Module 2: Developing Structural Similarities

TOPIC 3: RATIONAL FUNCTIONS

To start this topic, students explore and compare the graphs, tables, and values of a linear function, $f(x) = x$, and its reciprocal function, $g(x) = \frac{1}{x}$. Rational functions are defined. Students then explore transformations of rational functions. The term *removable discontinuity* is defined, and students graph several rational functions with holes or asymptotes in the graph. Operations with rational expressions are next introduced followed by problem solving, during which students create proportions, write rational expressions, describe the behavior of the ratios in the proportions, identify the domain and range, and calculate average costs.

Where have we been?

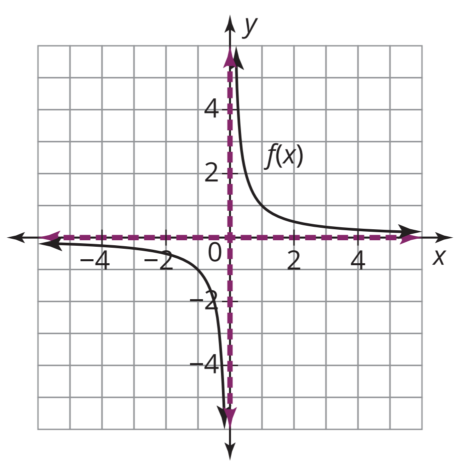
Students have been working with rational numbers since elementary school. They have extensive knowledge of function behaviors and characteristics to apply to the analysis of rational functions.

Where are we going?

As students experience in the final lesson of this topic, rational functions are helpfully applied in many real-world situations involving mixtures, distances, and costs. Rational functions are used heavily in medical and econometric modeling applications for analysis and prediction. Rational functions also have applications in image resolution and acoustics.

Asymptotes of Rational Functions

The basic rational function $f(x) = \frac{1}{x}$ has a vertical asymptote at $x = 0$ and a horizontal asymptote at $y = 0$. An asymptote is a line that a function gets closer and closer to, but never intersects.



An asymptote does not represent points on the graph of the function. It represents the output value that the graph approaches but never reaches.

Undefined

Consider the following mathematical explanation that 1 is equal to 2. Start by noting that any number multiplied by 0 is equal to 0.

$$1 \times 0 = 0$$
$$2 \times 0 = 0$$

Since 1×0 and 2×0 both equal 0, then they must be equal to each other.

$$1 \times 0 = 2 \times 0$$

Dividing both sides of an equation by the same value preserves equality.

$$\frac{1 \times 0}{0} = \frac{2 \times 0}{0}$$

Anything divided by itself is 1, so $\frac{0}{0} = 1$. This leaves $1 \times 1 = 2 \times 1$, or $1 = 2$.

What is wrong with this proof?

Talking Points

Rational equations can be an important topic to know about for college admissions tests.

Here is an example of a sample question:

Add the expression $\frac{x}{2x + 4} + \frac{x}{x + 2}$.

Determine a least common denominator: $2x + 4 = 2(x + 2)$, so the LCD is $2(x + 2)$.

Multiply the second fraction by $\frac{2}{2}$, which is the same as multiplying it by 1, and rewrite the sum as:

$$\frac{x}{2(x + 2)} + \frac{2x}{2(x + 2)} = \frac{3x}{2(x + 2)}$$

The given expression is equal to $\frac{3x}{2x + 4}$.

Key Terms

rational function

A rational function is any function that can be written as the ratio of two polynomials.

removable discontinuity

A removable discontinuity is a single point at which a function is not defined.

rational equation

A rational equation is an equation that contains one or more rational expressions. Rational equations are proportions.