Assignment

Write

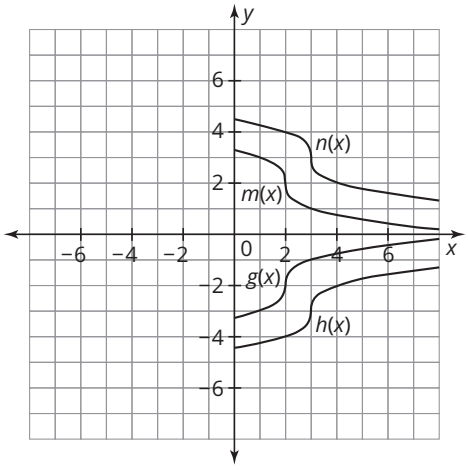
Describe how transformations affect the domain and range of radical functions.

Remember

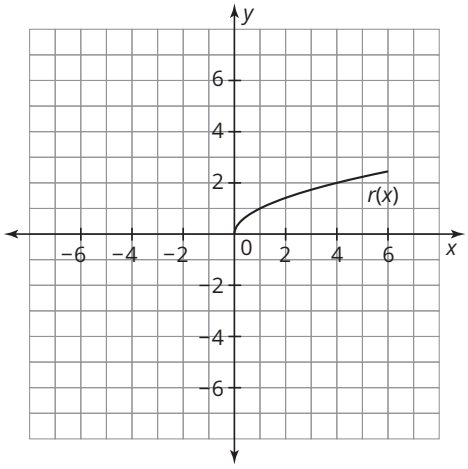
For the square root function and cube root function respectively, the transformation function can be written as $s(x) = A\sqrt{B(x - C)} + D$ or $c(x) = A\sqrt[3]{B(x - C)} + D$. Changes to the A- or D-values dilate, translate, or reflect a function vertically. Changes to the B- or C-values dilate, translate, or reflect a function horizontally.

Practice

1. Brandon, a graphic designer, designed the logo shown for the Lazy Y Ranch. Each curve in the design is a transformation of the cube root function $f(x) = \sqrt[3]{x}$ with a restricted domain.
   a. Describe each of the four transformations of $f(x) = \sqrt[3]{x}$ that were used to create the four functions in the design.
   b. Write each function used in the design. For each function, write the domain as an inequality.



2. Brandon is working on a logo for a publishing company. He starts by graphing the function $r(x) = \sqrt{x}$ with the restricted domain $0 \leq x \leq 6$. He plans to add the graphs of 5 more functions to complete the design.



   a. The next function Brandon adds is $s(x)$, which is the square root function after a vertical stretch by a factor of 2 and a translation 1 unit up. Write the function $s(x)$ and graph $s(x)$ with the domain $0 \leq x \leq 6$.
   b. Next, Brandon adds the function $t(x)$, which is the square root function after a vertical stretch by a factor of 3 and a translation 2 units up. Write the function $t(x)$ and graph $t(x)$ with the domain $0 \leq x \leq 6$.
   c. To complete the design, Brandon adds the functions $r'(x)$, $s'(x)$, and $t'(x)$ which are reflections of the original 3 functions across the y-axis. Write the functions $r'(x)$, $s'(x)$, and $t'(x)$ and graph each function with the domain $-6 \leq x \leq 0$.

Stretch

1. Consider the functions $f(x) = \sqrt[3]{x^4}$ and $g(x) = x\sqrt[3]{x}$.
   a. Determine $f(4)$ and $g(4)$ in decimal form. Show your work.
   b. Determine $f(-8)$ and $g(-8)$ in decimal form. Show your work.
   c. Rewrite $f(x)$ to show that it is equivalent to $g(x)$.

Review

1. Consider the function $f(x) = 4x^2$.
   a. Determine the domain and range of $f(x)$.
   b. Write the inverse function $f^{-1}(x)$.
   c. Determine the domain and range of $f^{-1}(x)$.
2. Algebraically determine whether $f(x) = 2x^3 + 15$ and $g(x) = \sqrt[3]{\frac{x - 15}{2}}$ are inverses. Show your work.
3. Consider the basic rational function $f(x) = \frac{1}{x}$. Explain how the graph of $g(x) = \frac{5}{x + 10} + 3$ compares to the graph of $f(x)$.
4. Consider the rational function $\frac{x}{x^3 - 5x^2 + 6x}$. Determine any vertical and horizontal asymptotes and any removable discontinuities of the graph of $f(x)$. Explain your reasoning.
5. Identify the number of real zeros of $x^3 - 7x^2 + 14x - 8 = 0$. Explain your reasoning.