CARNEGIE LEARNING

1 Analyzing Structure

Pacing: 42 Days

Topic 1: Exploring and Analyzing Patterns

Exploring and Analyzing Patterns begins with opportunities for students to analyze and describe various patterns. Questions ask students to represent algebraic expressions in different forms and use algebra and graphs to determine whether they are equivalent. They identify linear, exponential, and quadratic functions using multiple representations. The three forms of a quadratic equation are reviewed, and students learn to write quadratic equations given key points before using a system to write a quadratic equation given any three points. Finally, students recall how to solve quadratic equations; they consider quadratic equations with no real roots; and they then solve quadratic functions with imaginary roots.

Standards: N.CN.1, N.CN.2, N.CN.7, N.CN.8 (+), N.CN.9 (+), A.SSE.1a, A.SSE.1b, A.SSE.2, A.APR.1, A.CED.1, A.CED.2, A.REI.4, A.REI.4a, A.REI.4b, A.REI.7, F.IF.4, F.IF.8, F.IF.9, F.BF.1a **Pacing:** 19 Days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
1	**Patterns: They're Grrrrrowing!** Observing Patterns	F.IF.8	1	Tiling patterns on floors, keeping secrets, and patio designs are used to illustrate sequences described by observable patterns. Students will analyze sequences and describe observable patterns. They sketch other terms or designs in each sequence using their knowledge of the patterns, and then will answer questions relevant to the problem situation. In one situation, a table is used to organize data and help recognize patterns as they emerge.	• Sequences are used to show observable patterns. • Patterns are used to solve problems. • Functions can be used to describe patterns.
2	**The Cat's Out of the Bag!** Generating Algebraic Expressions	A.SSE.1b A.CED.1 F.IF.4 F.IF.8 F.BF.1a	2	This lesson revisits the three scenarios from the previous lesson. Students will write equivalent algebraic expressions for the tile pattern of a square floor to determine the number of new tiles that must be added to create the next square tile design. They then show that the expressions are equivalent using the distributive property and combining like terms. In the second activity, equivalent expressions are written to represent the exponential situation for keeping secrets. Students then prove the expressions to be equivalent algebraically and graphically. Next, using the patio design situation, students will determine the number of squares in the next two patio designs and write equivalent expressions that determine the total number of squares in any given design. Again, the expressions are proven equivalent algebraically and graphically. The last activity summarizes the lesson using a geometric pattern.	• Two or more algebraic expressions are equivalent if they produce the same output for all input values. • You can use the properties of a graph to prove two algebraic expressions are equivalent.
3	**Samesies** Comparing Multiple Representations of Functions	A.SSE.1b A.CED.1 F.IF.8 F.IF.9	2	The terms relation, function, and function notation are defined in this lesson. A sorting activity is presented that includes graphs, tables, equations, and contexts. Students sort the various representations into groups of equivalent relations. The various representations are then categorized with respect to their function families. Students then analyze a tile pattern and use a table to organize data, which leads to discovering additional patterns. Next, they create expressions that represent the number of white, gray and total tiles for any given design. Within the context of the problem situation, students use algebra to show different functions are equivalent and to identify them as quadratic functions.	• A relation is a mapping between a set of input values and a set of output values. • A function is a relation such that for each element of the domain there exists exactly one element in the range. • Function notation is a way to represent functions algebraically. The function $f(x)$ is read as "f of x" and indicates that x is the input and $f(x)$ is the output. • Tables, graphs, and equations are used to model function and non-function situations. • Equivalent expressions can be determined algebraically and graphically. • Graphing technology can be used to verify equivalent function representations.

*Pacing listed in 45-minute days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
4	**True to Form** Forms of Quadratic Functions	A.SSE.1a A.SSE.2 A.APR.1 A.CED.1 A.CED.2 F.IF.4 F.IF.9 F.BF.1a	2	Students match quadratic equations with their graphs using key characteristics. The standard form, the factored form, and the vertex form of a quadratic equation are reviewed as is the concavity of a parabola. Students then sort each of the equations with their graphs depending on the form in which the equation is written, while identifying key characteristics of each function such as the axis of symmetry, the *x*-intercept(s), concavity, the vertex, and the y-intercept. Next, students analyze graphs of parabolas in relation to a pair of numberless axes and select possible functions that could model the graph. A worked example shows that a unique quadratic function is determined when the vertex and a point on the parabola are known, or the roots and a point on the parabola are known. Students are given information about a function and use it to determine the most efficient form (standard, factored, vertex) to write the function. They then use the key characteristics of a graph and reference points to write a quadratic function, if possible. Finally, students analyze a worked example that demonstrates how to write and solve a system of equations to determine the unique quadratic function given three points on the graph. They then use this method to determine the quadratic function that models a problem situation and use it to answer a question about the situation.	• The standard form of a quadratic function is written as $f(x) = ax^2 + bx + c$, where a does not equal 0. • The factored form of a quadratic function is written as $f(x) = a(x - r_1)(x - r_2)$, where a does not equal 0. • The vertex form of a quadratic function is written as $f(x) = a(x - h)^2 + k$, where a does not equal 0. • The concavity of a parabola describes whether a parabola opens up or opens down. A parabola is concave down if it opens downward, and is concave up if it opens upward. • A graphical method to determine a unique quadratic function involves using key points and the vertical distance between each point in comparison to the points on the basic function. • An algebraic method to determine a unique quadratic function involves writing and solving a system of equations, given three reference points.
5	**The Root of the Problem** Solving Quadratic Equations	A.REI.4 A.REI.4a A.REI.4b A.REI.7	2	Students solve quadratic equations of the form $y - ax^2 + bx + c$. They first factor trinomials and use the Zero Product Property. Students then use the method of completing the square to determine the roots of a quadratic equation that cannot be factored. They use the Quadratic Formula to solve problems in real-world and mathematical problems. Finally, students solve a system composed of two quadratic equations using substitution and factoring.	• One method of solving quadratic equations in the form $0 = ax^2 + bx + c$ is to factor the trinomial expression and use the Zero Product Property. • When a quadratic equation in the form $0 = ax^2 + bx + c$ is not factorable, completing the square is an alternative method of solving of the equation. • The Quadratic Formula, $x = (-b\pm\sqrt{b^2 - 4ac})/2a$, can be used to solve to any quadratic equation written in general form, $0 = ax^2 + bx + c$, where a, b, and c represent real numbers and a (not equal to) 0. • A system of equations containing two quadratic equations can be solved algebraically and graphically. • The Quadratic Formula, substitution, and factoring are used to algebraically solve systems of equations. • A system of equations containing two quadratic equations may have no solution, one solution, two solutions, or infinite solutions.

*Pacing listed in 45-minute days

CARNEGIE LEARNING

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
6	***i* Want to Believe** Imaginary and Complex Numbers	N.CN.1 N.CN.2 N.CN.7 N.CN.8 (+) N.CN.9 (+) A.REI.4b	2	Students begin by analyzing a quadratic function that does not cross the *x*-axis and use the Properties of Equality and square roots to solve the corresponding equation for its roots. Students are introduced to the number *i*, imaginary roots, imaginary zeros, and the complex number system. They use a complex coordinate plane to plot complex numbers and then use the graphical representation to understand how to add, subtract, and multiply complex numbers. Students solve quadratic equations with complex solutions using any method of their choosing. Finally, the Fundamental Theorem of Algebra is introduced, and students analyze graphs to determine the number of real and imaginary roots each corresponding quadratic equation has.	• Equations with no solution in one number system may have solutions in a larger number system. • The number i is a number such that $i^2 = -1$. • The set of complex numbers is the set of all numbers written in the form $a + bi$, where a and b are real numbers and b is not equal to 0. • The Commutative Property, the Associative Property, and Distributive Properties apply to complex numbers. • Functions that do not intersect the *x*-axis have imaginary zeros. • When the discriminant of a quadratic equation is a negative number, the equation has two imaginary roots. • The Fundamental Theorem of Algebra states that any polynomial equation of degree *n* must have n complex roots or solutions.
Learning Individually with MATHia or Skills Practice		A.SSE.1 A.SSE.1b A.SSE.2 A.REI.4b F.IF.4 F.BF.3	8	Students watch a video and analyze three different patterns to generate linear, exponential, and quadratic algebraic expressions. They review three familiar function families—linear, quadratic, and exponential. They practice matching the equation of a function and the graph of a function to one of these function families. Students then identify key characteristics from the graph of a function, such as the intercepts, minimum and maximum *x*-values, minimum and maximum *y*-values, domain, and range. They use Explore Tools to investigate transformations of linear, exponential, and quadratic functions, including horizontal and vertical translations and dilations. Students use the Explore Tools to solve real-world problems modeling changes to an exponential function describing doubling and to a quadratic function describing the height of a jump. Students sort functions based upon whether they are written in standard, factored, or vertex form. They identify the concavity and *y*-intercept from functions in standard form, the concavity and *x*-intercepts from functions in factored form, and the concavity, vertex, and axis of symmetry from functions in vertex form. Given graphs, they use key characteristics to select the function that generates the graph. Students complete a table of values and graph from a scenario represented by a quadratic model. They construct the quadratic function for the scenario as a product of a monomial and a binomial or as the product of two binomials. They watch an animation introducing them to the imaginary number line and its relation to the real number line. They then practice identifying real and imaginary numbers through the sorting tool. Next, students are introduced to complex numbers and practice identifying them on the complex plane. They simplify radical expressions that result in complex numbers, identify expressions that are equivalent to *i*, -1, -*i*, and 1, and use the definition of *i* to rewrite higher powers of *i*. Students add, subtract, and multiply complex numbers. Finally, students solve quadratic equations, some of which have real solutions and some of which have complex solutions. **MATHia Unit:** Searching for Patterns **MATHia Workspaces:** Exploring and Analyzing Patterns / Comparing Familiar Function Representations **MATHia Unit:** Graphs of Functions **MATHia Workspaces:** Identifying Key Characteristics of Graphs of Functions / Transforming Functions **MATHia Unit:** Forms of Quadratic Functions **MATHia Workspaces:** Examining the Shape and Structure of Quadratic Functions / Quadratic Modeling / Quadratic Equation Solving / Quadratic Transformations **MATHia Unit:** Operations with Complex Numbers **MATHia Workspaces:** Introduction to Complex Numbers / Simplifying Radicals with Negative Radicands / Simplifying Powers of i / Adding and Subtracting Complex Numbers / Multiplying Complex Numbers / Solving Quadratic Equations with Complex Roots	

*Pacing listed in 45-minute days

CARNEGIE LEARNING

Topic 2: Composing and Decomposing Functions

Composing and Decomposing Functions introduces students to the concept of building new functions on the coordinate plane by operating on or translating functions. They build physical models of real-world scenarios and use what they know about linear functions to model linear dimensions. Students multiply these functions to build a quadratic function graphically and algebraically. Using what they already know about function transformations, students transform functions by variable amounts to build cubic functions. Students then consider new physical models and build cubic functions by multiplying three linear factors and by multiplying a linear factor by a quadratic factor. They are finally introduced to multiplicity, and they use the zero(s) of each factor and the signs of each linear function over given intervals of the x-value to sketch the graphs of functions.

Standards: N.CN.9 (+), A.SSE.1b, A.APR.1, A.APR.3, A.REI.10, A.REI.11, F.IF.4, F.IF.5, F.IF.7a, F.IF.7c, F.BF.3, G.GMD.3 **Pacing:** 10 Days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
1	**Blame It on the Rain** Modeling with Functions	A.SSE.1b A.APR.3 A.REI.11	1	This lesson presents a storm drain problem situation. Students use this situation to calculate the length of the drain, the width of the drain, and the maximum cross-sectional area of the drain in two different situations. They create tables of values, equations, and graphs to represent each situation. Students then identify the function that represents the cross-sectional area of the drain as quadratic and the two factors that represent the length and width of the drain as linear. Students analyze the graph by relating the intercepts and axis of symmetry to this problem situation. Students learn the steps of the mathematical modeling process and describe how they use these steps in modeling the drain problem.	• Tables, graphs, and equations can be used to model real-world situations. • A function created by the product of two linear factors is a quadratic function. • The steps of the modeling process are Notice and Wonder, Organize and Mathematize, Predict and Analyze, and Test and Interpret.
2	**Folds, Turns, and Zeros** Transforming Function Shapes	F.IF.7c F.BF.3	1	In this lesson, students dilate functions by non-constant values in order to create higher degree functions. They begin by adding the function $y = x$ to the constant function $y = 3$ and interpret that operation as a translation of all the points on the horizontal line $y = 3$ by x, or the x-coordinate of each point. Students observe the change in the function produced by the translation and identify the points that did not move, along with the zeros. Students then dilate linear functions with both positive and negative slopes by x and observe how the quadratic function is formed in each case. Again, students analyze how the new zeros are created by each transformation and observe how the factor functions affect the intervals of increase and decrease of the product functions. Students repeat this analysis when dilating a quadratic, or degree-2, function to create a degree-3 function. Finally, students summarize what they have observed regarding dilations, linear factors, zeros, and the	• Functions can be translated and dilated by non-constant values, which apply a different transformation to each point of the function. • The linear factors of a function indicate the locations of the zeros of the function composed of those functions. • When a linear function is dilated vertically by multiplying the function by another linear function, the resulting function is a degree-2 function. • When a quadratic function is dilated vertically by multiplying the function by a linear function, the resulting function is a degree-3 function. • The graph of a function behaves differently at zeros described by linear factors and factors of degree 2. • The linear factors of a function can be used to sketch the graph of a function.

*Pacing listed in 45-minute days

Algebra II Textbook
Table of Contents

CARNEGIE LEARNING

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
3	**Planting the Seeds** Exploring Cubic Functions	A.REI.10 F.IF.4 F.IF.5 F.IF.7a F.IF.7c G.GMD.3	2	A rectangular sheet of copper is used to create planters if squares are removed from each corner of the sheet and the sides are then folded upward. Students analyze several sizes of planters and calculate the volume of each size. They then write a volume function in terms of the height, length, and width and graph the function using a graphing calculator. Using key characteristics, students analyze the graph. They differentiate the domain and range of the problem situation from the domain and range of the cubic function. The second activity is similar, but uses a cylindrical planter.	• Cubic functions can be used to model real-world contexts such as volume. • The general form of a cubic function is written as $f(x) = ax^3 + bx^2 + cx + d$, where $a \neq$) 0. • A relative maximum is the highest point in a particular section of a graph, while a relative minimum is the lowest point in a particular section of a graph. • A cubic function may be created by the product of three linear functions or the product of a quadratic function and a linear function.
4	**The Zero's the Hero** Decomposing Cubic Functions	N.CN.9 (+) A.APR.1 A.APR.3 F.IF.7a F.IF.7c	2	Students investigate the multiplicity of the zeros of a polynomial function. They use these zeros, with multiplicity, to show the decompositions of quadratic and cubic functions into their linear and quadratic factors and reconstruct the product functions using these factors. Students review multiplying binomials in order to build polynomial expressions algebraically as well as graphically. They compare degree-1, degree-2, and degree-3 equations.	• The Fundamental Theorem of Algebra states that a degree n polynomial has, counted with multiplicity, exactly n zeros. • The Zero Product Property states that if the product of two or more factors is equal to zero, then at least one factor must be equal to zero. • The graph of a function written in factored form and the graph of a function written in general form is the same graph when the functions are equivalent. • Graphing is a strategy used to determine whether functions are equivalent. • The product of three linear functions is a cubic function, and the product of a quadratic function and a linear function is a cubic function. • Quadratic and cubic functions can be decomposed and analyzed in terms of their zeros
	Learning Individually with MATHia or Skills Practice	A.APR.1 A.APR.3 F.IF.4 F.IF.5 F.BF.1b	4	Students solve two contextual problems involving cubic functions. They first use a function to solve for the dependent variable, then use the graph to solve for the independent variable, and finally interpret a minimum or maximum point. Students explore power functions, concentrating on cubics and quartics and the key characteristics of end behavior and extrema. They determine end behavior based upon the function's degree and the sign of its leading term. They use graphs to determine extrema, relative maxima and minima, and absolute maxima and minima. Students classify graphs of polynomial functions by their key characteristics. Given graphs in and out of context, they interpret the meaning of relative maximum and minimum points, *y*-intercepts, *x*-intercepts, and increasing and decreasing intervals. Students are then introduced to real zeros, imaginary zeros, and zeros with multiplicity. They identify the number and types of zeros given graphs of cubic and quartic functions. For each given graph, they select the function that represents the graph based upon its degree and a-value; in one case, differentiation between the number of zeros is also required. Finally, given two polynomial functions in different representations—equation, graph, table, or description—with a contextual or non-contextual scenario, students compare the functions' degrees, extrema, rates of change, or zeros over a specific interval. **MATHia Unit:** Graphs of Polynomial Functions **MATHia Workspaces:** Modeling Polynomial Functions	

*Pacing listed in 45-minute days

07/23/19

Topic 3: Characteristics of Polynomial Functions

Students explore power functions to gain an understanding of end behavior and symmetry and their connection to even-degree and odd-degree functions. They then explore even and odd functions and determine whether several polynomial functions are even, odd, or neither. Questions ask students to graph, write, and explain the effects of transformations on cubic functions, and then draw conclusions about how symmetry is preserved in transformed functions. Questions ask students to compare and contrast the various polynomials to understand all the possible shapes and key characteristics for linear, quadratic, cubic, quartic, and quintic functions.

Standards: A.APR.3, A.CED.3, A.REI.11, F.IF.4, F.IF.6, F.IF.7c, F.IF.9, F.BF.1b, F.BF.3 **Pacing:** 13 Days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
1	**So Odd, I Can't Even** Power Functions	F.IF.7c F.BF.3	1	This lesson focuses on power functions described by the equation $P(x) = ax^n$. Students generalize the end behavior of even-degree and odd-degree power functions and sketch the graphs of power functions with negative coefficients. They explore the symmetry of graphs, concluding that even functions have line symmetry about $x = 0$ and odd functions have point symmetry about the origin. Students explore even and odd functions and determine algebraically whether given polynomial functions are even, odd, or neither.	• A power function is a function of the form $P(x) = ax^n$, where n is a non-negative integer. • For both odd- and even- degree functions, the graphs flatten as the degree increases for x-values between -1 and 1, and the graphs steepen as the degree increases for x-values less than -1 and greater than 1. • The end behavior of a graph of a function is the behavior of the graph as x approaches infinity and as x approaches negative infinity. • If a graph is symmetric about a line, the line divides the graph into two identical parts. • A function is symmetric about a point if each point on the graph has a point the same distance from the central point, but in the opposite direction. • When a point of symmetry is the origin, the graph is reflected across the x-axis and the y-axis. If (x, y) is replaced with $(-x, -y)$, the function remains the same. • The graph of an even function is symmetric about the y-axis, thus $f(x) = f(-x)$. • The graph of an odd function is symmetric about the origin, thus $f(x) = -f(-x)$.
2	**Math Class Needs a Makeover** Transformations of Polynomial Functions	F.BF.3	2	Using a table of values, reference points and symmetric properties, students will graph quadratic and cubic functions. Students recall the transformational function form $g(x) = Af(B(x - C)) + D$, and they use transformations to graph polynomial functions, write equations for these functions, and explain the effects of the transformations. The general form of a polynomial function is given, and quartic and quintic functions are defined. Students use the graphs of functions to determine whether the functions are odd, even, or neither. Tables are used to organize the effects of transformations on the basic cubic and quartic functions as well as simple polynomial functions. Graphs of functions that have undergone multiple transformations are given, and students write the appropriate equation to describe each graph.	• A quartic function is a fourth degree polynomial function, and a quintic function is a fifth degree polynomial function. • The function $g(x) = Af(B(x - C)) + D$ is the transformation function form, where the constants A and D affect the output values of the function and the constants B and C affect the input values of the function. • The general shape and end behavior is the same for all odd-degree power functions, and the general shape and end behavior is the same for all even-degree power functions. • The graph of even functions are symmetric about the y-axis. • The graph of odd functions are symmetric about the origin. • Some transformations affect the symmetry of the polynomial function.

*Pacing listed in 45-minute days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
3	**Poly-Wog** Key Characteristics of Polynomial Functions	A.APR.3 F.IF.4	2	Students analyze a quartic regression equation and the corresponding graph. They use the graph to state all relative maximums, relative minimums, absolute maximums, and absolute minimums of the function. Students then use graph parts to investigate key characteristics of polynomial functions with given end behavior. Next, students analyze tables listing all the possible combinations of real and imaginary zeros for a linear, quadratic, and cubic function, along with examples of graphs with the given combination of zeros. They then complete similar tables for a quartic and a quintic function. Finally, students are given sets of specific key characteristics and sketch a graph that encompasses these aspects for each situation when possible.	• A polynomial with an even power has end behavior that is the same in both directions. A polynomial with an odd power has end behavior that is opposite in each direction. • An nth-degree odd polynomial has zero or an even number of extrema. An *n*th-degree even polynomial has an odd number of extrema. In either case, the maximum number of extrema is $n - 1$. • A polynomial function changes direction at each of its extrema. For that reason, the number of extrema and the number of changes of direction in the graph of the function are equal. • A polynomial with an even power has an even number of intervals of increase or decrease. A polynomial with an odd power has an odd number of intervals of increase or decrease. • The combination of real and imaginary roots of a polynomial function are equal to the degree of the polynomial and can be used to help determine the shape of its graph.
4	**Function Construction** Building Cubic and Quartic Functions	A.APR.3 F.IF.7c F.BF.1b	2	Students analyze a set of linear and quadratic functions. They compose these functions to build cubic functions, given the three zeros of the function or other key characteristics of the function. Students reason that a cubic function may have 0 or 2 imaginary zeros and that multiple cubic functions can be written from a given set of zeros. Next, they describe different combinations of function types that build a quartic function. Students analyze tables representing three functions and determine whether the third function is quartic and identify the number of real and imaginary zeros and the end behavior of the function. Finally, they analyze a set of linear, quadratic, and cubic functions. Students sketch a combination of these functions whose product builds a quartic function when possible, given specific criteria. They build a polynomial function given a set of zeros and given a graph, describing the characteristics of the function and comparing both processes.	• Cubic functions can be the product of three linear functions or the product of a quadratic function and a linear function. • A cubic function may have 0 or 2 imaginary zeros. • Quartic functions can be the product of four linear functions, two quadratic functions, a quadratic function and two linear functions, or a cubic function and a linear function. • A quartic function may have 0, 2, or 4 imaginary zeros. • An infinite number of functions can be written from a given set of zeros. • A unique function can be written from the graph of a function. • Functions of degree *n* are composed of factors whose degree sum to *n*. • A polynomial function may have a combination of real and imaginary zeros.
5	**Level Up** Analyzing Polynomial Functions	A.CED.3 A.REI.11 F.IF.4 F.IF.6	1	A cubic function is used to model the profit of a business over a period of time. Students analyze the graph using key characteristics, and then use the graph to answer questions relevant to the problem situation. The average rate of change of a function is defined, and a worked example demonstrates how to calculate the average rate of change for a specified time interval, and students calculate an average rate of change over a different time interval.	• The average rate of change of a function is the ratio of the change in the dependent variable to the change in the independent variable over a specified interval. • The formula for average rate of change is $f(b) - f(a)$ / $b - a$ for an interval (a, b). The expression $b - a$ represents the change in the input of the function f. The expression $f(b) - f(a)$ represents the change in the function f as the input changes form a to b.

*Pacing listed in 45-minute days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
6	**To a Greater or Lesser Degree** Comparing Polynomial Functions	F.IF.9	2		• Polynomial functions can be compared using graphs, tables, and equations. • Analyzing key characteristics of polynomial functions allows for comparison of the functions.
Learning Individually with MATHia or Skills Practice		A.APR.3 F.IF.6	3	Students sketch third- and fourth-order polynomial functions. They are then provided the formula for the average rate of change for non-linear functions. The formula determines the rate of change of the line segment connecting the endpoints of a specified interval. Students estimate the average rate of change of polynomial functions for a specified interval when given a graph. They then use the formula to calculate the average rate of change of polynomial functions for a specified interval when given a graph. Finally, given two polynomial functions in different representations—equation, graph, table, or description—students compare the functions' degrees, extrema, rates of change, or zeros over a specific interval. **MATHia Unit:** Graphs of Polynomial Functions **MATHia Workspaces:** Analyzing Polynomial Functions / Classifying Polynomial Functions / Interpreting Key Features of Graphs in Terms of Quantities / Identifying Key Characteristics of Polynomial Functions / Identifying Zeros of Polynomials / Using Zeros to Sketch a Graph of Polynomial / Understanding Average Rate of Change of Polynomial Functions / Comparing Polynomial Functions in Different Forms	

*Pacing listed in 45-minute days

2 Developing Structural Similarities

Pacing: 37 Days

Topic 1: Relating Factors and Zeros

This topic presents opportunities for students to analyze, factor, solve, and expand polynomial functions. Relating Factors and Zeros begins with students expanding their knowledge of factoring quadratics to include polynomials. They use factors to determine zeros and sketch graphs of the functions. Students learn to divide polynomials using two methods and to expand on this knowledge to determine whether a divisor is a factor of the dividend. In addition, they determine that polynomial functions, just like the integers, are closed under addition, subtraction, and multiplication but not division. Finally, students solve polynomial inequalities graphically and algebraically.

Standards: A.APR.3, A.CED.3, A.REI.11, F.IF.4, F.IF.6, F.IF.7c, F.IF.9, F.BF.1b, F.BF.3 **Pacing:** 10 Days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
1	**Satisfactory Factoring** Factoring Polynomials to Identify Zeros	N.CN.8 (+) A.SSE.2 A.APR.3 F.IF.8a	2	Students recall factors of whole numbers in preparation for determining factors of polynomials. Methods of factoring polynomials are introduced, such as factoring out the Greatest Common Factor (GCF), chunking, recognizing perfect square trinomials, factoring by grouping, and factoring in quadratic form. Worked examples are used throughout the lesson to show the steps involved using the methods. Students write polynomials in factored form over the set of real numbers and over the set of complex numbers. They also determine the most efficient method of factoring several polynomials and explain their reasoning.	• The graphs of all polynomials that have a monomial GCF that includes a variable will pass through the origin. • Analyzing the structure of a polynomial may help you determine which factoring method may be most helpful. • Chunking is a method of factoring a polynomial in quadratic form that does not have common factors in all terms. Using this method, the terms are rewritten as a product of 2 terms, the common term is substituted with a variable, and then it is factored as is any polynomial in quadratic form. • Factoring a perfect square trinomial can occur in two forms: $a^2 - 2ab + b^2 = (a - b)^2$ or $a^2 + 2ab + b^2 = (a + b)^2$ • Factoring by grouping is a method of factoring a polynomial that has four terms in which not all terms have a common factor. The terms can be first grouped together in pairs that have a common factor, and then factored again. • Factoring by using quadratic form is a method of factoring a polynomial of degree 4 of the form, $ax^4 + bx^2 + c$. • Factoring the difference of squares is in the form: $a^2 - b^2 = (a + b)(a - b)$.
2	**Divide and Conquer** Polynomial Division	N.CN.8 (+) A.SSE.1a A.SSE.2 A.SSE.3a A.APR.1 A.APR.2	2	The algebraic representation of a cubic function is given and its graph is shown. Students determine the real factor of the function from the graph. They reason that the other factor must be a quadratic function with imaginary zeros, but they cannot represent it algebraically yet. A worked example of polynomial long division is provided and students determine the quadratic function that is the other factor. They distinguish between factoring over the real and the complex number system by determining the imaginary zeros of the quadratic function and rewriting the cubic function as a product of linear factors. Next, students investigate what the remainder means in terms of polynomial division. The Remainder Theorem is stated and students use the theorem to answer questions involving polynomial division with remainders. Finally, a worked example of synthetic division is provided. Students use the algorithm to determine the quotient in several problems.	• Factors of polynomials divide into a polynomial without a remainder. • A polynomial equation of degree n has n roots over the complex number system and can be written as the product of n factors of the form $(ax + b)$. • Polynomial long division is an algorithm for dividing one polynomial by another of equal or lesser degree. • Synthetic division is a shortcut method for dividing a polynomial by a linear expression of the form $(x - r)$. • The Factor Theorem states that a polynomial function $p(x)$ has $x - r$ as a factor if and only if the value of the function at r is 0, or $p(r) = 0$. • The Remainder Theorem states that when any polynomial equation or function $f(x)$ is divided by a linear expression of the form $(x - r)$, the remainder is $R = f(r)$ or the value of the function when $x = r$. • The difference of cubes can be written in factored form as: $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$. The sum of cubes can be written in factored form as: $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$.

*Pacing listed in 45-minute days

CARNEGIE LEARNING

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
3	**Closing Time** The Closure Property	A.APR.1	1	Students review the four basic operations over the set of natural numbers, whole numbers, integers, rational numbers, and irrational numbers and determine which operations are closed and not closed over which sets of numbers. They determine that integers and polynomials are not closed under division. The concept of polynomials closed under an operation is defined and students then prove that polynomials are closed under addition, subtraction, and multiplication. Students compare polynomials and use multiple representations to analyze and compare polynomial functions.	• When an operation is performed on any number or expression in a set and the result is in the same set, it is said to be closed under that operation. • Polynomials are closed under addition, subtraction, and multiplication. • Polynomials are not closed under division.
4	**Unequal Equals** Solving Polynomial Inequalities	A.CED.1 A.CED.3	1	Solving polynomial inequalities is very much like solving linear inequalities. Students solve polynomial inequalities both graphically and algebraically. Problem situations include profit models, vertical motion, glucose levels in the bloodstream, and volume. Graphing calculators are used in this lesson.	• Solving polynomial inequalities is similar to solving linear inequalities. • The solutions to a polynomial inequality are intervals of x-values that satisfy the inequality.
Learning Individually with MATHia or Skills Practice		A.APR.1	4	Students add and subtract higher order polynomials. They determine which factor table is appropriate for a given problem, set up the table, and then use the table to multiply polynomials. Students then use synthetic division as an efficient method to divide a higher-order polynomial by a linear divisor. They factor quadratic expressions using all known factoring methods. Students see the algebraic representations that determine the graphs of polynomial functions and make a connection between $f(x) = 0$ and a polynomial equation set equal to zero. They begin to solve polynomial equations by seeing both graphical and algebraic methods for the same equation. Students then focus on cubic functions with multiple or imaginary zeros. They practice solving quartic equations using these same skills. Finally, students solve polynomial inequalities graphically. **MATHia Unit:** Polynomial Operations **MATHia Workspaces:** Using a Factor Table to Multiply Polynomials / Multiplying Polynomials / Solving Quadratic Equations by Factoring / Synthetic Division **MATHia Unit:** Solving Polynomials **MATHia Workspaces:** Factoring Higher Order Polynomials / Solving Polynomial Functions	

*Pacing listed in 45-minute days

CARNEGIE LEARNING

Topic 2: Polynomial Models

In Polynomial Models, students use the concept of equality to express mathematical relationships using different representations. They begin by exploring polynomial identities, which are useful for showing the relationship between two seemingly unrelated expressions. Polynomial identities are used to perform calculations, verify Euclid's Formula, and generate Pythagorean triples. Students then explore patterns in Pascal's Triangle and use it to expand powers of binomials. They apply the Binomial Theorem and its combinatorics as an alternative method to expand powers of binomials. Finally, they move between function representations as they apply polynomial regressions to represent data in context.

Standards: A.APR.4, A.APR.5 (+), A.CED.3, F.IF.4, F.IF.5, F.BF.1, S.ID.6a **Pacing:** 6 Days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
1	**Not a Case of Mistaken Identity** Exploring Polynomial Identities	A.APR.4	1	Polynomial identities such as $(a + b)^2$, $(a - b)^2$, $a^2 - b^2$, $(a + b)^3$, $(a - b)^3$, $a^3 + b^3$, and $a^3 - b^3$ are used to perform calculations involving large numbers without a calculator. Euclid's Formula is stated and used to generate Pythagorean triples. In the last activity, students verify algebraic statements by transforming one side of the equation to show that it is equivalent to the other side of the equation.	• Polynomial identities such as $(a + b)^2 = a^2 + 2ab + b^2$ can be used to help perform calculations with large numbers. • Euclid's Formula can be used to generate Pythagorean triples given positive integers r and s, where $r > s$: $(r^2 + s^2)^2 = (r^2 - s^2)^2 + (2rs)^2$.
2	**Elegant Simplicity** Pascal's Triangle and the Binomial Theorem	A.APR.5 (+)	1	Students analyze and extend the patterns in the rows of Pascal's Triangle. They then explore a use of Pascal's Triangle when raising a binomial to a positive integer. Students expand several binomials using Pascal's Triangle. The combination formula is given and technology and Pascal's Triangle are used to calculate combinations. The Binomial Theorem is stated and students use it to expand $(a + b)^{15}$. Finally, students expand several binomials with coefficients other than 1.	• The Binomial Theorem states that it is possible to extend any power of $(a + b)$ into a sum of the form: $(a + b)^n =$ (n (over) 0) a^nb^0 + (n (over) 1) $a^{n-1}b^1$ + (n (over) 2) $a^{n-2}b^2$ + · · · + (n (over) (n – 1)) a^1b^{n-1} + (n (over) n) a^0b^n • The formula for a combination of k objects from a set of n objects for $n \geq k$ is: (n (over) k) = $_nC_k = n!/(k!(n - k)!)$
3	**Modeling Gig** Modeling with Polynomial Functions and Data	A.CED.3 F.IF.4 F.IF.5 F.BF.1 S.ID.6a	2	Traffic patterns in a downtown area, the federal minimum wage, monthly precipitation, and inflation are contexts modeled by polynomial functions. Data are organized in a table of values for each situation, and students use technology to create a scatter plot and determine polynomial regression equations. The coefficient of determination is used to determine which regression equation best describes the data. The regression equations are used to make predictions, and students then construct graphs to represent different periods of time.	• A regression equation is a function that models the relationship between two variables in a scatter plot. • The coefficient of determination, or R^2, measures the strength of the relationship between the original data and their regression equation. The value ranges from 0 to 1 with a value of 1 indicating a perfect fit between the regression equation and the original data. • Regression equations can be used to make predictions about future events.
Learning Individually with MATHia or Skills Practice		A.APR.1	2	Students can continue the development of factoring and solving polynomials through the MATHia aligned to the previous topic. **MATHia Unit:** Polynomial Models **MATHia Workspaces:** Pascal's Triangle / Binomial Theorem / Exploring Polynomial Regression / Solving Polynomial Inequalities	

*Pacing listed in 45-minute days

CARNEGIE LEARNING

Topic 3: Rational Functions

Students analyze, graph, and transform rational functions. The topic begins with an analysis of key characteristics of rational functions and graphs. Lessons then expand on this knowledge to transform rational functions. Students determine whether graphs of rational functions have vertical asymptotes, removable discontinuities, both, or neither, and then sketch graphs of rational functions detailing any holes and/or asymptotes. They then explore problem situations modeled by rational functions and answer questions related to each scenario. Rational Functions provides opportunities for students to connect their knowledge of operations with rational numbers to operations with rational expressions. They conclude that rational expressions are similar to rational numbers and are closed under all the operations. Students then write and solve rational equations and list restrictions, considering efficient ways to operate with rational expressions and to solve rational equations based on the structure of the original equation. The topic closes with problems related to work, mixture, cost, and distance.

Standards: A.SSE.2, A.APR.6, A.APR.7 (+), A.CED.1, A.REI.1, A.REI.2, A.REI.11, F.IF.5, F.IF.7d (+), F.BF.3, G.MG.2 **Pacing:** 21 Days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
1	**There's a Fine Line Between a Numerator and a Denominator** Introduction to Rational Functions	F.IF.7d (+)	2	Students explore and compare the graphs, tables, and values of two basic functions, $f(x) = x$ and $q(x) = x^2$, and their reciprocal functions, $g(x) = 1 / x$ and $r(x) = 1 / x^2$. Technology is used to explore the key characteristics of the reciprocals of all power functions, including horizontal and vertical asymptotes. Students then construct a Venn diagram to show the similarities and differences between the groups of reciprocal power functions.	• A rational function is any function that can be written as the ratio of two polynomials. It can be written in the form $f(x) = P(x) / Q(x)$, where $P(x)$ and $Q(x)$ are polynomial functions, and $Q(x) \neq 0$. • The reciprocals of power functions are rational functions. • A vertical asymptote is a vertical line that a function gets closer and closer to, but never intersects. • The reciprocals of all power functions have a vertical asymptote at $x = 0$, a horizontal asymptote at $y = 0$, and a domain of all real numbers except $x \neq 0$. • The reciprocals of power functions with an exponent that is an even number lie in Quadrants I and II, and their range is $y > 0$. • The reciprocals of power functions with an exponent that is an odd number lie in Quadrants I and III, and their range is all real numbers except y (not equal to) 0.
2	**Approaching Infinity** Transformations of Rational Functions	F.BF.3	2	Students explore transformations of rational functions. Without using technology, students sketch several rational functions and indicate the domain, range, vertical and horizontal asymptotes, and the y-intercept. They then match or sketch transformed rational functions with their graphs and vice versa.	• Translations of a rational function $f(x)$ are given in the form $g(x) = Af(B(x - C)) + D$, where a negative A-value reflects $f(x)$ vertically, the D-value translates $f(x)$ vertically, and a C-value translates $f(x)$ horizontally. • The C-value affects the vertical asymptote. The vertical asymptote affects the domain. • The D-value affects the horizontal asymptote. The horizontal asymptote affects the range. • Vertical asymptotes of a rational function can be determined by identifying values of x for which the denominator equals 0. • The reciprocal of a function of degree n can have at most n vertical asymptotes.

*Pacing listed in 45-minute days

CARNEGIE LEARNING

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
3	**There's a Hole in My Function!** Graphical Discontinuities	F.IF.7d (+) F.IF.8a	2	Students match rational functions with their graphs and recognize that functions with a common factor in the numerator and denominator have removable discontinuities in their graphs, whereas functions that have undefined values in the denominator have vertical asymptotes. They graph several rational functions containing holes or asymptotes. A table shows similarities between rational numbers and rational functions, and students list any restrictions in the domain for each example. They then analyze a worked example and explain why a hole and a vertical asymptote are both present in the graph of the function. Given the functions, students determine whether the graphs of rational functions have vertical asymptotes, removable discontinuity, both, or neither.	• A removable discontinuity is a single point at which the graph is not defined. • The graphs of rational functions have either a removable discontinuity or a vertical asymptote for all domain values that result in division by 0. • Holes are created in the graphs of rational functions when a common factor divides out of the numerator and denominator of the function.
4	**Must Be a Rational Explanation** Operations with Rational Expressions	A.SSE.2 A.APR.6 A.APR.7 (+)	3	The process for adding and subtracting rational expressions is compared to the process for adding and subtracting rational numbers. Students add and subtract several rational expressions by first determining common denominators and identifying restrictions on the domain of the function. The process for multiplying and dividing rational expressions is similar to the process for multiplying and dividing rational numbers. Students then multiply and divide several rational expressions and list restrictions on the variables, recognizing that the processes are similar to multiplying and dividing rational numbers. Students determine that the set of rational expressions is closed under addition, subtraction, multiplication, and division.	• The processes of adding, subtracting, multiplying, and dividing rational expressions are similar to the processes for rational numbers. • To determine the least common denominator of algebraic expressions, first factor the expressions and divide out common factors. • The domain restrictions for a rational expression must be based upon the original expressions. • Rational expressions are closed under the operations of addition, subtraction, multiplication, and division.
5	**Thunder. Thun-Thun- Thunder.** Solving Problems with Rational Equations	A.SSE.2 A.CED.1 A.REI.1 A.REI.2 F.IF.5	2	The average cost per month for cable television, grams of chocolate in trail mix, a thunderstorm, and the Golden Ratio are all situations students model using rational equations. They answer questions related to each scenario, create proportions, write rational expressions, describe the behavior of the ratios in the proportions, identify the domain and range, and calculate average costs. Students use multiple methods to solve rational equations, which are identified as proportions that students have solved in previous courses. A sorting activity is used to group and solve rational equations by different methods.	• Rational functions can be used to model real-world problems. • A rational equation is an equation that contains one or more rational expressions. Rational equations are proportions. • The structure of an equation often determines the most efficient method to solve the equation.

*Pacing listed in 45-minute days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
6	**16 Tons and What Do You Get?** Solving Work, Mixture, Distance, and Cost Problems	A.CED.1 A.CED.4 A.REI.1 A.REI.2 G.MG.2	2	Rational equations are used to model work problems, mixture problems, distance problems, and cost problems.	• Rational functions can be used to model real-world problems. • A work problem is a type of problem that involves the rates of several workers and the time it takes to complete a job. • A mixture problem is a type of problem that involves the combination of two or more liquids and the concentrations of those liquids. • A distance problem is a type of problem that involves distance, rate, and time. • A cost problem is a type of problem that involves the cost of ownership of an item over time.
Learning Individually with MATHia or Skills Practice		A.APR.6 A.CED.1 A.REI.2 F.IF.7d (+)	8	Students are given the definition of a rational function and use the definition to sort given functions as rational functions or not. They are then shown the graph of a rational function and introduced to horizontal and vertical asymptotes. Students use a function and its graph to describe the asymptotes. Next, they determine the asymptotes of rational functions using only the equation. Students solve rational equation problems using a worksheet format, with separate columns for the independent quantity, the numerator, the denominator, and the rational expressions. Students rewrite simple rational expressions, products and quotients of rational expressions, and sums and differences of rational expressions. They then solve rational equations and classify the solutions as valid or extraneous. Students either write expressions for given problem entity descriptions or equate two expressions to solve for an unknown. Finally, they solve work, mixture, and distance problems. **MATHia Unit:** Rational Functions **MATHia Workspaces:** Introduction to Rational Functions / Modeling Ratios as Rational Functions **MATHia Unit:** Rational Expressions and Equations **MATHia Workspaces:** Simplifying Rational Expressions / Adding and Subtracting Rational Expressions / Multiplying and Dividing Rational Expressions / Solving Rational Equations that Result in Linear Equations **MATHia Unit:** Rational Models **MATHia Workspaces:** Modeling Rational Functions / Using Rational Models / Solving Work, Mixture, and Distance Problems / Modeling and Solving with Rational Functions	

*Pacing listed in 45-minute days

CARNEGIE LEARNING

3 Inverting Functions

Pacing: 40 Days

Topic 1: Radical Functions

This topic presents opportunities for students to explore radical functions, simplify radical expressions, and solve radical equations. Radical Functions begins with an introduction to radical functions as inverses of power functions. Students graph radical functions, write their equations, and determine their key characteristics. Lessons then expand on this knowledge to explore transformations of radical functions. In the later part of the topic, students rewrite radicals using rational exponents and extract roots from radical expressions. Students also multiply, divide, add, and subtract radical expressions. Finally, students analyze solution strategies for radical equations, and use radical equations to solve real-world problem situations.

Standards: N.RN.1, N.RN.2, A.CED.4, A.REI.2, F.IF.4, F.IF.5, F.IF.7b, F.IF.9, F.BF.1c (+), F.BF.3, F.BF.4a, F.BF.4b (+), F.BF.4c (+), F.BF.4d (+), G.MG.2, G.MG.3 **Pacing:** 13 Days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
1	**Strike That, Invert It** Inverses of Power Functions	F.IF.4 F.IF.7b F.BF.4c (+)	1	Students trace the graphs of several power functions on patty paper. They transpose the independent and dependent quantities for each situation and determine whether the graph of the new function is also a function. The Vertical Line Test is used to determine whether or not the inverse of a power function is also a function. Students conclude that the Horizontal Line Test can be used to determine whether the inverse of a function is also a function. The term invertible function is defined.	• A function is the set of all ordered pairs (x, y), or $(x, f(x))$, where for every value of x there is one and only one value of y, or $f(x)$. • The inverse of a function is the set of all ordered pairs (y, x), or $(f(x), x)$. • If the inverse of a function is also a function, the function is said to be an invertible function, and its inverse is written as $f^{-1}(x)$. • A Horizontal Line Test is a visual method to determine whether a function has an inverse that is also a function. • A power function is a polynomial function of the form $P(x) = ax^n$, where n is a non-negative integer. When n is an odd number, the function is invertible, and when n is an even number, the function is not invertible.
2	**Such a Rad Lesson** Radical Functions	F.IF.4 F.IF.5 F.IF.7b F.BF.1c F.BF.4a F.BF.4b (+) F.BF.4d (+) G.MG.2	3	Students determine and graph the inverse of the power function $f(x) = x^2$ by transposing the coordinates of the points in a table of values. They restrict the domain to create a square root function—the inverse of a power function—and identify the key characteristics of each. They also determine the cube root function—the inverse of the power function $f(x) = x^3$—and conclude that the domain need not be restricted to create this inverse function. The general term radical function is introduced and defined. Students learn to use the composition of functions to determine algebraically whether pairs of functions are inverse functions. Students answer questions related to radical functions in real-world and mathematical problems.	• The square root function is the inverse of the power function $f(x) = x^2$ when the domain of the power function is restricted to values greater than or equal to 0. • The cube root function is the inverse of the power function $f(x) = x^3$. • Radical functions are inverses of power functions with exponents greater than or equal to 2. • For two functions f and g, the composition of functions uses the output of one as the input of the other. It is expressed as $f(g(x))$ or $g(f(x))$. If $f(g(x)) = g(f(x)) = x$, then $f(x)$ and $g(x)$ are inverse functions. • Radical functions may be used to model and solve real-world problems.
3	**Making Waves** Transformations of Radical Functions	F.IF.5 F.IF.7b F.IF.9 F.BF.3	1	Transformations of radical functions are used to create a graphic design that will serve as a logo for a surfing school. Students write equations and graph transformations of radical functions with restricted domains using the transformation function form. The effects of transformations on radical functions are identified, and students describe key characteristics and restrictions on the domains.	• Transformations of radical functions can be described by the transformation function form, $g(x) = Af(B(x - C)) + D$. • Transformations can be used to model graphic designs.

*Pacing listed in 45-minute days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
4	**Keepin' It Real** Rewriting Radical Expressions	N.RN.1 N.RN.2	2	Students analyze tables and graphs for different values of n in the expression $\sqrt[n]{x^n}$ and conclude that when extracting a variable from a radical expression, $\sqrt[n]{x^n}$ can be written as $\lvert x \rvert$ when n is even and as x when n is odd. Next, they use the rules of exponents to verify that a radical expression $\sqrt[n]{x^a}$ can be written as the exponential expression $x^{a/n}$. Students analyze examples of extracting roots and radicals with extracted roots. They multiply, divide, add, subtract, and simplify radical expressions. Finally, students complete a graphic organizer by providing examples of the sum, difference, product, and quotient of radical expressions.	• To extract a variable from a radical, the expression $\sqrt[n]{x^n}$ can be written as $\lvert x \rvert$, when n is even, or x, when n is odd. • A radical expression $\sqrt[n]{x^a}$ can be rewritten as an exponential expression $x^{a/n}$ for all real values of x if the index is n is odd, and for all real values of x greater than or equal to 0 if the index n is even. • The root of a product is equal to the product of its roots, $\sqrt[p]{a^m/b^n} = \sqrt[p]{a^m} \cdot \sqrt[p]{b^n}$. • The root of a quotient is equal to the quotient of its roots, $\sqrt[p]{a^m/b^n} = \sqrt[p]{a^m}/\sqrt[p]{b^n}$. • When multiplying or dividing radical expressions, multiply or divide numbers and variables separately and then extract roots. • Radical expressions with the same degree and same radicand are like terms. • When adding or subtracting radical expressions, add or subtract the coefficients of like terms.
5	**Into the Unknown** Solving Radical Equations	A.CED.4 A.REI.2	1	Students learn solution strategies to solve radical equations and check their answers to identify any extraneous solutions. They solve several radical equations both in and out of context.	• Strategies to solve equations, such as using the Properties of Equality and isolating the term containing the unknown, can be applied be solve radical equations. • Raising both sides of the equation to a power when solving a radical equation may introduce extraneous solutions. To identify extraneous solutions, you must substitute each solution into the original equation to determine whether it results in a true statement. • Radical equations can be used to model real-world problems.
Learning Individually with MATHia or Skills Practice		F.IF.7b F.BF.4 F.BF.4a N.RN.1 N.RN.2	5	Students watch an animation demonstrating that the inverse of a point is formed by reversing its x- and y-coordinates. Thus, the inverse of a function is a reflection of the original function across the line $y = x$. They use the Horizontal Line Test to identify the graphs of inverses of functions and determine whether a function is invertible. Students then learn how to write and graph the square root function as the inverse of the quadratic function $y = x^2$ with a domain restricted to $x > 0$. They identify simple transformations of the square root function and write equations for those transformations. Students graph inverses and reason about the domain and range. Given a function, they determine the equation of the inverse function and use composition of functions to verify that the functions are inverses. Students extract roots from numerical and algebraic radical expressions. They then add, subtract, multiply, and divide numeric and algebraic radical expressions. **MATHia Unit:** Inverses of Functions **MATHia Workspaces:** Investigating Inverses of Functions / Graphing Square Root Functions / Sketching Graphs of Inverses / Calculating Inverses of Linear Functions **MATHia Unit:** Rewriting and Operating with Radicals **MATHia Workspaces:** Simplifying Radicals / Adding and Subtracting Radicals / Multiplying Radicals / Dividing Radicals **MATHia Unit:** Radical Expressions with Variables **MATHia Workspaces:** Simplifying Radicals with Variables / Adding and Subtracting Radicals with Variables	

*Pacing listed in 45-minute days

CARNEGIE LEARNING

Topic 2: Exponential and Logarithmic Functions

Students analyze, graph, and transform exponential and logarithmic functions. Exponential and Logarithmic Functions begins with an exploration of exponential functions. Students analyze key characteristics of exponential functions and graphs. Lessons then expand on this knowledge for transformations of exponential functions. In the later part of the topic, lessons focus on logarithmic functions. Students determine key characteristics of logarithmic functions and graphs. They also transform logarithmic functions and make generalizations about the effect of a transformation on an inverse function.

Standards: A.SSE.3c , A.REI.11 , F.IF.4 , F.IF.5 , F.IF.7e , F.IF.8b , F.IF.9 , F.LE.5 , F.BF.3 , F.BF.4a **Pacing:** 10 Days

*Pacing listed in 45-minute days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
4	**I Like to Move It** Transformations of Exponential and Logarithmic Functions	A.SSE.3c F.BF.3	2	The general transformation function form $g(x) = Af(B(x - C)) + D$ is applied to both exponential and logarithmic functions. Students sketch the graphs of single transformations and multiple transformations of both exponential and logarithmic functions given an equation. They identify any effects that the transformations have on the domain, range, and asymptotes of the functions. Next, students use graphs of functions to write equations for transformed exponential and logarithmic functions. Students describe the graphical transformation(s) performed on an original function that produce a transformed function, given the equations of each. They write a transformed logarithmic function in terms of the original function given the equation of the original function and a key characteristic of the transformed function. Finally, students generalize the effect that a transformation on a function will have on its inverse.	• In the transformation function form $g(x) = Af(B(x - C)) + D$, the D-value translates the function $f(x)$ vertically, the C-value translates $f(x)$ horizontally, the A-value vertically stretches or compresses $f(x)$, and the B-value horizontally stretches or compresses $f(x)$. • Reflections of a basic exponential function do not affect the domain or horizontal asymptote. Reflections of a basic logarithmic function do not affect the range or vertical asymptote. • Vertical translations affect the range and the horizontal asymptote of exponential functions, while horizontal translations affect the domain and the vertical asymptote of logarithmic functions. • Transformations can be described through graphs, tables, key characteristics, writing an equation in terms of the original function, or by using a transformation equation. • A horizontal translation on a function produces a vertical translation on its inverse, while a vertical translation on a function produces a horizontal translation on its inverse. • A vertical dilation on a function produces a horizontal dilation the same number of units on its inverse, and a horizontal dilation on a function produces a vertical dilation the same number of units on its inverse.
Learning Individually with MATHia or Skills Practice		F.IF.7e	2	Students recall exponential functions and identify exponential growth and decay functions by the structure of their equations. Students watch an animation that demonstrates how to build an exponential expression modeling an account balance earning compound interest. They use the formula to determine compound interest amounts. Students then learn about the constant *e* and solve real-world problems about changes in populations using the formula for continuous exponential growth or decay. They then watch an animation demonstrating that a logarithm is an expression equal to the exponent of a corresponding exponential expression and that a logarithmic function is the inverse of the corresponding exponential function. Students evaluate logarithms and generalize about forms such as log(a) *a*, log(a) 1, and log(a) 1/*a*. They identify and analyze logarithmic functions of base 2, 10, and *e*. **MATHia Unit:** Exponential and Logarithmic Functions **MATHia Workspaces:** Properties of Exponential Graphs / Introduction to Logarithmic Functions	

*Pacing listed in 45-minute days

CARNEGIE LEARNING

Topic 3: Exponential and Logarithmic Equations

In Exponential and Logarithmic Equations, students use their understanding of exponential and logarithmic functions to solve exponential and logarithmic equations. Students begin by building understanding and fluency with exponential and logarithmic expressions, including estimating the values of logarithms on a number line and then deriving the properties of logarithms. Students explore alternative methods for solving logarithmic equations and solve exponential and logarithmic equations in context.

Standards: A.REI.11 , F.LE.2 , F.LE.4 , F.BF.5 (+) , S.ID.6a **Pacing:** 10 Days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
1	**All the Pieces of the Puzzle** Logarithmic Expressions	F.BF.5(+)	2	Students convert between exponential and logarithmic forms of an equation. They then use this relationship to solve for an unknown base, exponent, or argument in a logarithmic equation. Students use a number line to estimate logarithms that are irrational numbers. An always, sometimes, never activity is used to summarize the lesson.	• The value of a logarithmic expression is equal to the value of the exponent in the corresponding exponential expression. • For an exponential equation $a^c = b$ and its corresponding logarithmic equation $\log_a b = c$, the variables have the same restrictions. The base, a, must be greater than 0 but not equal to 1, the argument, b, must be greater than zero, and the value of the exponent/logarithm, c, has no restrictions. • A simple logarithmic equation can be solved by converting it to and exponential equation. To solve for an argument in a logarithmic equation, calculate the resulting expression. To solve for an exponent in a logarithmic equation, use like bases. To solve for a base in a logarithmic equation, use common exponents. • You can estimate the value of a logarithm using the relationship that exists between logarithms and exponents. • For a fixed base greater than 1, as the value of the argument increases, the value of the logarithm increases. For a fixed argument, when the value of the base is greater than 1 and increasing, the value of the logarithm is decreasing.
2	**Mad Props** Properties of Logarithms	F.BF.5(+)	1	Students develop rules and properties of logarithms based on their knowledge of various exponent rules and properties. They summarize the different properties by completing a table that defines each exponential and logarithmic property verbally and symbolically, providing examples for each instance.	• Logarithms by definition are exponents, so they have properties that are similar to those of exponents and powers. • The Zero Property of Logarithms states: "The logarithm of 1, with any base, is always equal to 0." • The Logarithm with Same Base and Argument Rule states: "When the base and argument are equal, the logarithm is always equal to 1." • The Product Rule of Logarithms states: "The logarithm of a product is equal to the sum of the logarithms of the factors." • The Quotient Rule of Logarithms states: "The logarithm of a quotient is equal to the difference of the logarithms of the dividend and divisor." • The Power Rule of Logarithms states: "The logarithm of a power is equal to the product of the exponent and the logarithm of the base of the power."

*Pacing listed in 45-minute days

CARNEGIE LEARNING

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
3	**More Than One Way to Crack an Egg** Solving Exponential Equations	F.BF.5(+) F.LE.4	1	Students analyze a real-world context to solve exponential equations by using the Change of Base Formula. They then derive the Change of Base of Formula. Students explore solving an exponential equation by taking the log of both sides of the equation. They analyze alternative solution methods and common misconceptions. Student practice solving exponential equations using methods of their choosing.	• The Change of Base Formula allows you to calculate an exact value for a logarithm by rewriting it in terms of a different base. By writing the logarithm using base 10 or base *e*, technology can be used to evaluate the expressions. • The Change of Base Formula states: $\log_b c = (\log_a c)/(\log_a b)$, where a, b, $c > 0$ and a, $b \neq 1$. • One method to solve an exponential equation is to write the equation in logarithmic form and then apply the Change of Base Formula. • Another method to solve an exponential equation is to take the common logarithm or natural logarithm of both sides of the exponential equation and then use the rules of logarithms to solve for x.
4	**Logging On** Solving Logarithmic Equations	F.BF.5(+) F.LE.4	2	Students solve logarithmic equations for the base, argument, or exponent by rewriting them as exponential equations or using the Change of Base Formula. Properties of logarithms are used to solve equations containing multiple logarithms, and students solve logarithmic equations in real-world contexts. A decision tree is created describing the first step used to solve each type of exponential and logarithmic equation.	• Logarithmic equations can be used to model real-world contexts. • When solving equations with more than one logarithmic expression, the rules of logarithms must first be applied to rewrite the equation as a single logarithm. • The structure of an exponential equation or logarithmic equation determines the most efficient solution strategy.
5	**What's the Use?** Applications of Exponential and Logarithmic Equations	F.BF.5(+) F.LE.2 F.LE.4 S.ID.6a	2	Students use exponential and logarithmic equations that model real-world situations to solve problems related to the situations. They write a function to model exponential decay from the description of a situation using their knowledge of transformation function form and use the function to answer a related question. Students use technology to write a regression equation for an exponential model and use it to solve problems.	• Exponential and logarithmic equations are used to model situations in the real-world. • Rounding too early in a series of calculations involving exponential or logarithmic equations has a great effect on the level of accuracy of the solution.
Learning Individually with MATHia or Skills Practice		F.LE.4	2	Students solve equations of the form $a \cdot b^x = c$ and $a \cdot \log_b x = c$, where b is either 2 or 10. They then solve equations of the form $a \cdot e^x = c$ and $a \cdot \ln x = c$. Finally, students solve equations of the form $a * b^x = c$ and $a \cdot \log_b x = c$, where b is one of 2, e, or 10. **MATHia Unit:** Solve Equations with Base 2, 10, or e **MATHia Workspaces:** Solving Base 2 and Base 10 Equations / Solving Base e Equations / Solving Any Base Equations **MATHia Unit:** Finite Geometric Solutions **MATHia Workspaces:** Introduction to Finite Geometric Series / Problem Solving using Finite Geometric Series	

*Pacing listed in 45-minute days

Topic 4: Applications of Growth Modeling

Students explore various real-world and mathematical situations that are modeled with exponential functions. Lessons provide opportunities for students to apply their understanding of geometric series to solve problems. They also use exponential functions to draw graphics, explore fractals, and study situations modeled by growth.

Standards: A.SSE.1a, A.SSE.4, F.IF.3, F.IF.7a, F.IF.7b, F.IF.7c, F.IF.7d (+), F.IF.7e, F.BF.1a, F.BF.2 **Pacing:** 7 Days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
1	**Series Are Sums** Geometric Series	A.SSE.1a A.SSE.4 F.BF.2	2	The term geometric series is defined, and students explore different methods to compute any geometric series. Worked examples are provided and Euclid's method is introduced. Next, students use the pattern generated from repeated polynomial long division to write a formula for the sum of any geometric series as $S_n = r^n - 1 / r - 1$. They learn that any series where $g1$ does not equal 1 can be rewritten by factoring out a greatest common factor. A second formula to compute any geometric series is derived, $S_n = g_1(r^n - 1) / r - 1$; the equivalence of the two formulas is verified. Students then rewrite a geometric series using summation notation and compute geometric series. Finally, students write explicit formulas to determine payoff amounts for credit cards.	• A geometric series is the sum of the terms of a geometric sequence. • The formula to compute any geometric series is $S_n = g_n(r) - g_1 / r - 1$, where g_n is the last term, r is the common ratio, and g_1 is the first term. • Another formula to calculate the sum of a geometric series is $S_n = g_n(r) - g_1 / r - 1$ where n is the number of terms, r is the common ratio, and g_1 is the first term. • Geometric series can be use to model real-world situations.
2	**Paint By Numbers** Art and Transformations	F.IF.7 F.IF.7a F.IF.7b F.IF.7c F.IF.7d F.IF.7e	1	Students use their knowledge of function transformations to create graphics on the coordinate plane. Fifteen different equations/ relations are graphed to create artwork on the coordinate plane. Students then write the equations/relations associated with a different created artwork. Finally, they create their own artwork and list all associated equations/ relations and domain restrictions.	• Basic functions and equations, along with their transformations and restricted domains, can be used to create graphics on the coordinate plane.
3	**This Is the Title of This Lesson** Fractals	F.IF.3 F.BF.1a F.BF.2`	2	The terms self-similar, fractal, and iterative process are defined. The Sierpinski Triangle, the Menger Sponge, and the Koch Snowflake are the self-similar objects that students explore. They construct different stages of the models, use the images to complete tables of values, use the tables of values to identify infinite geometric sequences and patterns, describe end behaviors, write formulas, and make predictions.	• A fractal is a complex geometric shape that is formed by an iterative process. Fractals are infinite and self-similar across different scales. • The Sierpinski Triangle, the Menger Sponge, the Koch Snowflake, and the Sierpinski Carpet are examples of fractals with characteristics that can be described by geometric sequences.
Learning Individually with MATHia or Skills Practice		A.SSE.4	2	Students review sequences and sort geometric sequences from all other types of sequences. The formula to calculate the sum of a finite geometric series is developed using Euler's Method. Students calculate sums of finite series when given a series or summation notation for a series. They then solve problems in real-world contexts requiring the calculation of the sum of the geometric series. Students practice solving problems given scenarios where the appropriate model is a finite geometric series. **MATHia Unit:** Graphs of Trigonometric Functions **MATHia Workspaces:** Understanding the Unit Circle / Representing Periodic Behavior	

*Pacing listed in 45-minute days

4 Investigating Periodic Functions

Pacing: 18 Days

Topic 1: Trigonometric Relationships

Students begin this topic by exploring how periodic functions are built. They analyze the graphs of periodic functions for characteristics such as the maximum, minimum, period, amplitude, and midline. Students explore the unit circle to understand radian measure and convert between angle measures in degrees and radians. Using an understanding of the unit circle, radian measure, and periodic functions, students investigate the sine and cosine functions as well as their key characteristics and graphs. They then recall the transformation function form $g(x) = Af(B(x - C) + D$ to graph and analyze transformations of the sine and cosine functions and build a graph of the tangent function using a context. Students then analyze the characteristics of the tangent graph and apply their knowledge of transformations to sketch graphs of transformed tangent functions.

Standards: F.IF.4 , F.IF.7e , F.BF.3 , F.TF.1 , F.TF.2, F.TF.3 (+) , F.TF.4 (+) , F.TF.5 **Pacing:** 9 Days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
1	**A Sense of Déjà Vu** Periodic Functions	N.Q.1 F.IF.4 F.IF.7e	2	The term periodic function is defined. Periodic functions are used to model two situations related to a Ferris wheel: the height of a person on a Ferris wheel as a function of the number of revolutions of the wheel, and the height of a rider above ground with respect to the number of revolutions of an underground Ferris wheel. Students also use a protractor and graph to create a periodic function related to the height of a rider above ground on an underground Ferris wheel as a function of the measure of an angle in standard position and answer questions related to each problem situation.	• Periodic functions are used to model real-world situations. • A periodic function is a function whose values repeat over regular intervals. • The period of a periodic function is the length of the smallest interval over which the function repeats. • An angle is in standard position when the vertex is at the origin and one ray of the angle is on the x-axis. The ray on the x-axis is the initial ray and the other ray is the terminal ray. • The amplitude of a periodic function is one-half the absolute value of the difference between the maximum and minimum values of the function. • The midline of a periodic function is a reference line whose equation is the average of the minimum and the maximum values of the function.
2	**The Knights of the Round Table** Radian Measure	F.TF.1	1	The terms unit circle and radian measure are introduced. Students develop the concept of radian measure and label the measures of several central angles of a unit circle in both degree measure and radian measure. Formulas are given and used to convert degree measure to radian measure and vice versa.	• A radian is a unit that describes the measure of an angle in terms of the radius and arc length of a unit circle. • The measure of a central angle, θ, in radians, is the ratio intercepted arc length of a central angle/ length of the radius. • There are 2π radians in 360° and π radians in 180°. • To convert the units to measure angles from radians to degrees, multiply the radian measure by 180° / π radians. • To convert the units to measure angles from degrees to radians, multiply the degree measure by π radians/ 180°

*Pacing listed in 45-minute days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
3	**What Goes Around** The Sine and Cosine Functions	F.IF.7e F.TF.2 F.TF.3 (+) F.TF.4 (+)	1	The trigonometric functions sine and cosine and the periodicity identities are introduced in this lesson. Students explore the values of the sine and cosine functions using the unit circle and reference triangles centered at the origin in each of the four quadrants, such that θ = 30°, 45°, 60°, etc. The sine and cosine of an angle are identified as the coordinates of any point on the unit circle, and the coordinate values are used to graph the functions on a coordinate plane that is extended to 8 radians. Students calculate the values of the functions for different values of x and use the information to conclude that for all x, sin(x + 2) = sin(x) and cos(x + 2) = cos(x). The measures of the angles in both radians and degrees and the value of the function related to each angle measure are summarized in a table. Students compare the functions and identify the key characteristics.	• The coordinates of the point where the terminal ray of a central angle θ intersects the unit circle can be written as (cos θ, sin θ). • The coordinates of the point where the terminal ray intersects the unit circle is determined by the quadrant where the point lies and the reference angle that corresponds to the measure of θ. • The periodicity identities state that sin(x + 2π) = sin x and cos(x + 2π) = cos x. • The functions y = sin x and y = cos x are periodic trigonometric functions with a period of 2π. • The cosine function is a translation of the sine function by π/ 2 radians to the left, so cos x = sin(x + π/2).
4	**The Sines They Are A-Changin'** Transformations of Sine and Cosine Functions	F.BF.3 F.TF.3 (+) F.TF.5	1	The key characteristics of the sine and cosine functions, as well as the general transformation function form of an equation, are reviewed. Students explore the key characteristics and graphs of transformed functions, such as $y = A$ sin(x), $y = A$ cos(x), y = sin(Bx), and y = cos(Bx), y = sin($x + C$), y = cos($x + C$), y = sin(x) + D, and y = cos(x) + D, where A, B, C, and D are constants. They compare, contrast, and summarize the effects of each constant on functions both graphically and algebraically.	• Multiplying a function y = sin x or y = cos x by a constant A such that $y = A$ sin x or $y = A$ cos x dilates the function vertically by a factor of $\|A\|$. If $\|A\| > 1$, the graph is stretched vertically by a factor of $\|A\|$, if $0 < \|A\| < 1$, the graph is compressed vertically by a factor of $\|A\|$, and if $A < 0$, the graph is reflected across the x-axis. • The amplitude of a sine or cosine function is one-half the absolute value of the difference between the maximum and minimum values of the function. • Multiplying the argument of a function y = sin x or y = cos x by a constant B such that y = sin(Bx) or y = cos(Bx) dilates the function horizontally by a factor of $\|B\|$. If $\|B\| > 1$, the graph is horizontally compressed by a factor of 1 / $\|B\|$, if $0 < \|B\| < 1$, the graph is horizontally stretched by a factor of 1 / $\|B\|$, and if $B < 0$, the graph is reflected across the y-axis. • The period of the functions y = sin(Bx) and y = cos(Bx) is 2π / $\|B\|$. • The frequency of a periodic function is the reciprocal of the period and specifies the number of repetitions of the graph of a periodic function per unit. • Adding a constant C to the argument of a function y = sin x or y = cos x such that y = sin($x + C$) or y = cos($x + C$) translates the function horizontally. If $C < 0$, the graph is shifted to the left C units, and if C , 0, the graph is shifted to the right C units. • A horizontal translation of a periodic function is called a phase shift. • Adding a constant D to the argument of a function y = sin x or y = cos x such that y = sin $x + D$ or y = cos $x + D$ shifts the midline of the function vertically. If $D < 0$, the midline is shifted up D units, and if $D < 0$, the midline is shifted down D units.

*Pacing listed in 45-minute days

CARNEGIE LEARNING

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
5	**Farmer's Tan** The Tangent Function	F.IF.4 F.IF.7e F.TF.3 (+)	2	Students determine the slope for a central angle formed as the terminal ray transverses in a unit circle. They use the information to graph the changing slope and conclude the graph represents a periodic function. The tangent ratio is reviewed, and students explore the key characteristics of the tangent function and how it is defined in the unit circle in terms of the sine and cosine functions. Students state the periodicity identity for the tangent function and label the tangent values on the unit circle. Students also graph transformed tangent functions and match functions with their appropriate graphs.	• The tangent ratio opposite /adjacent can also be expressed as sin θ / cos θ . • The tangent ratio is equal to the slope of the hypotenuse, when the hypotenuse lies along the terminal ray of the central angle that lies on the unit circle. • The tangent function is a periodic function with a period of π radians. • The periodicity identity for the tangent function is written as $\tan(x + \pi) = \tan x$. • The tangent function has asymptotes at $n\pi / 2$, where n is any odd integer. • The tangent function has no maximum or minimum points.
Learning Individually with MATHia or Skills Practice		F.TF.1 F.TF.2 F.TF.5	2	Students watch an animation that demonstrates how to determine and use radian measures around the unit circle. The animation shows how the sine and cosine functions are constructed, mapping radian measures as inputs to the outputs of the sine or cosine of a central angle measure on the unit circle. Students derive how to convert between radian measures and degree measures. Finally, students analyze the sine and cosine functions, as members of a family of periodic trigonometric functions, and identify the amplitude, midline, and period of each function. They use the period to evaluate each function for different radian measures. Then, given a scenario that can be modeled by the sine function, students extract the values of A, p, k and h to create the function using $f(x) = A \cdot \sin(2\pi / p\,(x - k)) + h$. **MATHia Unit:** Graphs of Trigonometric Functions **MATHia Workspaces:** Understanding the Unit Circle / Representing Periodic Behavior	

*Pacing listed in 45-minute days

CARNEGIE LEARNING

Topic 2: Trigonometric Equations

Students are introduced to solving trigonometric equations. They use their knowledge of the unit circle, radian measures, and the graphical behaviors of trigonometric functions to solve sine, cosine, and tangent equations. Students then apply all that they have learned to model various situations with trigonometric functions, including circular motion. Finally, students explore the damping function and modeling with trigonometric transformations.

Standards: A.SSE.2, A.REI.1, F.IF.4, F.BF.1b, F.TF.1 , F.TF.2, F.TF.5, F.TF.8 **Pacing:** 9 Days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
1	**$\sin^2\theta$ Plus $\cos^2\theta$ Equals 1^2** The Pythagorean Identity	F.TF.8	1	Students investigate the sine, cosine, and tangent of angle measures which form right triangles in other quadrants: Quadrants II, III, and IV. They use a unit circle to determine the signs of each trigonometric ratio in each quadrant of the coordinate plane. Students then use what they know about the trigonometric ratios, the unit circle, and the Pythagorean Theorem to prove the Pythagorean identity, $\sin^2\theta + \cos^2\theta = 1^2$, and write this identity in different forms. Students use the Pythagorean identity to determine the sine, cosine, and tangent of angle measures in given quadrants. Students summarize their work at the end of the lesson by labeling different angle measures on the unit circle with degrees and radians, identifying the sign of the sine, cosine, and tangent of each angle measure, and then calculating each trigonometric ratio.	• The Pythagorean identity states that $\sin^2\theta + \cos^2\theta = 1$, where θ represents an angle measure. • The ratios sine, cosine, and tangent may be positive or negative, depending on which quadrant of the coordinate plane the reference angle and reference triangle are drawn. • The sines and cosines of angle measures in different quadrants of the coordinate plane are related by symmetry.
2	**Chasing Theta** Solving Trigonometric Equations	A.SSE.2 A.REI.1 F.TF.1 F.TF.2	2	Trigonometric equations are solved using a variety of strategies. Graphs and periodicity identities are used to determine multiple solutions. Worked examples are provided throughout the lesson. Students solve trigonometric equations involving transformations of the basic function, and inverse trigonometric functions are used in conjunction with a graphing calculator to solve equations. Trigonometric equations for all real numbers written in quadratic form are solved by factoring or using the Quadratic Formula.	• A trigonometric equation is an equation in which the unknown is associated with a trigonometric function. • Trigonometric equations can be solved using a graph and by synthesizing algebra skills and knowledge of trigonometric functions. • Inverse functions and the unit circle can be used to solve for x, the angle measure that results in a given value. • The number of solutions to a trigonometric equation is based upon the restrictions of the domain. An unrestricted domain has infinite solutions which can be identified using the periodicity of the function.
3	**Wascally Wabbits** Modeling with Periodic Functions	F.IF.4 F.TF.5	1	A periodic function is used to model the seasonal fluctuation in the rabbit population. Students are given the model and equation and use them to construct a graph and answer questions related to the situation. In the second situation, students model the seasonal amount of daylight in various locations. They are given a table of data and use the values to graph the situation. The graph is then used to determine a model equation, and that model is compared to the regression equation derived by the use of statistical technology. Students use the regression equation to answer related questions.	• Trigonometric functions may be used to model some real-world contexts. • Key characteristics of trigonometric functions, including period, amplitude, midline, phase shift, and vertical shift are evident in its algebraic representation, table, and graph.

*Pacing listed in 45-minute days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
4	**The Wheel Deal** Modeling Motion with a Trigonometric Function	F.TF.5	1	A periodic function is used to model the height from the street of a point on the circumference of a wheel as a function of time. Students build a trigonometric function by first using the $y = \sin(x)$ function to represent a situation, then students use the $y = \cos(x)$ function to model the height from the street of a point on the circumference of a wheel as a function of time. They use these functions to answer questions about the situations.	• Trigonometric functions can be used to model circular motion in real-world problems. • Trigonometric functions can express height in terms of an angle measure or in terms of time when given a rate of rotation. • Transformations of periodic functions can be used to map function behavior to the behavior of periodic phenomena.
5	**Springs Eternal** The Damping Function	F.IF.4 F.TF.5	1	A damping function is used to model the height of an object suspended on a spring bouncing up and down. Students build a trigonometric function by first using the $y = \cos(x)$ function to represent a situation where the object connected to the spring bounces up and down the same amount forever. Then the situation is altered to be more realistic, and students develop a trigonometric function that combines a cosine function and an exponential function to model the height of the object on the spring over time.	• A damping function is a function multiplied to a periodic function to decrease the amplitude over time. • While a periodic function may be used to model a real-world situation, a damping function is sometimes applied to the periodic function for a more accurate model.
Learning Individually with MATHia or Skills Practice		F.TF.8	3	Students combine their knowledge of the Pythagorean Theorem and the unit circle to complete a proof of the Pythagorean identity $\sin^2 x + \cos^2 x = 1$. They use this identity to solve problems where the value of sine or cosine in a specific quadrant is provided, and they must solve for the value of the other trigonometric function. A proof is provided for $\tan^2 x + 1 = \sec^2 x$, and students duplicate the process to prove $1 + \cot^2 x = \csc^2 x$. Students use identities to solve problems where the value of tangent or cosine in a specific quadrant is provided, and they must solve for the value of the other trigonometric function. Students then use the Pythagorean identity $\sin^2 x + \cos^2 x = 1$ to solve for sin or cos of an angle given sin, cos, or tan of that angle. **MATHia Unit:** Pythagorean Identity **MATHia Workspaces:** Proving the Pythagorean Identity / Using the Pythagorean Identity to Determine Sine, Cosine, or Tangent **MATHia Unit:** Solving Trigonometric Equations **MATHia Workspaces:** Solving Sine and Cosine Equations (No Type In) / Solving Tangent Equations (No Type In) / Solving Tangent, Since, and Cosine Equations (No Type In)	

*Pacing listed in 45-minute days

CARNEGIE LEARNING

5 Relating Data and Decisions

Pacing: 18 Days

Topic 1: Interpreting Data in Normal Distributions

The beginning of this topic leverages student knowledge of relative frequency histograms to introduce normal distributions. Students explore the characteristics of normal distributions. They then build their knowledge of normal distributions using the Empirical Rule for Normal Distributions to determine the percent of data between given intervals that are bounded by integer multiples of the standard deviation from the mean. They use a z-score table and technology to determine the percent of data in given intervals that is bounded by non-integer multiples of the standard deviation from the mean. Finally, students use their knowledge of probability and normal distributions to analyze scenarios and make decisions.

Standards: S.ID.1, S.ID.2, S.ID.4, S.MD.6 (+), S.MD.7 (+) **Pacing:** 7 Days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
1	**Recharge It!** Normal Distributions	S.ID.1 S.ID.2 S.ID.4	1	Data sets are given and students calculate the relative frequency which is used to create relative frequency histograms within the context of a situation. They explore how relative frequency histograms begin to resemble a bell-shaped curve when the sample size increases and the interval size of the bars decreases. The terms mean, normal curve, normal distribution, and standard deviation are defined. Sets of normal curves are given and students identify the mean and standard deviation related to each curve.	• Discrete data are data whose possible values are countable, while continuous data are data which can take any numeric value within a range. • A population represents all the possible data that are of interest in a study or survey. A sample is a subset of data selected from a population. • A normal curve is a bell-shaped curve symmetric about the mean of a data set. Sometimes a relative frequency histogram begins to resemble a normal curve when the sample size increases and the interval size of the bars decrease. • A data set that can be modeled using a normal curve has a normal distribution. • The mean is identified by the value on the horizontal axis associated with the peak value on a normal curve. • The standard deviation of data is the measure of how spread out the data is from the mean. A lower standard deviation represents data more tightly clustered near the mean, while a higher standard deviation represents data that are more spread out from the mean.

2	**The Form of Norm**
	The Empirical Rule for Normal Distributions
		S.ID.1
		S.ID.4
			2	Two histograms from the previous lesson are revisited. Students estimate the percent of data within each standard deviation on the histograms, which leads to the Empirical Rule for Normal Distributions. The Empirical Rule for Normal Distributions is depicted with an illustration of a normal curve. In different problem situations, students use the Empirical Rule for Normal Distributions to estimate the percent of data within specific intervals.	• The Empirical Rule of Normal Distributions states:
					  • Approximately 68% of the data in a normal distribution is within one standard deviation of the mean.
					  • Approximately 95% of the data in a normal distribution is within two standard deviations of the mean.
					  • Approximately 99.7% of the data in a normal distribution is within three standard deviations of the mean.
					• The Empirical Rule of Normal Distributions is used to estimate the percent of data within specific intervals of a normal distribution.
					• The standard normal distribution is a normal distribution with a mean value of 0 and a standard deviation of 1. Positive integers represent standard deviations greater than the mean and negative integers represent standard deviations less than the mean.
					• The total area under a standard normal curve is 1 square unit. This corresponds to the sum of all the relative frequencies of a histogram of a normally distributed set of data.

*Pacing listed in 45-minute days

CARNEGIE LEARNING

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
3	**Above, Below, and Between the Lines** Z-Scores and Percentiles	S.ID.4	1	The fuel efficiency of hybrid cars is modeled by a normal curve. Students use the model to answer questions about the percent of data in given intervals based upon integer multiples of the standard deviation from the mean. They then consider data values that are not aligned with integer multiples of the standard deviation from the mean, and use a *z*-score table and technology to determine percents. The term percentile is defined and students relate what they know about *z*-scores to make sense of percentiles.	• A *z*-score is a number that describes a specific data value's distance from the mean, in terms of standard deviation units. • The Empirical Rule for Normal Distributions can only be used to determine the percent of data that lies between standard deviations. A *z*-score table or technology can be used to calculate the percent of data values that lies within any interval. • A percentile is a data value for which a certain percent of the data is below the data value. • A *z*-score table or technology can be used to calculate a percentile. It is the inverse operation of determining the percent of data that is below a given data value.
4	**Toh-May-Toh, Toh-Mah-Toh** Normal Distributions and Probability	S.MD.6 (+) S.MD.7 (+)	1	Three different real-world situations are given. Students use a normal curve to determine the probabilities of randomly selected students sending and receiving text messages, the likelihood of delivering a pizza within a given time frame, and having a prize-winning tomato.	• A normal distribution curve can be used to model real-world situations. • Normal distributions are used to determine probabilities and make decisions.
Learning Individually with MATHia or Skills Practice		S.ID.4	2	Students investigate the properties of a distribution by plotting the mean, plotting an observed value, and shading the area under the curve. They first use the graph to calculate a percentile score and then when necessary calculate a *z*-score and translate it into a percentile score. Students then compute the *z*-scores for two separate distributions and compare the probability of the observed value occurring in each distribution. Finally, they make a statistical decision based on the data and calculate the margin of error. **MATHia Unit:** Normal Distributions **MATHia Workspaces:** Applying the Empirical Rule for Normal Distributions / Z-Scores and Percentiles / Normal Distributions and Probability	

*Pacing listed in 45-minute days

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Topic 2: Making Inferences and Justifying Conclusions

Students learn data collection methods to analyze a characteristic of interest, specific sampling methods, and the significance of randomization. They then use data from samples to estimate population means and proportions and determine whether results are statistically significant. Finally, students complete a culminating project based on concepts from throughout the topic.

Standards: S.IC.1, S.IC.2, S.IC.3, S.IC.4, S.IC.5, S.IC.6 **Pacing:** 11 Days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
1	**Data, Data Everywhere** Sample Surveys, Observational Studies, and Experiments	S.IC.1 S.IC.3	1	Examples of a sample survey, an observational study, and an experiment are provided. Students answer questions related to each method and differentiate between the methods. Vocabulary terms associated with each type of data collection are defined. Students design a data collection plan to learn how much time fellow students spend online each day. Finally, they classify scenarios as sample surveys, observational studies, or experiments and list the characteristic of interest, the population, the sample, factors that contribute to confounding, and ways to prevent bias.	• A characteristic of interest is a specific question that you are trying to answer or the specific information that you are trying to gather. • Methods of data collection include a sample survey, an observational study, and an experiment. • A sample survey poses a question of interest to a sample of a targeted population. • An observational study gathers data about a characteristic of the population in its natural setting. • An experiment gathers data on the effect of one or more treatments, or experimental conditions, on the characteristic of interest. • A random sample is a sample that is selected from the population in such a way that every member of the population has the same chance of being selected. • A biased sample is a sample that is not representative of the population. • Confounding occurs when there are other possible reasons for the results to have occurred that were not identified prior to the study.
2	**Ample Sample Examples** Sampling Methods and Randomization	S.IC.1 S.IC.3	3	Three sampling methods are introduced: convenience sampling, subjective sampling, and volunteer sampling. Students perform a simple random sampling using a random digit table and technology. They then use other types of random sampling methods such as stratified random samples, cluster samples, and systematic samples.	• Collecting data using a convenience sample, a subjective sample, or a volunteer sample will likely result in a biased sample. • Simple random sampling, stratified random sampling, cluster sampling, and systematic sampling are sampling methods in which all members of a population have an equal chance of being selected. • A simple random sample can be created using a random number table or technology. • A stratified random sample is a random sample obtained by dividing a population into different groups, or strata, according to a characteristic and randomly selecting data from each group. • A cluster sample is a random sample that is obtained by creating clusters. Each cluster contains the characteristics of the population. Then, one cluster is randomly selected for the sample. • A systematic sample is a random sample obtained by selecting every nth data value in a population.
3	**A Vote of Confidence** Using Confidence Intervals to Estimate Unknown Population Means	S.IC.1 S.IC.4 S.IC.6	2	Given a scenario about a political poll, students conduct a simulation using a random number generator to increase the sample size and compare the results to the original poll. They then apply the formula to calculate the standard deviation of a sampling distribution, and determine the confidence interval for different problem situations. The formula is given to calculate the standard deviation for the population mean in another scenario, and students use this formula to determine a range of values for population means associated with a confidence interval of 95%.	• A sampling distribution is the set of sample proportions for all possible equally-sized samples. • The formula for calculating the standard deviation of a sampling distribution of categorical data is $\sqrt{\hat{p}(1-\hat{p})/n}$, where $\hat{p}$ (p-hat) is the sample proportion and n is the sample size. • The formula for calculating the standard deviation of a sampling distribution for continuous data is $s/\sqrt{n}$, where s is the standard deviation of the original sample and n is the sample size. • The confidence interval gives an estimated range of values that will likely include a population proportion or population mean. When stating the margin of error, a 95% confidence interval is typically used.

*Pacing listed in 45-minute days

CARNEGIE LEARNING

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
4	**How Much Different?** Using Statistical Significance to Make Inferences About Populations	S.IC.1 S.IC.2 S.IC.4 S.IC.5 S.IC.6	2	Within the context of various situations, students use a sample proportion or mean and a 95% confidence interval to determine the margin of error. The sample proportion or mean and standard deviation of its sampling distribution are used to label the axis of a normal curve and students determine whether or not results are statistically significant. A random number generator is used to create an additional sample for the purpose of comparison.	• Statistically significant results of a study are unlikely to have occurred by chance. • Sample proportions or sample means that are more than 2 standard deviations from the sample proportion or sample mean of its sampling distribution are considered statistically significant. • When considering the statistical significance of categorical data with two choices and 50% occurs outside of the 95% confidence interval, then the data is considered to be statistically significant. • When considering the statistical significance of continuous data and the new sample mean is outside of the 95% confidence interval, then the data is considered to be statistically significant. • When comparing the statistical significance of two data sets from the same population and there is an overlap in their confidence intervals, then the data is not considered to be statistically significant.
5	**DIY** Designing a Study and Analyzing the Results	S.IC.1 S.IC.2 S.IC.3 S.IC.4 S.IC.5 S.IC.6	1	Students are given the guidelines developed throughout the topic for designing and conducting a sample survey, observational study, or an experiment for a characteristic of interest of their choice.	• A sample survey, observational study, or experiment may be useful in drawing a conclusion regarding a characteristic of interest. • There is a structure to design, conduct, summarize, and analyze the data of a sample survey, observational study, or experiment.
Learning Individually with MATHia or Skills Practice		S.ID.4	2	Students can continue the development of normal distributions through the MATHia content aligned to the previous topic.	

Total Days: 155

Learning Together: 106
Learning Individually: 49

*Pacing listed in 45-minute days