Assignment

Write

Explain how the natural base e is similar to and different from π.

Remember

The natural base $e \approx 2.7182818$ is an irrational number that represents continuous growth and is used to model population changes as well as continuously compounded interest.

For continuous compounding, the compound interest formula is $A = Pe^{rt}$.

Practice

1. Caleb wants to invest $1000 in a savings account. Lincoln Federal Bank is offering 6% interest compounded yearly. Washington National Bank is offering 5.5% interest compounded daily.
   a. For each bank, write an exponential function to represent the amount of money Caleb would have in the account after t years.
   b. Determine which bank Caleb should choose if he plans to invest his money for 5 years. If Caleb decides to leave the money in the bank for a longer period of time, will the other bank be a better deal in the long run? Explain your reasoning.
   c. Discuss the domain, range, asymptotes, intercepts, end behavior, and intervals of increase and decrease for each function as they relate to this problem situation.
2. In 2010, Bolivia had a population of 10.5 million people and an annual growth rate of 1.6%.
   a. Write a function to model Bolivia's population with respect to t, the number of years since 2010. Write your function in the form $N(t) = N_0e^{rt}$.
   b. Use your model to predict what Bolivia's population will be in the year 2030.
   c. Use your model to estimate Bolivia's population in the years 1990 and 1970.
   d. Use technology to estimate when Bolivia's population will reach 20 million people.
   e. Discuss the domain, range, asymptotes, intercepts, end behavior, and intervals of increase and decrease for your population model as they relate to this problem situation.

Stretch

It has been called the most beautiful equation in all of mathematics. This equation links together four of the most important constants in math, including e and the imaginary number i:

$$e^{i\pi} + 1 = ?$$

Three of these constants are shown. The right-hand side of the equation is the fourth constant. What is it? What does the expression $e^{i\pi}$ tell us? Research this question.

Review

1. Nadine bought a house in 1995 for $155,000. The table displays the value of the house over several years. The year 0 corresponds to the date of the year Nadine bought the house.
   a. Does the value of the house represent an example of exponential decay, exponential growth, or neither? Explain your reasoning.
   b. Write a function to represent the value of the house, V, as a function of the year, n.
   c. What was Nadine's house worth in 2010, assuming the value of the house increased each year at the same rate?

Year	Value of House ($)
0	155,000
1	162,750
2	170,887.50
3	179,431.875

2. The population of a small town in 2015 was 8,455. The table displays the population of the town for several years. The year 0 corresponds to the the year 2015.
   a. Does the population represent an example of exponential decay, exponential growth, or neither? Explain your reasoning.
   b. Write a function to represent the population of the town, P, as a function of the year, t.
   c. Use the function you wrote in part (a) to complete the table. Round all answers to the nearest whole number.

Year	Population
0	8455
1	6764
2	5411
3	
4	
5	

3. Solve each equation and check your solution.
   a. $\sqrt[3]{4x + 3} = -2$
   b. $\sqrt{x + 2} + 4 = x$
   c. $\frac{m - 4}{4} + \frac{m}{3} = 6$