Module 3: Inverting Functions

TOPIC 3: EXPONENTIAL AND LOGARITHMIC EQUATIONS

In this topic, students first convert between exponential and logarithmic forms of an equation, and then use this relationship to solve for an unknown base, exponent, or argument in a logarithmic equation. They then develop rules and properties of logarithms based on their prior knowledge of various exponent rules and properties. Students derive the Change of Base Formula and solve logarithmic equations for the base, argument, or exponent using the formula or by rewriting them as exponential equations.

Where have we been?

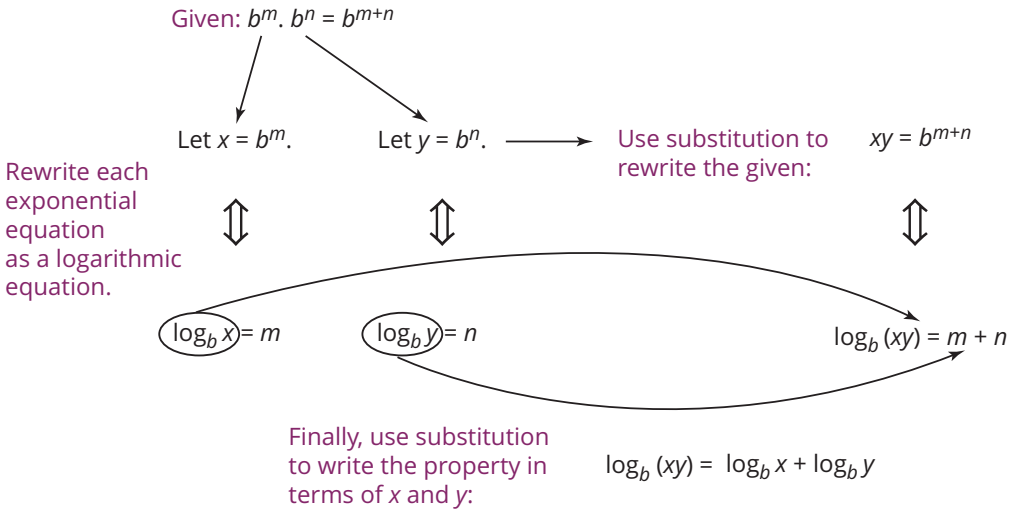
Students oriented their thinking around exponential and logarithmic functions in the previous topic. From their prior work with quadratic functions, students have experience with manipulating equations of function inverses.

Where are we going?

In this topic, students use exponential and logarithmic equations that model real-world situations to solve problems. They use technology to determine logarithmic regressions. As evidenced by the scenarios in this topic, logarithmic equations are used to represent a large variety of complex real-world situations.

Properties of Logarithms

The properties of powers that you know can be applied to derive properties of logarithm operations. For example, you can use the Product Rule of Powers to derive a similar property for logarithms called the Product Rule of Logarithms.



Newton's Law of Cooling

Crime investigators use logarithmic equations to estimate a body's time of death based on two temperature readings of the body. Specifically, investigators use what's known as Newton's Law of Cooling, which states that an object cools down at a rate that is proportional to the temperature difference between the object and the environment.

Coroners—government officials who are responsible for verifying deaths—often use a rule of thumb to estimate the time of death: subtract 2 degrees from normal body temperature for the first hour after death and then 1 degree for each hour after that.

Talking Points

Logarithmic equations can be an important topic to know about for college admissions tests.

Here is an example of a sample question:

Solve $5^x = 432$ for x. Write your answer to the nearest hundredth.

If you take the log of both sides, you get $\log 5^x = \log 432$. Using the Power Rule of Logarithms, you can rewrite $\log 5^x$ as $x \cdot \log 5$.

So, $x = \frac{\log 432}{\log 5} \approx 3.77$.

Key Terms

Zero Property of Logarithms

The Zero Property of Logarithms states that $\log_b 0 = 1$.

Power Rule of Logarithms

The Power Rule of Logarithms states that $\log_b a^m = m \cdot \log_b a$.

Change of Base Formula

The Change of Base Formula allows you to calculate an exact value for a logarithm by rewriting it in terms of a different base:

$\log_b c = \frac{\log_a c}{\log_a b}$, where $a, b, c > 0$ and $a, b \neq 1$.