Assignment

Write

Write an example for each term using the dividend $x^2 - 2x + 4$ and the divisor $x + 1$.

1. Factor Theorem
2. Polynomial long division
3. Remainder Theorem
4. Synthetic division

Remember

A polynomial function $p(x)$ has $(x - r)$ as a factor if and only if the value of the function at r is 0, or $p(r) = 0$. When any polynomial equation or function $f(x)$ is divided by a linear expression of the form $(x - r)$, the remainder is $R = f(r)$ or the value of the equation or function when $x = r$.

Practice

1. Use the Factor Theorem to determine whether each linear expression is a factor of the polynomial $x^4 + x^3 - 17x^2 + 15x$.
   a. $x + 3$
   b. $x + 5$
   c. $x - 1$
2. Factor each binomial over the set of real numbers.
   a. $4x^2 - 9y^2$
   b. $x^3 + 216$
3. The Polynomial Pool Company offers 10 different pool designs numbered 1 through 10. Each pool is in the shape of a rectangular prism. The volume of water in Pool Design x can be determined using the function $V(x) = l(x) \cdot w(x) \cdot d(x) = 2x^3 + 18x^2 + 46x + 30$, where $l(x)$, $w(x)$, and $d(x)$ represent the length, width, and depth of the pool in feet.
   a. Determine the expressions for the functions $w(x)$ and $d(x)$ if $l(x) = 2x + 2$ and the width of each pool is greater than the depth. Do not use a calculator.
   b. Determine the length, width, and depth of Pool Design 9.
4. The function $m(x) = 2x^2 + 6x - 7$ generates the same remainder when divided by $(x - a)$ and $(x - 2a)$ when $a \neq 0$. Calculate the value(s) of a and determine the corresponding factors.
5. The given table of values represents the function $f(x) = x^3 + 9x^2 + 14x - 24$.

(x)	-2	-1	0	1	2
$f(x)$	-24	-30	-24	0	48

   a. Determine one of the factors of $f(x)$ without using a calculator. Explain your reasoning.
   b. Completely factor $f(x)$ without using a calculator.
   c. Determine all of the zeros of $f(x)$ without using a calculator.

Stretch

The Rational Root Theorem states that if the polynomial

$P(n) = a_n x^n + a_{1-n} x^{n-1} + \ldots + a_2 x^2 + a_1 x + a_0$ has any rational roots, then they must be of the form $\pm\frac{p}{q}$, where p is factors of a_0 and q is factors of a_n.

1. Consider the function $f(x) = 3x^3 - 4x^2 - 17x + 6$.
   a. Determine the values of a_0 and a_n for this polynomial function.
   b. Determine the values of p, or the factors of a_0.
   c. Determine the values of q, or the factors of a_n.
   d. Determine the possible rational zeros of the function.
   e. Check all the possible rational zeros to determine whether any of them are roots of the function $f(x)$.

Review

1. Determine the zeros of each function.
   a. $f(x) = 3(x - 1)^4 - 9$
   b. $f(x) = -2(x + 2)^3 - 1$
   c. $f(x) = 5(x + 3)^3$
2. Factor each polynomial over the set of real numbers.
   a. $16x^2 - 3x - 3$
   b. $x^4 - 10x^2 + 25$
3. Sketch the graph of $f(x) = x^{17}$ and describe the end behavior of the graph.