Module 4: Investigating Periodic Functions

TOPIC 1: TRIGONOMETRIC RELATIONSHIPS

Students begin this topic by examining a reference angle and understanding how an angle opens up on the unit circle, defining *standard position, initial ray*, and *terminal ray.* The unit circle is then unrolled along the *x*-axis to demonstrate the key characteristics of periodic functions. Using new understanding of the unit circle, radian measure, and periodic functions, students investigate the sine and cosine functions as well as their characteristics and graphs. Finally, students analyze the characteristics of the tangent graph.

Where have we been?

In the previous course, students learned the geometric definitions of the sine, cosine, and tangent ratios and used them to solve problems. Students also defined radian measures and explored the side lengths of special right triangles. They have extensive experience with graphing functions, identifying key characteristics, and performing function transformations.

Where are we going?

Students will use their understanding of the key characteristics of trigonometric functions to model periodic functions from the real world and solve for unknown values. Trigonometric functions are widely used in advanced calculus courses to model real-world scenarios involving circular motion. In these courses, the use of radian measures leads to simple formulas for derivatives and integrals.

Properties of Periodic Functions

The graphs of periodic functions have characteristics that are given special names, such as amplitude and midline.
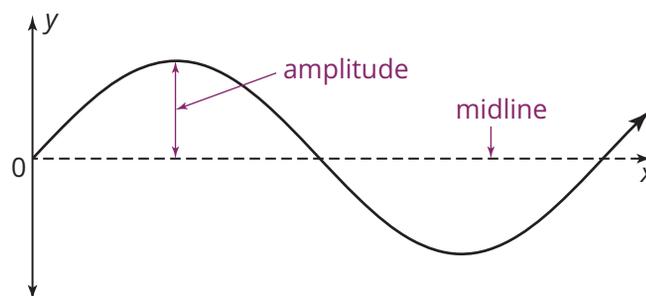
Wheel. Of. Ferris.

The Ferris wheel is named after George Washington Gale Ferris, Jr., a Pittsburgh bridge-builder. The first Ferris wheel was designed and constructed by Ferris for the World's Columbian Exposition in Chicago in 1893. It stood 264 feet tall, contained 36 cars, took 20 minutes to complete 2 rotations, and cost 50 cents per ride.

The record for the world's tallest Ferris wheel has been broken many times since 1893. As of 2014, the High Roller in Las Vegas, Nevada, holds the record. It stands a whopping 550 feet tall—over twice as tall as Ferris' original wheel.

Talking Points

Trigonometric functions can be an important topic to know about for college admissions tests.

Here is an example of a sample question:

Identify the amplitude of the graph of the function $a(x) = 2\pi \cdot \sin(2\pi x)$.

The amplitude of the basic sine function, $\sin(x)$, is 1. You know that changing the A-value of the function stretches or shrinks the graph vertically.

Thus, the amplitude of $a(x) = 2\pi \cdot \sin(2\pi x)$ is equal to the A-value, which is 2π.

Key Terms

periodic function
A periodic function is a function whose values repeat over regular intervals.

unit circle
A unit circle has a radius of 1 unit.

radians
The unit that describes the measure of an angle theta, θ, in terms of the radius and arc length of a unit circle is called a radian. The ratio of the intercepted arc length of a central angle to the radius is the measure of the central angle in radians.

trigonometric function
Trigonometric functions take angle measures (θ values) as inputs and output real number values, which correspond to coordinates of points on the unit circle.