Module 4: Connecting Geometric and Algebraic Descriptions

TOPIC 2: CONIC SECTIONS

This topic explores circles and parabolas on the coordinate plane as well as conic sections, which include parabolas, ellipses, and hyperbolas. Equations for these geometric figures can be written using their key characteristics. Students begin by using the Distance Formula to derive equations for a circle centered at the origin and a circle centered at point (h, k). Students explore how to use the Pythagorean Theorem, the Distance Formula, and symmetry to determine whether a given point lies on a circle centered on or off the origin. Finally, the focus and directrix of a parabola are introduced through an exploratory activity.

Where have we been?

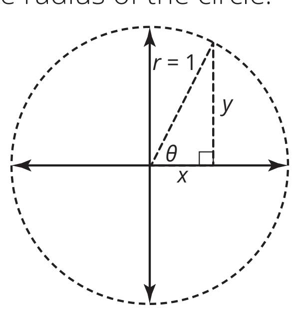
Students first learned the Pythagorean Theorem in middle school. They have used it to solve for distances on the coordinate plane, to derive the Distance Formula, and to verify properties of triangles and quadrilaterals on a coordinate plane. Development of the Pythagorean Identity recalls students' understanding of trigonometric ratios.

Where are we going?

Together with parabolas, conic sections like ellipses and hyperbolas will be useful in studying three-dimensional geometry. Conic sections model important physical processes in nature, including the trajectories of objects in space and of charged particles, making them an important tool not only in mathematics but also in physics, astronomy, and engineering.

Deriving the Equation for a Circle on the Coordinate Plane

A right triangle can be situated at the origin of a coordinate plane. As the reference angle θ changes, the end of the hypotenuse can trace out a circle. The coordinates of this endpoint are always $(r \cdot \cos\theta, r \cdot \sin\theta)$, where r is the radius of the circle.



When r is 1, as it is in the unit circle, then the Pythagorean Theorem tells us that $\cos^2\theta \cdot \sin^2\theta = 1$.

Take Us to Your Leader!

Scientists explore deep space by using large antennas to listen for distant radio waves. Parabolic antennas amplify faint signals by using the properties of parabolas to focus them onto a receiver.

The worldwide Deep Space Network also helps to keep track of exploratory spacecraft like the two Voyager spacecrafts, which have left our Solar System!

Talking Points

It can be helpful to understand concepts involved in circles on a coordinate plane for college admissions tests.

Here is an example of a sample question:

$$x^2 + (y + 1)^2 = 4$$

The graph of the equation above on the coordinate plane is a circle. If the center of the circle is translated 2 units down and the radius is increased by 1, what is the equation of the resulting circle?

A circle with its center at (h, k) is described by the equation $(x - h)^2 + (y - k)^2 = r^2$, where r is the radius. So, the center of the original circle is at $(0, -1)$ and it has a radius of 2. The new center will be at $(0, -3)$ and its radius will be 3.

So, the equation for the resulting circle will be $x^2 + (y + 3) = 9$.

Key Terms

Pythagorean identity

A Pythagorean identity is a trigonometric equation that expresses the Pythagorean Theorem in terms of trigonometric ratios.

focus of a parabola

A parabola is the set of all points in a plane that are equidistant from a fixed point and a fixed line. The focus of a parabola is the fixed point.

directrix of a parabola

A parabola is the set of all points in a plane that are equidistant from a fixed point and a fixed line. The directrix of a parabola is the fixed line.

concavity

The concavity of a parabola describes the orientation of the curvature of the parabola.

ellipse

An ellipse is the locus of all points in a plane for which the sum of whose distances from two given points is a constant.