Module 2: Developing Structural Similarities

TOPIC 2: POLYNOMIAL MODELS

To begin this topic, Euclid's Formula is stated and used to generate Pythagorean triples. Students then analyze specific patterns in the rows of Pascal's Triangle and look for other observable patterns. The Binomial Theorem is provided and students use it to expand binomials with coefficients equal to 1 and with coefficients other than 1. They use the Binomial Theorem to determine specific terms for binomials written in the form $(x + y)^n$ and $(x - y)^n$ with the same value for n. Finally, students use polynomials to model data, such as traffic patterns, monthly precipitation, and minimum wages.

Where have we been?

The previous topic in this module has prepared students to analyze real-world data using polynomial functions. Throughout this course, pattern identification is emphasized as a key step in mathematical modeling and thinking. In previous courses, students have used linear and nonlinear regressions to analyze and make predictions about data.

Where are we going?

The Binomial Theorem is used repeatedly in combinatorial mathematics to analyze sample spaces, permutations, and combinations. Polynomial models are used throughout industry and the sciences to analyze complex phenomena and make predictions about real-world situations.

Pascal's Triangle

A large variety of different patterns can be perceived using Pascal's Triangle. The first six rows of Pascal's Triangle are shown, where $n = 0$ represents the first row, $n = 1$ represents the second row, and so on.
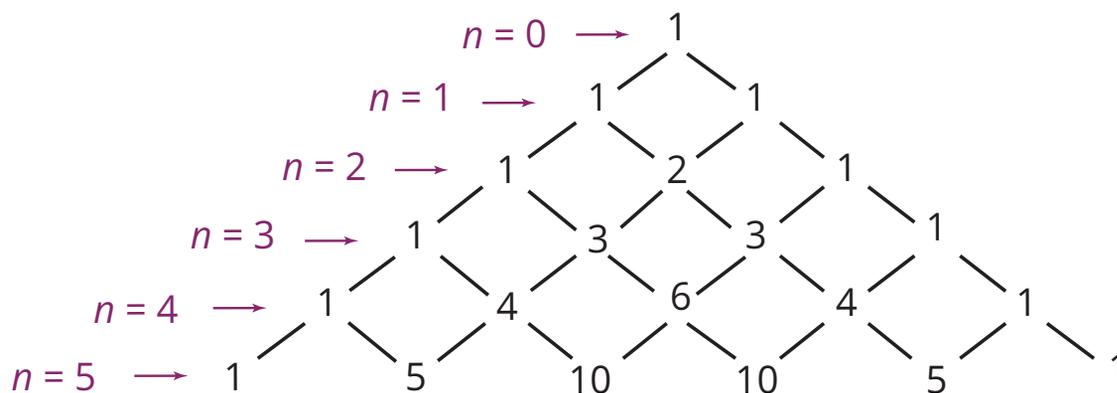
Even though Pascal's triangle has been around for centuries—even before the time of mathematician Blaise Pascal for whom it is named—new patterns are still being discovered.

Planes, Trains, and Automobiles

Transportation plans are an essential part of any large urban development project. Whether designing residential blocks, shopping districts, or stadiums, part of the planning process is determining how to move large groups of people in and out of an area quickly. Building new highways, bus stations, bike lanes, or railways may be necessary for some large-scale developments.

Part of urban development projects is monitoring existing conditions in a specific area. Planners must determine how well the current traffic infrastructure meets the community's needs before modeling and predicting what transportation processes may work best for a future project.

Talking Points

The Binomial Theorem can be an important topic to know about for college admissions tests.

Here is an example of a sample question:

What is the fifth term of the expansion of $(a + b)^8$?

The coefficients of the expansion given by Pascal's triangle are 1, 8, 28, 56, 70, 56, 28, 8, 1. The fifth of these is 70.

The expansion proceeds as $a^8 + 8a^7b + 28a^6b^2$... The fifth term in the expansion, then, is $70a^4b^4$.

Key Terms

Euclid's Formula

Given positive integers r and s, where $r > s$, Euclid's Formula states that $(r^2 + s^2)^2 = (r^2 - s^2)^2 + (2rs)^2$.

Binomial Theorem

The Binomial Theorem states that it is possible to extend any power of $(a + b)$ into a sum of the form
$$(a + b)^n = \binom{n}{0}a^nb^0 + \binom{n}{1}a^{n-1}b^1 + \binom{n}{2}a^{n-2}b^2 + \dots$$
$$+ \binom{n}{n-2}a^1b^{n-1} + \binom{n}{n}a^0b^n$$

coefficient of determination

The coefficient of determination (R^2) measures the strength of the relationship between the original data and their regression equation.