Module 4: Investigating Periodic Functions

TOPIC 2: TRIGONOMETRIC EQUATIONS

Now that students are equipped with an understanding of trigonometric functions and their key characteristics, they are able to model periodic phenomena that occur in the real-world. They are introduced to strategies for solving trigonometric equations, and then use their knowledge of the unit circle, radian measures, and the graphical behaviors of trigonometric functions to solve sine, cosine, and tangent equations. Students apply all that they have learned to model various situations with trigonometric functions, including circular motion. They model real-world problems with sine or cosine functions and interpret the key characteristics in terms of the problem situation.

Where have we been?

Students have had significant experience solving equations. They know how to solve an equation using graphs, the Properties of Equality, square roots, inverse operations, and factoring. In a previous course, students used the inverses of sine, cosine, and tangent to solve simple trigonometric equations. From their work in the previous topic, they understand periodic functions, their key characteristics, and the periodicity identities.

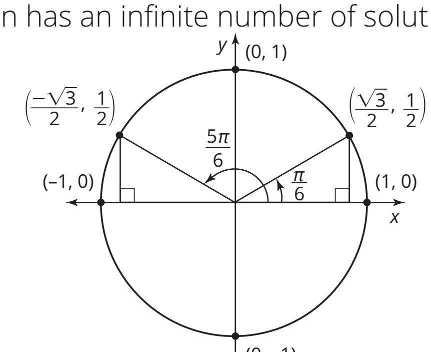
Where are we going?

The ability to solve trigonometric equations is important in modeling any phenomena involving circular motion. Students who pursue mathematics at the post-secondary level will use trigonometric equations extensively in calculus classes, where they will calculate derivatives to determine the velocity and acceleration of particles in motion. This knowledge is used in physics and electrical engineering.

Trigonometric Values in Different Quadrants

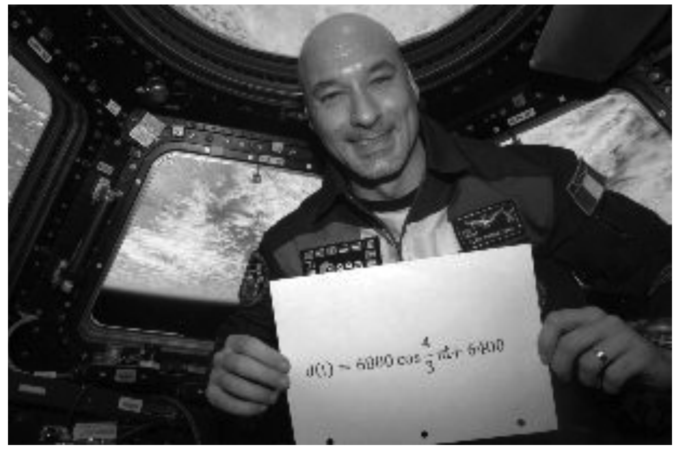
What is the value of x when $\sin x = \frac{1}{2}$?

On the unit circle, you can see that $\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$ and $\sin\left(\frac{5\pi}{6}\right) = \frac{1}{2}$. So, $x = \frac{\pi}{6}$ or $\frac{5\pi}{6}$. When the domain is restricted to $0 \le x \le 2\pi$, these are the only 2 solutions to the equation. When there are no domain restrictions, the equation has an infinite number of solutions.



Trig's in Space!

On October 14, 2013, astronaut Luca Parmitano sent us this picture from aboard the International Space Station (ISS). During his mission, Luca became the first Italian astronaut to take part in a spacewalk. The trigonometric function he is holding represents the position of the ISS in orbit over time.



Talking Points

Trigonometric equations can be an important topic to know about for college admissions tests.

Here is an example of a sample question:

Let $\sin x = \frac{1}{2}$. If $0° < x° < 90°$, what is $\cos x$?

The variable *x* represents an acute angle measure of a right triangle in the first quadrant. The sine of *x* (opposite over hypotenuse) is $\frac{1}{2}$. This means that the adjacent side of the right triangle, using the Pythagorean Theorem, is $\sqrt{3}$.

Thus, the cosine of *x* is $\frac{\sqrt{3}}{2}$.

Key Terms

trigonometric equation

A trigonometric equation is an equation in which the unknown is associated with a trigonometric function. The number of solutions of a trigonometric equation can vary depending on how the domain of the function is restricted.

inverse sine ($\sin^{-1}$)

The inverse sine function is used to determine solutions to sine equations.