Distance Formula

The Distance Formula states that if  $(x_1, y_1)$  and  $(x_2, y_2)$  are two points on the coordinate plane, then the distance  $d$  between  $(x_1, y_1)$  and  $(x_2, y_2)$  is given by

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

#### composite figure

A composite figure is a figure that is formed by combining different shapes.