# High School Math Solution Integrated Math 2 **MATHia**® Table of Contents
2020-2021 School Year

CARNEGIE LEARNING

# 1 Reasoning with Shapes

| | | | | | | Strategies | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **MATHia Unit** | **MATHia Workspace** | **Overview** | **CCSS** | **Concept Builder** | **Mastery** | Animations | Classifications | Explore Tools | Graphing Tools | Interactive Diagrams | Interactive Worksheets | Proof | Real-World Scenarios | Solvers | Worked Example |
| **Topic 1 Composing and Decomposing Shapes** | | | | | | | | | | | | | | | |
| **From Informal to Formal Geometric Thinking** | Introduction to Geometric Figures | Students watch an animation defining some of these basic geometric figures: point, line, line segment, ray, and angle. They identify these figures highlighted in a diagram. Students learn the symbols used to name these geometric figures. They analyze when these figures have more than one name. | G.CO.1 | ✓ | | | | | | | | | | | |
| | Naming Lines, Rays, Segments, and Angles | Students practice identifying geometric entities from their names, writing names for various geometric entities, and identifying when an entity has multiple possible names. | G.CO.1 | | ✓ | | | | | • | | | | | |
| | Working with Measures of Segments and Angles | Students practice writing measure statements for segments and angles using appropriate notation. | G.CO.1 | | ✓ | | | | | • | | | | | |
| **Using Circles to Make Conjectures** | Introduction to Circles | Students watch an animation defining some of the terminology of circle parts. They then identify chords, tangents, points of tangency, and secants of circles. Next, students sort inscribed and central angles. Finally, they classify minor and major arcs as well as semicircles. | G.C.1 G.C.2 | ✓ | | • | • | | | | | | | | |
| | Exploring the Inscribed Angle Theorem | Student use an Explore Tool to determine the measures of major and minor arcs. They investigate the measure of inscribed angles whose sides intersect the endpoints of a circle's diameter. Students investigate the measure of inscribed angles that intercept the same arc. They investigate the measures of central angles and inscribed angles that have the same intercepted arc. Students use the Inscribed Angle Theorem and the Explore Tool to determine the measure of inscribed angles that intercept a given arc and the measure of an inscribed angle's intercepted arc. | G.C.2 | ✓ | | | | • | | | | | | | |

# 1 Reasoning with Shapes

| MATHia Unit | MATHia Workspace | Overview | CCSS | Concept Builder | Mastery | Animations | Classifications | Explore Tools | Graphing Tools | Interactive Diagrams | Interactive Worksheets | Proof | Real-World Scenarios | Solvers | Worked Example |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | Strategies | | | | | | | | | |
| **Using Circles to Make Conjectures (continued)** | Determining Central and Inscribed Angles in Circles | Students calculate the measure of an arc or an angle using the definition of a central angle, the Arc Addition Postulate, or the Inscribed Angle Theorem. | G.C.2 | | ✓ | | | | | • | | | | | |
| **Conjectures About Quadrilaterals** | Using Circles to Draw Quadrilaterals | Students use an Explore Tool to investigate different quadrilaterals that can be drawn using two concentric circles. They answer questions about the diagonals of special quadrilaterals and use the relationship of the diagonals to draw a quadrilateral with a given diagonal. Students then complete tables determining whether or not special quadrilaterals have given properties. | G.CO.11 | ✓ | | | | • | | | • | | | | |
| | Angles of an Inscribed Quadrilateral | Students are shown an inscribed quadrilateral and prove the Inscribed Quadrilateral-Opposite Angles Conjecture. They then use the theorem to determine the measure of an angle in an inscribed quadrilateral given the measure of the opposite angle. | G.C.3 | ✓ | | | | | | | • | • | | | • |
| **Points of Concurrency** | Points of Concurrency | Students watch animations about the points of concurrency and answer questions about these points. They analyze a table showing the point of concurrency for different types of triangles and complete a table identifying the location of a point of concurrency for each type of triangle.. | G.C.3 | ✓ | | • | | | | | • | | | | |
| **Topic 2 Justifying Line and Angle Relationships** | | | | | | | | | | | | | | | |
| **Forms of Proof** | Introduction to Proofs | Students are introduced to proof by answering questions related to two animations demonstrating the Triangle Sum Theorem and the Vertical Angle Theorem. | G.CO.1 | ✓ | | • | • | | | | | • | | | |
| | Completing Measure Proofs | Students complete the steps in a scaffolded proof, supplying appropriate statements and reasons to prove a variety of fundamental angle and segment theorems. | G.CO.1 | | ✓ | | | | | | | • | | | |
| | Connecting Steps in Angle Proofs | Students arrange the steps of more complex proofs into logical order. | G.CO.9 | | ✓ | | | | | | | • | | | |

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| | | | | | | Strategies | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **MATHia Unit** | **MATHia Workspace** | **Overview** | **CCSS** | **Concept Builder** | **Mastery** | Animations | Classifications | Explore Tools | Graphing Tools | Interactive Diagrams | Interactive Worksheets | Proof | Real-World Scenarios | Solvers | Worked Example |
| **Forms of Proof (continued)** | Using Angle Theorems | Students use a wide variety of postulates, properties, and theorems to solve mathematical problems related to angles in geometrical figures and diagrams. The Congruent Complements Theorem, Congruent Supplements Theorem, Angle Addition Postulate, angle bisection, Vertical Angle Theorem, and the Transitive Property are all discussed. | G.CO.9 | ✓ | | | | | | | | | | | |
| **Lines Cut by a Transversal** | Classifying Angles Formed by Transversals | Students follow worked examples and complete sorting activities as they learn to identify angles and angle pairs formed by lines cut by a transversal. | G.CO.9 | ✓ | | | • | | | | | | • | | |
| | Calculating Angle Measures Formed by Transversals | Calculate the measure of the sought angle by using angle relationships formed by two lines cut by a single transversal. | G.CO.9 | | ✓ | | | | | • | | | | | |
| | Calculating Angles Formed by Multiple Transversals | Calculate the measure of the sought angle by using angle relationships formed by three parallel lines cut by a single transversal or two parallel lines cut by two transversals. | G.CO.9 | | ✓ | | | | | • | | | | | |
| **Proving Parallel Lines Theorems** | Proving Parallel Lines Theorems | Students apply basic angle theorems to prove the alternate interior, alternate exterior, same side interior, and side side exterior parallel line theorems. | G.CO.9 | | ✓ | | | | | | | • | | | |
| | Proving the Converses of Parallel Lines Theorems | Students apply basic angle theorems to prove the alternate interior converse, alternate exterior converse, same side interior converse, and side side exterior converse parallel line theorems. | G.CO.9 | | ✓ | | | | | | | • | | | |
| **Interior and Exterior Angles of Polygons** | Proving Triangle Theorems | Students apply previously proved theorems to prove the triangle sum and exterior angle theorems. | G.CO.10 | | ✓ | | | | | | | • | | | |
| **Proving Triangles Congruent** | Proving Triangles Congruent using SAS and SSS | Students prove triangles congruent using the side-angle-side and side-side-side congruence theorems in a variety of diagrams. | G.CO.10 | | ✓ | | | | | | | • | | | |
| | Proving Triangles Congruent using AAS and ASA | Students prove triangles congruent using the angle-angle-side and angle-side-angle congruence theorems in a variety of diagrams. | G.CO.10 | | ✓ | | | | | | | • | | | |

CARNEGIE LEARNING

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| | | | | | | Strategies | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **MATHia Unit** | **MATHia Workspace** | **Overview** | **CCSS** | **Concept Builder** | **Mastery** | Animations | Classifications | Explore Tools | Graphing Tools | Interactive Diagrams | Interactive Worksheets | Proof | Real-World Scenarios | Solvers | Worked Example |
| **Proving Triangles Congruent (continued)** | Proving Theorems using Congruent Triangles | Students use congruent triangle theorems to prove the perpendicular bisector theorem, isosceles triangle base angle theorem and its converse, and the angle bisector theorem. | G.CO.10 | | ✔ | | | | | | | • | | | |
| **Special Right Triangles** | Introduction to Special Right Triangles | Students use what they know about the Pythagorean Theorem to identify patterns in 45-45-90 and 30-60-90 triangles. They use the relationships between the side lengths of the special right triangles to solve for unknown side lengths. Students sort triangles according to whether they are a 45-45-90 triangle, a 30-60-90 triangle, or neither. | G.SRT.4 | ✔ | | | • | | | | | | • | | • |
| | Calculating the Lengths of Sides of Special Right Triangles | | G.SRT.4<br>G.SRT.5 | | ✔ | | | | | • | | | | | |
| **Solving Problems with Congruence** | Using Triangle Theorems | Students apply angle, parallel line, and triangle theorems to prove relationships between elements in more complex diagrams. | G.CO.10 | | ✔ | | | | | | | • | | | |
| **Angle Relationships Inside and Outside Circles** | Determining Interior and Exterior Angles in Circles | Students calculate the measure of an arc or an angle using Interior Angles of a Circle Theorem and Exterior Angles of a Circle Theorem. | G.C.2 | | ✔ | | | | | • | | | | | |

## Topic 3 Using Congruence Theorems

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Extending Triangle Congruence Theorems** | Proving Triangles Congruent using HL and HA | Students prove triangles congruent using the hypotenuse-leg and hypotenuse-angle congruence theorems in a variety of diagrams. | G.CO.10 | | ✔ | | | | | | | • | | | |

# 1 Reasoning with Shapes

| MATHia Unit | MATHia Workspace | Overview | CCSS | Concept Builder | Mastery | Strategies: Animations | Classifications | Explore Tools | Graphing Tools | Interactive Diagrams | Interactive Worksheets | Proof | Real-World Scenarios | Solvers | Worked Example |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Properties of Quadrilaterals** | Understanding Parallelograms | Students are given the properties of parallelograms and use the information to determine the side parallel to a given side of a parallelogram as well as the sides or angles that are congruent to a given side or angle of a parallelogram. They then determine a missing statement to prove a quadrilateral is a parallelogram using the Parallelogram/ Congruent-Parallel Side Theorem. Finally, students identify quadrilaterals by properties of their sides, angles, and diagonals. | G.CO.11 | ✓ | | | | | | • | | | • | | • |
| | Determining Parts of Quadrilaterals and Parallelograms | Students are given a parallelogram and asked to calculate the length of the bisected diagonals, the measure of the angles, and the length of the opposite side and base. | G.CO.11 | | ✓ | | | | | • | | | | | |
| **Parallelogram Proofs** | Proofs about Parallelograms | Students apply their knowledge of congruent triangles and parallel lines in order to prove several theorems about parallelograms. | G.CO.11 | | ✓ | | | | | | | • | | | |

CARNEGIE LEARNING

# 2 Investigating Proportionality

## Topic 1 Similarity

| MATHia Unit | MATHia Workspace | Overview | CCSS | Concept Builder | Mastery | Strategies: Animations | Classifications | Explore Tools | Graphing Tools | Interactive Diagrams | Interactive Worksheets | Proof | Real-World Scenarios | Solvers | Worked Example |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Dilating Figures to Create Similar Figures** | Understanding Similarity | Students watch an animation which demonstrates that when figures are similar, a series of rigid motions and dilations can transform one figure on top of the other to match exactly. Students recall that similar figures have corresponding side lengths that are proportional and congruent corresponding angles. Students identify similar figures and determine corresponding side lengths and corresponding angle measures, given similar figures. | G.SRT.2 | ✓ | | • | | | | | | | | | |
| | Specifying a Sequence of Transformations | Students select multiple transformations from translation, rotation, dilation, and reflection about any line to match a pre-image to a target image, given a reference point. | G.CO.5 | | ✓ | | | | • | | | | | | |
| **Theorems About Proportionality** | Proofs Using Similar Triangles | Students use the AA Similarity Postulate, SSS Similarity Theorem, and SAS Similarity Theorem to prove the parallel segment proportionality theorem and triangle midsegment theorem. | G.SRT.5 | | ✓ | | | | | | | • | | | |
| **Application of Similar Triangles** | Calculating Corresponding Parts of Similar Triangles | Students calculate corresponding parts of similar triangles, both in context and out of context. | G.SRT.5 | | ✓ | | | | | • | | | • | • | |
| **Partitioning Segments in Given Ratios** | Partitioning Segments in Given Ratios | Students watch an animation describing the usefulness of directed line segments and how to interpret fractions of directed segments. Students observe directed line segments divided into two lengths by a point and determine the ratio of those lengths, starting with horizontal and vertical line segments and then moving to non-vertical and non-horizontal segments. | G.GPE.6 | ✓ | | • | | | • | | | | | | |
| | Partitioning Segments Proportionately | Students determine the coordinates of points that partition given line segments into different ratios. | G.GPE.6 | | ✓ | | | | | • | | | | | |

CARNEGIE LEARNING

# 2 Investigating Proportionality

## Topic 2 Trigonometry

| MATHia Unit | MATHia Workspace | Overview | CCSS | Concept Builder | Mastery | Strategies: Animations | Classifications | Explore Tools | Graphing Tools | Interactive Diagrams | Interactive Worksheets | Proof | Real-World Scenarios | Solvers | Worked Example |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Trigonometric Ratios** | Introduction to Trigonometric Ratios | Students use similar triangles to define and understand the trigonometric ratios sine, cosine, and tangent. Students then explore the sine, cosine, and tangent and estimating these ratios using an interactive Explore Tool with a unit circle, including describing the ratios as percents of different lengths. Students solve problems in various contexts using the trigonometric ratios and the Explore Tool. | G.SRT.6 | | | | | • | | • | • | | | | |
| | Relating Sines and Cosines of Complementary Angles | Students use the interactive unit circle trig ratio Explore Tool to explore complementary angles and to see that the sine of an angle is equal to the cosine of its complement, and vice versa. | G.SRT.7 | ✔ | | | | • | | | | | | | |
| | Using One Trigonometric Ratio to Solve Problems | Students calculate the measures of sides and angles of a right triangle using trigonometric ratios, the Pythagorean Theorem, and/or the Triangle Sum Theorem in both contextual and abstract problems. | G.SRT.8 | | ✔ | | | | | • | | | • | • | |
| | Using Multiple Trigonometric Ratios to Solve Problems | Students calculate the measures of sides and angles of two right triangles that share a side using trigonometric ratios, the Pythagorean Theorem, and/or the Triangle Sum Theorem in both contextual and abstract problems. | G.SRT.8 | | ✔ | | | | | • | | | • | • | |

# 2 Investigating Proportionality

## Topic3 Circles and Volume

| MATHia Unit | MATHia Workspace | Overview | CCSS | Concept Builder | Mastery | Strategies: Animations | Classifications | Explore Tools | Graphing Tools | Interactive Diagrams | Interactive Worksheets | Proof | Real-World Scenarios | Solvers | Worked Example |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Similarity Relationships in Circles** | Relating Arc Length and Radius | Students explore the difference between the degree measure of an arc and the length of an arc. They then practice calculating the fraction of a circle's circumference that an arc occupies and writing an expression that can be used to calculate an arc's length. Students then calculate the arc length given the radius or diameter of the circle. Next, they relate the arc length to the circle's radius and are introduced to the units radians and the theta symbol. Finally, students practice determining different measurements of a circle using the formula theta = s/r. | G.C.5 | ✓ | | | | | | | | | | | • |
| | Determining Chords in Circles | Students calculate the length of an arc using the radius or diameter, the circumference, and the arc-to-circle ratio. | G.C.2 | | ✓ | | | | | • | | | | | |
| | Calculating the Area of a Sector | Students are given the definition of a sector of a circle and practice identifying sectors. They then work through an example that develops the formula for determining the area of a sector of a circle before using the formula to find areas of different sectors of circles. | G.C.5 | ✓ | | | • | | | | | | | | • |
| **Volume** | Calculating Volume of Cylinders | Students use mathematical and real-world objects to determine the volume of cylinders. | G.GMD.3 | | ✓ | | | | | | • | | • | • | |
| | Calculating Volume of Pyramids | Students calculate the volume of pyramids in mathematical and real-world contexts using given measurements. | G.GMD.3 | | ✓ | | | | | | • | | • | • | |
| | Calculating Volume of Cones | Students use mathematical and real-world objects to determine the volume of cones. | G.GMD.3 | | ✓ | | | | | | • | | • | • | |
| | Calculating Volume of Spheres | Students use mathematical and real-world objects to determine the volume of spheres. | G.GMD.3 | | ✓ | | | | | | • | | • | • | |

CARNEGIE LEARNING

# 2 Investigating Proportionality

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|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Surface Area** | Introduction to Formulas for the Surface Area of Solids | Students use the faces of a cylinder, square pyramid, and cone to determine surface area formulas for the solids rewritten in different ways. They use the formulas to determine the surface area of each solid. | 7.G.6 G.GMD.3 | ✓ | | | | | | | | | | | |
| | Calculating Surface Area of Solids | Students use mathematical and real-world objects to determine the surface areas of cylinders, pyramids, cones, and spheres. | 7.G.6 G.GMD.3 | | ✓ | | | | | | ● | | ● | | |

CARNEGIE LEARNING

# 3 Exploring Functions

## Topic 1 Functions Derived from Linear Relationships

| MATHia Unit | MATHia Workspace | Overview | CCSS | Concept Builder | Mastery | Strategies: Animations | Classifications | Explore Tools | Graphing Tools | Interactive Diagrams | Interactive Worksheets | Proof | Real-World Scenarios | Solvers | Worked Example |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Defining Absolute Value Functions and Transformations** | Building Absolute Value Functions | Students watch an animation showing how taking the absolute value of a number reflects that number across 0, or across the x-axis. Students use reflections to create the basic absolute value function from the functions f(x) = x and f(x) = -x. They show that the two functions f(x) = \|x\| and f(x) = \|-x\| are equivalent. | F.IF.7b | ✓ | | | • | | | | • | | | | |
| | Vertically Dilating Absolute Value Functions | | F.BF.3 | | ✓ | | | | | | | | | | |
| | Vertically Translating Absolute Value Functions | | F.BF.3 | | ✓ | | | | | | | | | | |
| | Horizontally Translating Absolute Value Functions | | F.BF.3 | | ✓ | | | | | | | | | | |
| | Multiple Transformations of Absolute Value Functions | | F.BF.3 | | ✓ | | | | | | | | | | |
| **Absolute Value Equations and Inequalities** | Reasoning About Absolute Value Functions | Students determine the solutions to absolute value equations by graphing each side of the equation as a separate function and looking for the points of intersection. They determine if an absolute value equation has 0, 1, or 2 solutions. They use a graph to solve a real-world problem modeled by an absolute value function. | A.REI.11 | ✓ | | | | | | | | | | • | • |
| | Graphing Simple Absolute Value Equations Using Number Lines | Students write a simple absolute value equation from a verbal statement, determine the number of solutions, and then represent the solution on a number line. | A.CED.3 | | ✓ | | | | • | | | | | | |

# 3 Exploring Functions

| MATHia Unit | MATHia Workspace | Overview | CCSS | Concept Builder | Mastery | Strategies: Animations | Classifications | Explore Tools | Graphing Tools | Interactive Diagrams | Interactive Worksheets | Proof | Real-World Scenarios | Solvers | Worked Example |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Absolute Value Equations and Inequalities (continued)** | Introduction to Absolute Value Equations | Students practice rewriting absolute value equations as two linear equations. They solve absolute value equations algebraically using properties of equality. Students sort absolute value equations by their number of solutions. They use absolute value equations to solve a contextual problem and consider the reasonableness of the solutions. | A.CED.3 A.REI.3 | ✓ | | | | • | | | | | | • | • |
| | Solving Absolute Value Equations | Students solve multi-step absolute value equations, determine the number of solutions, and then represent the solution on a number line | A.CED.3 | | ✓ | | | | • | | | | • | | |
| | Reasoning About Absolute Value Inequalities | Students use graphical representations to solve absolute value inequalities. They learn to write equivalent compound inequalities for absolute value inequalities. | A.CED.3 | ✓ | | | | • | | | | | | | • |
| **Linear Piecewise Functions** | Introduction to Piecewise Functions | Students are introduced to a linear piecewise function through a real-world scenario and giving the definition of a piecewise function. They then sort sketches of graphs of linear piecewise functions to given scenarios. Finally, students identify the graph of a linear piecewise function after being given the function's equation. | F.IF.C.7b | ✓ | | | | • | | | | | | • | • |
| | Graphing Linear Piecewise Functions | Given a linear piecewise function definition, students represent its piece domain boundaries on a number line and then graph the function. | F.IF.C.7b | | ✓ | | | | • | | | | | • | |
| | Interpreting Piecewise Functions | Students identify the domain in both non-continuous and continuous piecewise functions given an equation and the graph of the function. They are then given a domain and a graph of a piecewise function and are asked to determine the equation the graph with that domain represents. | F.IF.C.7b | ✓ | | | | | • | | | | | | • |
| | Using Linear Piecewise Functions | Students use graphs of linear piecewise functions to answer questions about scenarios in context. | F.IF.C.7b | | ✓ | | | | • | | | | | • | |

# 3 Exploring Functions

| MATHia Unit | MATHia Workspace | Overview | CCSS | Concept Builder | Mastery | Strategies: Animations | Classifications | Explore Tools | Graphing Tools | Interactive Diagrams | Interactive Worksheets | Proof | Real-World Scenarios | Solvers | Worked Example |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Step Functions** | Analyzing Step Functions | Students are introduced to step functions in the first problem. They then identify the domain of a given equation of a step function using a problem situation and graph. Next students are asked to identfiy the step function that represents a given problem situation and graph. | F.IF.C.7b | ✔ | | | | | | | | | | | • |

## Topic 2 Exponentials

| MATHia Unit | MATHia Workspace | Overview | CCSS | Concept Builder | Mastery | Strategies: Animations | Classifications | Explore Tools | Graphing Tools | Interactive Diagrams | Interactive Worksheets | Proof | Real-World Scenarios | Solvers | Worked Example |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Rational Exponents** | Using the Properties of Exponents | Students review the properties of powers and identify simplified versions of expressions with numeric powers. They deal with more complex exponential expressions with variables, sorting them into groups that are equivalent expressions. | 8.EE.1 | ✔ | | | | • | | | | | | | |
| | Properties of Rational Exponents | Students learn the names of the components of radical notation (radical, radicand, index and nth root). They use the properties of powers to make sense of the fact that x to the one-half power and the square root of x are equivalent. Students practice rewrite expressions with radical notation using rational exponents, and then reverse the process and rewrite expressions with rational exponents using radical notation. In these problems, all rational exponents are positive fractions with one as a numerator. | N.RN.1 | ✔ | | | | | | | | | | | • |
| | Rewriting Expressions with Radical and Rational Exponents | Students expand their understanding of rational exponents to include making sense of fractional exponents with a numerator other than one and negative exponents. Given various expressions with exponents with fractions, exponents with negative values and powers raised to a power, they select a equivalent radical expressions. The process is then reversed, and students convert radical expressions to expressions with positive or negative fractional exponents. | N.RN.2 | ✔ | | | | • | | | | | | | • |

# 3 Exploring Functions

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|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Growth and Decay Functions** | Recognizing Linear and Exponential Models | Students compare linear and exponential functions and their graphs in the context of simple interest (linear) and compound interest (exponential). Students solve problems related to the independent and dependent variables of both linear and exponential functions using the graphs and equations. | F.LE.1b F.LE.1c | ✔ | | | | | • | | | | | • | • |
| | Calculating and Interpreting Average Rate of Change | A formula is provided to calculate the average rate of change for a specified interval of an exponential function, and the reason it is considered an average is explained. Students are provided contexts along with either a graph or table. They must calculate the average rate of change for specified intervals of the function, and make an estimation for another interval based upon their results. | F.IF.6 | ✔ | | | | | | | • | | | | • |
| | Recognizing Growth and Decay | Students watch two different animations: one shows a model of exponential growth and one shows a model of exponential decay. They analyze how to recognize the difference between the two exponential models before interpreting exponential functions using scenarios of population increase and decrease. | F.LE.1c | ✔ | | | • | | | | | | | • | |
| **Transformations of Exponential Functions** | Introduction to Transforming Exponential Functions | Students use four animations, demonstrating the different ways of transforming an exponential function, to investigate how changing the equation for an exponential function changes the graph of the function. Students answer questions related to horizontal and vertical translations and dilations of exponential functions. | F.BF.3 | ✔ | | | • | | | | | | | | |
| | Vertically Translating Exponential Functions | Students vertically shift graphs of linear and exponential functions. Students use verbal descriptions, graphs, and algebraic representations. | F.BF.3 | | ✔ | | | | • | | | | | | |
| | Horizontaly Translating Exponential Functions | Students horizontally shift graphs of linear and exponential functions. Students use verbal descriptions, graphs, and algebraic representations. | F.BF.3 | | ✔ | | | | • | | | | | | |

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| | | | | | | Strategies | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **MATHia Unit** | **MATHia Workspace** | **Overview** | **CCSS** | **Concept Builder** | **Mastery** | Animations | Classifications | Explore Tools | Graphing Tools | Interactive Diagrams | Interactive Worksheets | Proof | Real-World Scenarios | Solvers | Worked Example |
| **Transformations of Exponential Functions (continued)** | Reflecting and Dilating Exponential Functions Using Graphs | Students reflect and dilate graphs of linear and exponential functions. Students use verbal descriptions, graphs, and algebraic representations. | F.BF.3 | | ✔ | | | | • | | | | | | |
| | Transforming Exponential Functions Using Tables of Values | Given a table of values and a table of transformed values, students determine how the basic linear and exponential functions were transformed to create the new functions. | F.BF.3 | | ✔ | | | | • | | | | | | |
| | Multiple Transformations of Exponential Functions | Given a representation of a transformed function, students determine how the basic linear and exponential functions were transformed to create the new functions. | F.BF.3 | | ✔ | | | | • | | | | | | |

## Topic 3 Introduction to Quadratic Functions

| **MATHia Unit** | **MATHia Workspace** | **Overview** | **CCSS** | **Concept Builder** | **Mastery** | Animations | Classifications | Explore Tools | Graphing Tools | Interactive Diagrams | Interactive Worksheets | Proof | Real-World Scenarios | Solvers | Worked Example |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Exploring Quadratic Functions** | Introduction to a Quadratic Function | Students watch an animation introducing them to a quadratic function. They sort functions modeled by an equation or by a graph as quadratic or not. Students use a table of values to graph a basic quadratic function. They sort graphs of quadratic functions by their number of x-intercepts. Students determine the range of quadratic functions given their graphs. | F.IF.7a | ✔ | | | • | • | | | • | | | | |
| | Modeling Area as Product of Monomial and Binomial | Students complete a table of values and graph from a scenario represented by a quadratic model. Students construct the quadratic function for the scenario as a product of a monomial and a binomial. | A.CED.1 | | ✔ | | | | • | | • | | • | • | |
| | Modeling Area as Product of Two Binomials | Students complete a table of values and graph from a scenario represented by a quadratic model. Students construct the quadratic function for the scenario as the product of two binomials. | A.CED.1 | | ✔ | | | | • | | • | | • | • | |
| | Modeling Projectile Motion | Students use quadratic functions to model projectile motion, and use the solver and the graphs to answer questions. | F.IF.4 | | ✔ | | | | • | | • | | • | • | |

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# 3 Exploring Functions

| MATHia Unit | MATHia Workspace | Overview | CCSS | Concept Builder | Mastery | Strategies: Animations | Classifications | Explore Tools | Graphing Tools | Interactive Diagrams | Interactive Worksheets | Proof | Real-World Scenarios | Solvers | Worked Example |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Exploring Quadratic Functions (continued) | Recognizing Key Features of Vertical Motion Graphs | Students use an interactive Explore Tool to investigate how a vertical motion graph changes when the different values in the vertex, factored, and general form of the quadratic function change. They then use vertical motion graphs to identify the maximum, x-intercepts, y-intercept, domain, and range of a quadratic function. Finally, students use a vertical motion graph to determine the axis of symmetry and vertex of a quadratic function. | F.IF.4 | ✓ | | • | | | | | | | | • | • |
| | Interpreting Maximums of Quadratic Models | Students analyze the graphs of functions modeling scenarios of area and vertical motion to identify the maximum and interpret what it means in terms of the problem. | A.CED.1 | ✓ | | | | | | | | | | | • |
| Key Characteristics of Quadratic Functions | Recognizing Quadratic Functions from Tables | Students recall first differences and are introduced to second differences. They calculate and analyze the first and second differences of linear and quadratic functions, comparing the values to the equations and graphs of the functions. Students then determine first and second differences in a table of values and identify the function represented by the table. | F.IF.6 | ✓ | | | | | | | • | | | | |

CARNEGIE LEARNING

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| **Key Characteristics of Quadratic Functions (continued)** | Identifying Properties of Quadratic Functions | Students differentiate among general form, factored form, and vertex form of a quadratic function. They learn the characteristics of the graph that are visible from each form: y-intercept from general form, x-intercepts from factored form, and vertex from vertex form, and practice identifying these characteristics from the algebraic representations. The axis of symmetry is introduced as an aid in graphing, and students determine the vertex and axis of symmetry from the vertex form and factored form of a quadratic function. They use the concept of symmetry to determine an additional point that lies on a parabola. Lastly, students identify whether a parabola is concave up or down based upon the sign of the x-squared term when the function is written in any form. | F.IF.C.8a | ✓ | | | | • | | | • | | | | • |
| **Transformations of Quadratic Functions** | Vertically Translating Quadratic Functions | Students vertically shift graphs of linear and quadratic functions. Students use verbal descriptions, graphs, and algebraic representations. | F.BF.3 | | ✓ | | | | • | | | | | | |
| | Horizontally Translating Quadratic Functions | Students horizontally shift graphs of linear and quadratic functions. Students use verbal descriptions, graphs, and algebraic representations. | F.BF.3 | | ✓ | | | | • | | | | | | |
| | Reflecting and Dilating Quadratic Functions using Graphs | Students reflect and dilate graphs of linear and quadratic functions. Students use verbal descriptions, graphs, and algebraic representations. | F.BF.3 | | ✓ | | | | • | | | | | | |
| | Transforming Quadratic Functions Using Tables | Given a table of values and a table of transformed values, students determine how the basic linear and quadratic functions were transformed to create the new functions. | F.BF.3 | | ✓ | | | | • | | | | | | |
| | Multiple Transformations of Quadratic Functions | Given a representation of a transformed function, students determine how the basic linear and quadratic functions were transformed to create the new functions. | F.BF.3 | | ✓ | | | | • | | | | | | |

CARNEGIE LEARNING

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|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Sketching and Comparing Quadratic Functions** | Comparing Increasing Linear, Exponential, and Quadratic Functions | Students use graphs and tables to observe that an increasing exponential function will eventually exceed an increasing linear or quadratic function. They determine the average rate of change for a linear, quadratic, and exponential function over different intervals. Students compare an increasing linear, quadratic, and exponential model in context to determine that the exponential model has the best output over time. | F.LE.3 | ✓ | | | | | | | • | | | • | |
| | Sketching Quadratic Functions | Sketch a quadratic function given factored, standard or vertex form | F.IF.C.7.a | | ✓ | | | | • | | | | | | |
| | Comparing Quadratic Functions in Different Forms | Given two quadratic functions in different representations -- equation, graph, table, or description -- with a contextual or noncontextual scenario, students compare the functions' y-intercepts, zeros, absolute maximums/minimums, or rates of change over a specific interval. | F.IF.C.9 | | ✓ | | | | | | | | | • | |

CARNEGIE LEARNING

# 4 Seeing Structure

## Topic 1 Solving Quadratic Equations

| MATHia Unit | MATHia Workspace | Overview | CCSS | Concept Builder | Mastery | Strategies: Animations | Classifications | Explore Tools | Graphing Tools | Interactive Diagrams | Interactive Worksheets | Proof | Real-World Scenarios | Solvers | Worked Example |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Adding, Subtracting, and Multiplying Polynomials** | Introduction to Polynomial Arithmetic | Students are introduced to polynomials and identify the difference between different types of polynomials as well as non-polynomials. They then use an Explore Tool to investigate combining like terms when adding polynomial expressions. Finally, students examine the steps to simplifying polynomial expressions that are either added or subtracted before simplifying on their own. | A.APR.1 | ✓ | | • | | • | | | • | | | | • |
| | Identifying Parts of Complex Algebraic Expressions | Students identify the parts of complex algebraic expressions, including terms, coefficients, sums, factors, products, differences, and quotients. | A.SSE.1a | ✓ | | | | • | | | | | | | • |
| | Operating with Functions on the Coordinate Plane | Students watch an animation about operating with functions on the coordinate plane before examining adding and subtracting constant functions, linear functions, and a linear and a quadratic function. | F.BF.3 | ✓ | | | • | | | | • | | | | |
| | Adding Polynomials | Students add quadratic expressions. | A.APR.1 | | ✓ | | | | | | | | • | | |
| | Subtracting Polynomials | Students subtract polynomials. | A.APR.1 | | ✓ | | | | | | | | • | | |
| | Using a Factor Table to Multiply Binomials | Students use factor tables to multiply linear expressions. Students combine like terms. | A.APR.1 | | ✓ | | | | | | • | | | | |
| | Multiplying Binomials | Students determine which factor table is appropriate for a given problem, set up the table, and then use the table to multiply linear expressions. | A.APR.1 | | ✓ | | | | | | • | | | | |

CARNEGIE LEARNING

# 4 Seeing Structure

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|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Representing Solutions to Quadratic Equations** | Making Sense of Roots and Zeros | Students experiment with patterns relating two lines and the parabola that is generated by the product of their two linear functions. The first pattern solidifies the fact that the two expressions are factors of the quadratic function. The second pattern guides students to the Zero Product Property, an underpinning for determining the zeros of a quadratic function written in factored form. | A.REI.11 | ✓ | | | | | | | | | | | |
| | Factoring using Difference of Squares | Students factor quadratic expressions using difference to two squares. | A.SSE.2 | | ✓ | | | | | | | | • | | |
| **Solutions to Quadratic Equations in Vertex Form** | Using Properties of Equality to Solve Quadratic Equations | Students use the Properties of Equality to solve quadratic equations in the form $y = ax^2$, $y = ax^2 + d$, $y = a(x - c)^2$, and $y = a(x - c)^2 + d$ where a, c, and d are constants. | A.REI.4b | | ✓ | | | | | | | | • | | |
| **Factoring and Completing the Square** | Introduction to Factoring | Students are introduced to factoring trinomials first using factor tables. They analyze patterns in the operations of binomial factors. Students factor the GCF from quadratic expressions. They practice factoring quadratic trinomials with and without first factoring out a GCF. Students then use factoring as a method to solve a quadratic equation. | A.SEE.3a | ✓ | | | | | | | • | | | | • |
| | Factoring Trinomials with Coefficients of One | Students factor quadratic trinomials with a coefficient of one. | A.SSE.3a | | ✓ | | | | | | | | • | | |
| | Factoring Trinomials with Coefficients Other than One | Students factor quadratic trinomials with a coefficient other than one. | A.SSE.3a | | ✓ | | | | | | | | • | | |
| | Factoring Quadratic Expressions | Students factor quadratic expressions using all known factoring methods. | A.SSE.3a | | ✓ | | | | | | | | • | | |
| | Solving Quadratic Equations by Factoring | Students solve quadratic equations by factoring and applying the zero-product property. | A.REI.4b | | ✓ | | | | | | | | • | | |

CARNEGIE LEARNING

# 4 Seeing Structure

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|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | Animations | Classifications | Explore Tools | Graphing Tools | Interactive Diagrams | Interactive Worksheets | Proof | Real-World Scenarios | Solvers | Worked Example |
| **Factoring and Completing the Square (continued)** | Problem Solving Using Factoring | Students create quadratic equations to represent mathematical and real-world situations. They then factor these equations and determine zeros to reveal different structures and quantities that can help them relate quantities and solve problems. | A.SSE.3a | ✓ | | | | | | | | | | | • |
| | Completing the Square | Students analyze a worked example of a quadratic function in general form being written in vertex form through the process of completing the square. They then practice completing the square using polynomials and area models before filling in unknown values in trinomials that create perfect square trinomials. Finally, students are shown the algebraic method of changing a quadratic function in general form to vertex form by completing the square. They use the algebra shown to determine the axis of symmetry and vertex of quadratic functions in general form. | A.REI.4a<br>F.IF.C.8a | ✓ | | | | | | | | | | | • |
| | Problem Solving Using Completing the Square | Students use the method of Completing the Square to convert quadratic equations to vertex form in order to solve real-world problems in different situations by revealing maxima of quadratic functions. | A.SSE.3b | ✓ | | | | | | | | | | | • |
| **The Quadratic Formula** | Deriving the Quadratic Formula | Students use the completing the square method to determine the roots of a given quadratic equation. They then analyze the method of completing the square for any quadratic equation in general form from which the Quadratic Formula is derived. They practice using the Quadratic Formula to calculate the roots of quadratic equations in general form. | A.REI.4a<br>A.REI.4b | ✓ | | | | | | | | | | | |
| | Solving Quadratic Equations | Students solve quadratic equations by using factoring or the quadratic formula. | A.REI.4b | | ✓ | | | | | | | | • | | |

# 4 Seeing Structure

## Topic 2 Applications of Quadratics

| MATHia Unit | MATHia Workspace | Overview | CCSS | Concept Builder | Mastery | Animations | Classifications | Explore Tools | Graphing Tools | Interactive Diagrams | Interactive Worksheets | Proof | Real-World Scenarios | Solvers | Worked Example |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Imaginary and Complex Numbers** | Introduction to Complex Numbers | Students watch a video introducing them to the imaginary number line and its relation to the real number line. They then practice identifying real and imaginary numbers through the sorting tool. Finally, students are introduced to complex numbers and practice identifying them on the complex plane to help them understand that all numbers are complex, but some are real and some are purely imaginary. | N.CN.1 | ✓ | | • | • | | | • | | | | | |
| | Simplifying Radicals with Negative Radicands | Students simplify radical expressions that result in complex numbers. | N.CN.1 | | ✓ | | | | | | | | | • | |
| | Simplifying Powers of *i* | Students identify expressions that are equivalent to i, -1, -i, and 1. They use the definition of i to rewrite higher powers of i. | N.CN.1 | ✓ | | | • | | | | • | | | | • |
| | Adding and Subtracting Complex Numbers | Students add and subtract complex numbers. | N.CN.2 | | ✓ | | | | | | | | | • | |
| | Multiplying Complex Numbers | Students multiply complex numbers, including both problems where the two complex numbers are complex conjugates, and problems where they are not. | N.CN.2 | | ✓ | | | | | | | | | • | |
| | Solving Quadratic Equations with Complex Roots | Students solve quadratic equations, some of which have real solutions and some of which have complex solutions. | N.CN.C.7 | | ✓ | | | | | | | | | • | |
| **Using Quadratic Functions to Model Data** | Using Regression Models | Students use equations of quadratic regression models, the solver, and graphs to answer questions. | S.ID.6a | | ✓ | | | | • | | • | | • | • | |

CARNEGIE LEARNING

# 4 Seeing Structure

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|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Using Quadratic Functions to Model Data (continued)** | Introduction to Inverses | Students watch an animation that shows the graph of the inverse of a function is a reflection of the function across the line y = x. They determine whether a function is a one-to-one function given its graph and the graph of its inverse. Students identify the graph of the inverse of a function by considering its reflection across y = x. They complete a table of values for the inverse of a function and determine its graph. | F.BF.4 | ✓ | | | • | • | | | | | | | |
| | Recognizing Graphs of Inverses | Given the graphs of two relations, students decide if the relations are inverses. | F.BF.4 | | ✓ | | | | | | | | | | |

## Topic 3 Circles on a Coordinate Plane

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|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Equation of a Circle** | Deriving the Equation of a Circle | Students are given a circle on the coordinate plane with a defined center. They use the Pythagorean Theorem to derive the standard form for the equation of a circle. | G.GPE.1 | ✓ | | | | | | | | | | | • |
| | Determining the Radius and Center of a Circle | Students are given an equation for a circle. They then rewrite the equation if necessary in standard form to identify the radius and center of the circle. | G.GPE.1 | ✓ | | | • | | | | | | | | • |
| **The Pythagorean Identity** | Proving the Pythagorean Identity | Students combine their knowledge of the Pythagorean Theorem and the unit circle to complete a proof of the Pythagorean identity $(\sin(x))^2 + (\cos(x))^2 = 1$. They use this identity to solve for solve problems where the value of sine or cosine in a specific quadrant is provided, and they must solve for the value of the other trigonometric function. A proof is provided for $(\tan(x))^2 + 1 = (\sec(x))^2$, and students duplicate the process to prove $1 + (\cot(x)^2) = (\csc(x))^2$. Students use the identity $(\tan(x))^2 + 1 = (\sec(x))^2$ to solve problems where the value of tangent or cosine in a specific quadrant is provided, and they must solve for the value of the other trigonometric function. | F.TF.C.8 | ✓ | | | | | | | • | | | | • |

CARNEGIE LEARNING

# 4 Seeing Structure

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|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **The Pythagorean Identity (continued)** | Using the Pythagorean Identity to Determine Sine, Cosine or Tangent | Students use the Pythagorean identity to solve for sin or cos of an angle given sin, cos, or tan of that angle. | F.TF.C.8 | | ✔ | | | | | | | | | | |

CARNEGIE LEARNING

# 5 Making Informed Decisions

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|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Topic 1 Independence and Conditional Probability** | | | | | | | | | | | | | | | |
| **Independence and Conditional Probablility** | Independent Events | Students define "independent events." They investigate different scenarios to determine whether the events given are independent or not independent. Students then investigate compound probability with "and" and use the equation P(A and B) = P(A) x P(B) to verify whether two events are independent or not. | S.CP.2 | ✔ | | | | | | | | | • | | • |
| **Topic 2 Computing Probabilities** | | | | | | | | | | | | | | | |
| **Computing Probabilities** | Understanding Frequency Tables | Students review how to read a two-way frequency table and construct a relative frequency table. Students then use two-way frequency tables to determine probabilities, including conditional and other compound probabilities, and they use information from frequency tables to check for the independence of events. | S.CP.4 | ✔ | | | | | | | • | | • | | • |
| | Calculating Compound Probabilities from Two-Way Tables | Students determine probabilities of compound events from two-way frequency tables via the Addition Rule. | S.CP.7 | | ✔ | | | | | | | | • | | |
| | Conditional Probability | Students use an interactive Explore Tool to explore probability using area and random points. Students then explore the idea of conditional probability, using the interactive tool to visualize the conditional probability formula P(A \| B) = P(A and B) / P(B). Students apply what they know about conditional probability to make predictions and check for independence of events using the Explore Tool. | S.CP.3 S.CP.6 | ✔ | | | | • | | | | | | | |
| | Recognizing Concepts of Conditional Probability | Students investigate conditional probabilities using two-way frequency tables. They apply the concept of conditional probability in a variety of different situations involving a change in the sample space as a result of an event occurring. | S.CP.5 | ✔ | | | | | | | | | • | | • |