CARNEGIE LEARNING

1 Exploring Relationships

Pacing: 33 Days

Topic 1: Applying Linear Expressions and Equations

Students rewrite literal equations using what they know about the Properties of Equality. They recall relationships formed by arithmetic and geometric sequences and write recursive and explicit formulas to represent these relationships. Finally, students use linear inequalities in one and two variables to represent constraints in real-world scenarios. They learn to use linear programming to optimize results.

Standards: ACE.3, ACE.4, AREI.11, AREI.12, NQ.1, FBF.1.a, FIF.2 **Pacing:** 5 Days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
1	**It's Literally About Literal Equations** Literal Equations	ACE.4 NQ.1	1	Students identify the slope and intercepts of functions in general, factored, and standard form. They determine the same characteristics for the equation $Ax + By = C$. They then explain which form is more efficient in determining the slope and the *x*- and *y*-intercepts. Next, the term literal equation is defined. Students rewrite different literal equations to solve for given variables.	• The general form of a linear equation is $y = ax + b$, where a and b are real numbers; a represents the slope, and b represents the *y*-intercept. • The factored form of a linear equation is $y = a(x - c)$, where a and b are real numbers; a represents the slope, and c represents the *x*-intercept. • The standard form of a linear equation is $Ax + By = C$, where A is a positive integer, B and C are integers, and both A and $B \neq 0$. It can be rewritten in general form as $y = -(A/B)x + C/B$; $-A/B$ represents the slope, C/B represents the *y*-intercept, and C/A represents the *x*-intercept. • General form of a linear equation is most useful form to identify the slope and *y*-intercept. Factored form of a linear equation is the most useful form to identify the slope and *x*-intercept. • Literal equations can be rewritten to highlight a specific variable.
2	**Did You Mean: Recursion?** Determining Recursive and Explicit Expressions from Contexts	FBF.1.a	1	Scenarios are presented that can be represented by arithmetic and geometric sequences. Students determine the value of different terms in each sequence. As the term number increases it becomes more time-consuming to generate the term value, which sets the stage for explicit formulas to be defined and used. Students practice using these formulas to determine the values of terms in both arithmetic and geometric sequences.	• A recursive formula expresses each new term of a sequence based on a preceding term of the sequence. • An explicit formula for a sequence is a formula for calculating each term of the sequence using the term's position in the sequence. • The explicit formula for determining the *n*th term of an arithmetic sequence is $a_n = a_1 + d(n-1)$, where n is the term number, a_1 is the first term in the sequence, a_n is the *n*th term in the sequence, and d is the common difference. • The explicit formula for determining the *n*th term of a geometric sequence is $g_n = g_1 \cdot r^{(n-1)}$, where n is the term number, g_1 is the first term in the sequence, g_n is the *n*th term in the sequence, and r is the common ratio.
3	**Take It to the Max … or Min** Linear Programming	ACE.3 AREI.11 AREI.12 FIF.2	1	Students are introduced to function notation for two variables and the term linear programming is defined. They define variables and identify the constraints as a system of linear inequalities for different scenarios. Students then graph the solution region of the system and label all points of intersection of the boundary lines, identifying the vertices of the solution region. They write a function to represent the profit or cost and substitute each of the four vertices into the equation of the function to determine a maximum profit or a minimum cost.	• Linear programming is a branch of mathematics that determines the maximum and minimum value of linear expressions on a region produced by a system of linear inequalities. • Real-world problems that involve determining maximum profit or minimum costs may be solved using linear programming. • Linear programming involves determining the solution to a system of linear inequalities, identifying the vertices of its solution region, and substituting the coordinates of each vertex into an algebraic expression to determine a maximum or minimum value.

*Pacing listed in 45-minute days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
Learning Individually with MATHia or Skills Practice		ACE.4 FBF.1a FIF.3	2	In the MATHia software, students practice solving literal equations. They then determine and classify the patterns in sequences and identify the next terms. Students write recursive and explicit formulas for given sequences. **MATHia Unit:** Literal Equations **MATHia Workspaces:** Solving Literal Equations / Describing Patterns in Sequences / Writing Recursive Formulas / Writing Explicit Formulas	

*Pacing listed in 45-minute days

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Topic 2: Exploring and Analyzing Patterns

Exploring and Analyzing Patterns begins with opportunities for students to analyze and describe various patterns. Questions ask students to represent algebraic expressions in different forms and use algebra and graphs to determine whether they are equivalent. They identify linear, exponential, and quadratic functions using multiple representations. The three forms of a quadratic equation are reviewed, and students learn to write quadratic equations given key points before using a system to write a quadratic equation given any three points. Finally, students recall how to solve quadratic equations; they consider quadratic equations with no real roots; and they then solve quadratic functions with imaginary roots.

Standards: AAPR.1, ACE.1, ACE.2, AREI.4, AREI.4.a, AREI.4.b, AREI.7, ASE.1, ASE.2, FBF.1.a, FIF.4, FIF.8, FIF.9, NCNS.1, NCNS.2, NCNS.7, NCNS.8, NCNS.9 **Pacing:** 19 Days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
1	**Patterns: They're Grrrrrowing!** Observing Patterns	FIF.8 ASE.1 ACE.1	1	Tiling patterns on floors, keeping secrets, and patio designs are used to illustrate sequences described by observable patterns. Students will analyze sequences and describe observable patterns. They sketch other terms or designs in each sequence using their knowledge of the patterns, and then will answer questions relevant to the problem situation. In one situation, a table is used to organize data and help recognize patterns as they emerge.	• Sequences are used to show observable patterns. • Patterns are used to solve problems. • Functions can be used to describe patterns.
2	**The Cat's Out of the Bag!** Generating Algebraic Expressions	ACE.1 ASE.1 FBF.1.a FIF.4 FIF.8	2	This lesson revisits the three scenarios from the previous lesson. Students will write equivalent algebraic expressions for the tile pattern of a square floor to determine the number of new tiles that must be added to create the next square tile design. They then show that the expressions are equivalent using the distributive property and combining like terms. In the second activity, equivalent expressions are written to represent the exponential situation for keeping secrets. Students then prove the expressions to be equivalent algebraically and graphically. Next, using the patio design situation, students will determine the number of squares in the next two patio designs and write equivalent expressions that determine the total number of squares in any given design. Again, the expressions are proven equivalent algebraically and graphically. The last activity summarizes the lesson using a geometric pattern.	• Two or more algebraic expressions are equivalent if they produce the same output for all input values. • You can use the properties of a graph to prove two algebraic expressions are equivalent.
3	**Samesies** Comparing Multiple Representations of Functions	ACE.1 ASE.1 FIF.8 FIF.9	2	The terms relation, function, and function notation are defined in this lesson. A sorting activity is presented that includes graphs, tables, equations, and contexts. Students sort the various representations into groups of equivalent relations. The various representations are then categorized with respect to their function families. Students then analyze a tile pattern and use a table to organize data, which leads to discovering additional patterns. Next, they create expressions that represent the number of white, gray and total tiles for any given design. Within the context of the problem situation, students use algebra to show different functions are equivalent and to identify them as quadratic functions.	• A relation is a mapping between a set of input values and a set of output values. • A function is a relation such that for each element of the domain there exists exactly one element in the range. • Function notation is a way to represent functions algebraically. The function $f(x)$ is read as "f of x" and indicates that x is the input and $f(x)$ is the output. • Tables, graphs, and equations are used to model function and non-function situations. • Equivalent expressions can be determined algebraically and graphically. • Graphing technology can be used to verify equivalent function representations.

*Pacing listed in 45-minute days

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Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
4	**True to Form** Forms of Quadratic Functions	AAPR.1 ACE.1 ACE.2 ASE.1 ASE.2 FBF.1.a FIF.4 FIF.9	2	Students match quadratic equations with their graphs using key characteristics. The standard form, the factored form, and the vertex form of a quadratic equation are reviewed as is the concavity of a parabola. Students then sort each of the equations with their graphs depending on the form in which the equation is written, while identifying key characteristics of each function such as the axis of symmetry, the *x*-intercept(s), concavity, the vertex, and the y-intercept. Next, students analyze graphs of parabolas in relation to a pair of numberless axes and select possible functions that could model the graph. A worked example shows that a unique quadratic function is determined when the vertex and a point on the parabola are known, or the roots and a point on the parabola are known. Students are given information about a function and use it to determine the most efficient form (standard, factored, vertex) to write the function. They then use the key characteristics of a graph and reference points to write a quadratic function, if possible. Finally, students analyze a worked example that demonstrates how to write and solve a system of equations to determine the unique quadratic function given three points on the graph. They then use this method to determine the quadratic function that models a problem situation and use it to answer a question about the situation.	• The standard form of a quadratic function is written as $f(x) = ax^2 + bx + c$, where a does not equal 0. • The factored form of a quadratic function is written as $f(x) = a(x - r_1)(x - r_2)$, where a does not equal 0. • The vertex form of a quadratic function is written as $f(x) = a(x - h)^2 + k$, where a does not equal 0. • The concavity of a parabola describes whether a parabola opens up or opens down. A parabola is concave down if it opens downward, and is concave up if it opens upward. • A graphical method to determine a unique quadratic function involves using key points and the vertical distance between each point in comparison to the points on the basic function. • An algebraic method to determine a unique quadratic function involves writing and solving a system of equations, given three reference points.
5	**The Root of the Problem** Solving Quadratic Equations	ASE.3.b AREI.4 AREI.7	2	Students solve quadratic equations of the form $y - ax^2 + bx + c$. They first factor trinomials and use the Zero Product Property. Students then use the method of completing the square to determine the roots of a quadratic equation that cannot be factored. They use the Quadratic Formula to solve problems in real-world and mathematical problems. Finally, students solve a system composed of two quadratic equations using substitution and factoring.	• One method of solving quadratic equations in the form $0 = ax^2 + bx + c$ is to factor the trinomial expression and use the Zero Product Property. • When a quadratic equation in the form $0 = ax^2 + bx + c$ is not factorable, completing the square is an alternative method of solving of the equation. • The Quadratic Formula, $x = (-b\pm\sqrt{}\ b^2 - 4ac)/2a$, can be used to solve to any quadratic equation written in general form, $0 = ax^2 + bx + c$, where a, b, and c represent real numbers and a (not equal to) 0. • A system of equations containing two quadratic equations can be solved algebraically and graphically. • The Quadratic Formula, substitution, and factoring are used to algebraically solve systems of equations. • A system of equations containing two quadratic equations may have no solution, one solution, two solutions, or infinite solutions.

*Pacing listed in 45-minute days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
6	***i* Want to Believe** Imaginary and Complex Numbers	AREI.4.b NCNS.1 NCNS.2 NCNS.7 NCNS.8 NCNS.9	2	Students begin by analyzing a quadratic function that does not cross the *x*-axis and use the Properties of Equality and square roots to solve the corresponding equation for its roots. Students are introduced to the number *i*, imaginary roots, imaginary zeros, and the complex number system. They use a complex coordinate plane to plot complex numbers and then use the graphical representation to understand how to add, subtract, and multiply complex numbers. Students solve quadratic equations with complex solutions using any method of their choosing. Finally, the Fundamental Theorem of Algebra is introduced, and students analyze graphs to determine the number of real and imaginary roots each corresponding quadratic equation has.	• Equations with no solution in one number system may have solutions in a larger number system. • The number i is a number such that $i^2 = -1$. • The set of complex numbers is the set of all numbers written in the form $a + bi$, where a and b are real numbers and b is not equal to 0. • The Commutative Property, the Associative Property, and Distributive Properties apply to complex numbers. • Functions that do not intersect the *x*-axis have imaginary zeros. • When the discriminant of a quadratic equation is a negative number, the equation has two imaginary roots. • The Fundamental Theorem of Algebra states that any polynomial equation of degree n must have n complex roots or solutions.
Learning Individually with MATHia or Skills Practice		AREI.4.b ASE.1 ASE.2 FBF.3 FIF.4 NCNS.1 NCNS.2 NCNS.7	8	In the MATHia software, students analyze patterns to generate algebraic expressions. They compare representations of, identify key characteristics of graphs of, and determine transformations performed on linear, quadratic, and exponential functions. Students examine the shape and structure of quadratic functions. They model with quadratics and solve quadratic equations and systems of quadratic equations. Students operate with complex numbers. **MATHia Unit:** Searching for Patterns **MATHia Workspaces:** Exploring and Analyzing Patterns / Comparing Familiar Function Representations **MATHia Unit:** Graphs of Functions **MATHia Workspaces:** Identifying Key Characteristics of Graphs of Functions / Transforming Functions **MATHia Unit:** Forms of Quadratic Functions **MATHia Workspaces:** Examining the Shape and Structure of Quadratic Functions / Quadratic Modeling / Quadratic Equation Solving / Quadratic Transformations **MATHia Unit:** Operations with Complex Numbers **MATHia Workspaces:** Introduction to Complex Numbers / Simplifying Radicals with Negative Radicands / Simplifying Powers of i / Adding and Subtracting Complex Numbers / Multiplying Complex Numbers / Solving Quadratic Equations with Complex Roots	

*Pacing listed in 45-minute days

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Topic 3: Applications of Quadratics

Students use the structure of a parabola and a given context to solve quadratic inequalities. They use what they know about solutions to functions on a graphical representation to solve systems of equations comprised of a quadratic and a linear function or two quadratic functions. Given a data set, students use technology to determine a regression curve that best fits the data and to make predictions for given input values. Finally, students reflect quadratics across $y = x$ to identify the graphical inverse of a function and then learn how to determine the equation of the inverse algebraically. Because quadratic functions are not one-to-one, students restrict the domain of quadratic functions to write their inverse functions.

Standards: ACE.1, ACE.2, ACE.3, AREI.4, AREI.7, AREI.11, FIF.7, SPID.6 **Pacing:** 9 Days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
1	**Ahead of the Curve** Solving Quadratic Inequalities	ACE.1 ACE.2 ACE.3 AREI.4	1	Students use the graph of a vertical motion model to approximate the times when an object is at given heights. They identify regions on the graph that are less than or greater than a given height and write a quadratic inequality to represent the situation. Next, students are shown how to solve a quadratic inequality algebraically. They determine the solution set of the inequality by dividing the graph into intervals defined by the roots of the quadratic equation, and then test values in each interval to determine which intervals satisfy the inequality. Finally, with a second scenario, students write the function that represents the situation, sketch a graph of the function, and write and solve a quadratic inequality related to the solution set of the quadratic function.	• A horizontal line drawn across the graph of a quadratic function intersects the parabola at exactly two points, except at the vertex, where it intersects the parabola at exactly one point. • The solution set of a quadratic inequality is determined by first solving for the roots of the quadratic equation, and then determining which interval(s) created by the roots will satisfy the inequality. A combination of algebraic and graphical methods may be the most efficient solution method. • Quadratic inequalities can be used to model some real-world contexts. The effects of translations of quadratic functions can be used to make comparisons within a context.
2	**All Systems Are Go!** Systems of Quadratic Equations	ACE.2 ACE.3 AREI.7 AREI.11	2	Students are presented with a scenario that can be modeled with a quadratic and a linear equation and reason about the intersections of the two equations in the context of the problem. Next, they solve systems of equations composed of a linear equation and a quadratic equation algebraically using substitution, factoring, and the Quadratic Formula. They then verify their algebraic solutions graphically by determining the coordinates of the points of intersection. Finally, students solve a system composed of two quadratic equations using the same methods. They conclude that a system of equations consisting of a linear and a quadratic equation can have one solution, two solutions, or no solutions, while a system of two quadratic equations can have one solution, two solutions, no solutions, or infinite solutions.	• Systems of equations involving a linear equation and a quadratic equation or two quadratic equations can be solved both algebraically and graphically. • A system of equations containing a linear equation and a quadratic equation may have no solution, one solution, or two solutions. • A system of equations containing two quadratic equations may have no solution, one solution, two solution, or an infinite number of solutions. • The number of solutions for a system of equations depends on the number of points where the graphs of the two equations intersect. • A system of equations involving a linear equation and a quadratic equation may be used to model real-world problems.

*Pacing listed in 45-minute days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
3	**Model Behavior** Using Quadratic Functions to Model Data	FIF.7 SPID.6	2	Students determine a quadratic regression that best models a table of data. They answer questions and make predictions using the regression equation. Students then analyze another data set and determine the quadratic regression inverse relation of the quadratic function they wrote. Next, the same algebraic process is used to determine the inverse of a quadratic function. Students graph equations containing square roots, identify the domain and range of each graph, and determine which graphs describe functions. Using only the equation of the inverse of a function, students then determine the original function and identify its domain and range. The term restrict the domain is introduced. Students determine the restrictions on the domain of a quadratic function based on the problem situation and graph the function with the restricted domain. They write the equation for the inverse function and interpret it with respect to the problem situation. Finally, students determine whether certain types of functions are one-to-one functions.	• Some data in context can be modeled by a quadratic regression equation. The regression equation can be used to make predictions; however there may be limitations on the domain depending on the context. • To determine the inverse of a function, replace $f(x)$ with y, switch the x and y variables, then solve for y. • When a problem requires using a given function to determine the independent quantity when a dependent quantity is given, determining the inverse of the original function may be a more efficient way to handle the situation. • The inverse of a function may or may not be a function. A function is a one-to-one function if its inverse is also a function. • To restrict the domain of a function means to define a new domain for the function that is a subset of the original domain.
Learning Individually with MATHia or Skills Practice		FBF.1.b F.BF.4 SPID.6	4	In the MATHia software, students solve systems comprising a linear and quadratic equation. They solve problems involving quadratic inequalities. Students use quadratic regression models to answer questions. They add and subtract linear functions. Students determine if two relations are inverses from their graphs. **MATHia Unit:** Graphs of Polynomial Functions **MATHia Workspaces:** Analyzing Polynomial Functions / Classifying Polynomial Functions / Interpreting Key Features of Graphs in Terms of Quantities / Identifying Key Characteristics of Polynomial Functions / Identifying Zeros of Polynomials / Using Zeros to Sketch a Graph of Polynomial / Understanding Average Rate of Change of Polynomial Functions / Comparing Polynomial Functions in Different Forms	

*Pacing listed in 45-minute days

CARNEGIE LEARNING

2 Analyzing Structure

Pacing: 23 Days

Topic 1: Composing and Decomposing Functions

Composing and Decomposing Functions introduces students to the concept of building new functions on the coordinate plane by operating on or translating functions. They build physical models of real-world scenarios and use what they know about linear functions to model linear dimensions. Students multiply these functions to build a quadratic function graphically and algebraically. Using what they already know about function transformations, students transform functions by variable amounts to build cubic functions. Students then consider new physical models and build cubic functions by multiplying three linear factors and by multiplying a linear factor by a quadratic factor. They are finally introduced to multiplicity, and they use the zero(s) of each factor and the signs of each linear function over given intervals of the x-value to sketch the graphs of functions.

Standards: N.CN.9 (+), A.SSE.1b, A.APR.1, A.APR.3, A.REI.10, A.REI.11, F.IF.4, F.IF.5, F.IF.7a, F.IF.7c, F.BF.3, G.GMD.3 **Pacing:** 10 Days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
1	**Blame It on the Rain** Modeling with Functions	ASE.1 AAPR.3 AREI.11 FBF.1.b	1	This lesson presents a storm drain problem situation. Students use this situation to calculate the length of the drain, the width of the drain, and the maximum cross-sectional area of the drain in two different situations. They create tables of values, equations, and graphs to represent each situation. Students then identify the function that represents the cross-sectional area of the drain as quadratic and the two factors that represent the length and width of the drain as linear. Students analyze the graph by relating the intercepts and axis of symmetry to this problem situation. Students learn the steps of the mathematical modeling process and describe how they use these steps in modeling the drain problem.	• Tables, graphs, and equations can be used to model real-world situations. • A function created by the product of two linear factors is a quadratic function. • The steps of the modeling process are Notice and Wonder, Organize and Mathematize, Predict and Analyze, and Test and Interpret.
2	**Folds, Turns, and Zeros** Transforming Function Shapes	FIF.7	1	In this lesson, students dilate functions by non-constant values in order to create higher degree functions. They begin by adding the function $y = x$ to the constant function $y = 3$ and interpret that operation as a translation of all the points on the horizontal line $y = 3$ by x, or the x-coordinate of each point. Students observe the change in the function produced by the translation and identify the points that did not move, along with the zeros. Students then dilate linear functions with both positive and negative slopes by x and observe how the quadratic function is formed in each case. Again, students analyze how the new zeros are created by each transformation and observe how the factor functions affect the intervals of increase and decrease of the product functions. Students repeat this analysis when dilating a quadratic, or degree-2, function to create a degree-3 function. Finally, students summarize what they have observed regarding dilations, linear factors, zeros, and the	• Functions can be translated and dilated by non-constant values, which apply a different transformation to each point of the function. • The linear factors of a function indicate the locations of the zeros of the function composed of those functions. • When a linear function is dilated vertically by multiplying the function by another linear function, the resulting function is a degree-2 function. • When a quadratic function is dilated vertically by multiplying the function by a linear function, the resulting function is a degree-3 function. • The graph of a function behaves differently at zeros described by linear factors and factors of degree 2. • The linear factors of a function can be used to sketch the graph of a function.

*Pacing listed in 45-minute days

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Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
3	**Planting the Seeds** Exploring Cubic Functions	AREI.10 FIF.4 FIF.5 FIF.7 GGMD.3	2	A rectangular sheet of copper is used to create planters if squares are removed from each corner of the sheet and the sides are then folded upward. Students analyze several sizes of planters and calculate the volume of each size. They then write a volume function in terms of the height, length, and width and graph the function using a graphing calculator. Using key characteristics, students analyze the graph. They differentiate the domain and range of the problem situation from the domain and range of the cubic function. The second activity is similar, but uses a cylindrical planter.	• Cubic functions can be used to model real-world contexts such as volume. • The general form of a cubic function is written as $f(x) = ax^3 + bx^2 + cx + d$, where $a \neq 0$. • A relative maximum is the highest point in a particular section of a graph, while a relative minimum is the lowest point in a particular section of a graph. • A cubic function may be created by the product of three linear functions or the product of a quadratic function and a linear function.
4	**The Zero's the Hero** Decomposing Cubic Functions	AAPR.1 AAPR.3 FIF.7 NCNS.9	2	Students investigate the multiplicity of the zeros of a polynomial function. They use these zeros, with multiplicity, to show the decompositions of quadratic and cubic functions into their linear and quadratic factors and reconstruct the product functions using these factors. Students review multiplying binomials in order to build polynomial expressions algebraically as well as graphically. They compare degree-1, degree-2, and degree-3 equations.	• The Fundamental Theorem of Algebra states that a degree n polynomial has, counted with multiplicity, exactly n zeros. • The Zero Product Property states that if the product of two or more factors is equal to zero, then at least one factor must be equal to zero. • The graph of a function written in factored form and the graph of a function written in general form is the same graph when the functions are equivalent. • Graphing is a strategy used to determine whether functions are equivalent. • The product of three linear functions is a cubic function, and the product of a quadratic function and a linear function is a cubic function. • Quadratic and cubic functions can be decomposed and analyzed in terms of their zeros
Learning Individually with MATHia or Skills Practice		FIF.5	4	In the MATHia software, students solve contextual problems involving cubic functions. They use a cubic function to solve for a dependent variable. Students then use the graph of the function to solve for the independent variable and interpret the maximum or minimum point of the graph. **MATHia Unit:** Graphs of Polynomial Functions **MATHia Workspaces:** Modeling Polynomial Functions	

*Pacing listed in 45-minute days

Topic 2: Characteristics of Polynomial Functions

Students explore power functions to gain an understanding of end behavior and symmetry and their connection to even-degree and odd-degree functions. They then explore even and odd functions and determine whether several polynomial functions are even, odd, or neither. Questions ask students to graph, write, and explain the effects of transformations on cubic functions, and then draw conclusions about how symmetry is preserved in transformed functions. Questions ask students to compare and contrast the various polynomials to understand all the possible shapes and key characteristics for linear, quadratic, cubic, quartic, and quintic functions.

Standards: AAPR.3, ACE.3, AREI.11, FBF.1.b, FBF.3, FIF.4, FIF.6, FIF.7, FIF.9 **Pacing:** 13 Days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
1	**So Odd, I Can't Even** Power Functions	AAPR.3 FBF.3	1	This lesson focuses on power functions described by the equation $P(x) = ax^n$. Students generalize the end behavior of even-degree and odd-degree power functions and sketch the graphs of power functions with negative coefficients. They explore the symmetry of graphs, concluding that even functions have line symmetry about $x = 0$ and odd functions have point symmetry about the origin. Students explore even and odd functions and determine algebraically whether given polynomial functions are even, odd, or neither.	• A power function is a function of the form $P(x) = ax^n$, where n is a non-negative integer. • For both odd- and even- degree functions, the graphs flatten as the degree increases for x-values between -1 and 1, and the graphs steepen as the degree increases for x-values less than -1 and greater than 1. • The end behavior of a graph of a function is the behavior of the graph as x approaches infinity and as x approaches negative infinity. • If a graph is symmetric about a line, the line divides the graph into two identical parts. • A function is symmetric about a point if each point on the graph has a point the same distance from the central point, but in the opposite direction. • When a point of symmetry is the origin, the graph is reflected across the x-axis and the y-axis. If (x, y) is replaced with $(-x, -y)$, the function remains the same. • The graph of an even function is symmetric about the y-axis, thus $f(x) = f(-x)$. • The graph of an odd function is symmetric about the origin, thus $f(x) = -f(-x)$.
2	**Math Class Needs a Makeover** Transformations of Polynomial Functions	FBF.3	2	Using a table of values, reference points and symmetric properties, students will graph quadratic and cubic functions. Students recall the transformational function form $g(x) = Af(B(x – C)) + D$, and they use transformations to graph polynomial functions, write equations for these functions, and explain the effects of the transformations. The general form of a polynomial function is given, and quartic and quintic functions are defined. Students use the graphs of functions to determine whether the functions are odd, even, or neither. Tables are used to organize the effects of transformations on the basic cubic and quartic functions as well as simple polynomial functions. Graphs of functions that have undergone multiple transformations are given, and students write the appropriate equation to describe each graph.	• A quartic function is a fourth degree polynomial function, and a quintic function is a fifth degree polynomial function. • The function $g(x) = Af(B(x – C)) + D$ is the transformation function form, where the constants A and D affect the output values of the function and the constants B and C affect the input values of the function. • The general shape and end behavior is the same for all odd-degree power functions, and the general shape and end behavior is the same for all even-degree power functions. • The graph of even functions are symmetric about the y-axis. • The graph of odd functions are symmetric about the origin. • Some transformations affect the symmetry of the polynomial function.

*Pacing listed in 45-minute days

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Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
3	**Poly-Wog** Key Characteristics of Polynomial Functions	AAPR.3 FIF.4	2	Students analyze a quartic regression equation and the corresponding graph. They use the graph to state all relative maximums, relative minimums, absolute maximums, and absolute minimums of the function. Students then use graph parts to investigate key characteristics of polynomial functions with given end behavior. Next, students analyze tables listing all the possible combinations of real and imaginary zeros for a linear, quadratic, and cubic function, along with examples of graphs with the given combination of zeros. They then complete similar tables for a quartic and a quintic function. Finally, students are given sets of specific key characteristics and sketch a graph that encompasses these aspects for each situation when possible.	• A polynomial with an even power has end behavior that is the same in both directions. A polynomial with an odd power has end behavior that is opposite in each direction. • An nth-degree odd polynomial has zero or an even number of extrema. An *n*th-degree even polynomial has an odd number of extrema. In either case, the maximum number of extrema is $n - 1$. • A polynomial function changes direction at each of its extrema. For that reason, the number of extrema and the number of changes of direction in the graph of the function are equal. • A polynomial with an even power has an even number of intervals of increase or decrease. A polynomial with an odd power has an odd number of intervals of increase or decrease. • The combination of real and imaginary roots of a polynomial function are equal to the degree of the polynomial and can be used to help determine the shape of its graph.
4	**Function Construction** Building Cubic and Quartic Functions	AAPR.3 FBF.1.b FIF.7	2	Students analyze a set of linear and quadratic functions. They compose these functions to build cubic functions, given the three zeros of the function or other key characteristics of the function. Students reason that a cubic function may have 0 or 2 imaginary zeros and that multiple cubic functions can be written from a given set of zeros. Next, they describe different combinations of function types that build a quartic function. Students analyze tables representing three functions and determine whether the third function is quartic and identify the number of real and imaginary zeros and the end behavior of the function. Finally, they analyze a set of linear, quadratic, and cubic functions. Students sketch a combination of these functions whose product builds a quartic function when possible, given specific criteria. They build a polynomial function given a set of zeros and given a graph, describing the characteristics of the function and comparing both processes.	• Cubic functions can be the product of three linear functions or the product of a quadratic function and a linear function. • A cubic function may have 0 or 2 imaginary zeros. • Quartic functions can be the product of four linear functions, two quadratic functions, a quadratic function and two linear functions, or a cubic function and a linear function. • A quartic function may have 0, 2, or 4 imaginary zeros. • An infinite number of functions can be written from a given set of zeros. • A unique function can be written from the graph of a function. • Functions of degree n are composed of factors whose degree sum to n. • A polynomial function may have a combination of real and imaginary zeros.
5	**Level Up** Analyzing Polynomial Functions	ACE.3 AREI.11 FIF.4 FIF.6	1	A cubic function is used to model the profit of a business over a period of time. Students analyze the graph using key characteristics, and then use the graph to answer questions relevant to the problem situation. The average rate of change of a function is defined, and a worked example demonstrates how to calculate the average rate of change for a specified time interval, and students calculate an average rate of change over a different time interval.	• The average rate of change of a function is the ratio of the change in the dependent variable to the change in the independent variable over a specified interval. • The formula for average rate of change is $f(b) - f(a) / b - a$ for an interval (a, b). The expression $b - a$ represents the change in the input of the function f. The expression $f(b) - f(a)$ represents the change in the function f as the input changes form a to b.

*Pacing listed in 45-minute days

CARNEGIE LEARNING

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
6	**To a Greater or Lesser Degree** Comparing Polynomial Functions	FIF.9	2		• Polynomial functions can be compared using graphs, tables, and equations. • Analyzing key characteristics of polynomial functions allows for comparison of the functions.
Learning Individually with MATHia or Skills Practice		AAPR.1 AAPR.3 FBF.1.b FIF.4 FIF.6 FIF.9	3	In the MATHia software, students identify the key characteristics of polynomial functions from their equations and graphs and interpret them in context. They identify the zeros of a polynomial and use the zeros to sketch its graph. Students use a graph to calculate the average rate of change of a polynomial function for a specified interval. They compare polynomial functions in different representations. **MATHia Unit:** Graphs of Polynomial Functions **MATHia Workspaces:** Analyzing Polynomial Functions / Classifying Polynomial Functions / Interpreting Key Features of Graphs in Terms of Quantities / Identifying Key Characteristics of Polynomial Functions / Identifying Zeros of Polynomials / Using Zeros to Sketch a Graph of Polynomial / Understanding Average Rate of Change of Polynomial Functions / Comparing Polynomial Functions in Different Forms	

*Pacing listed in 45-minute days

CARNEGIE LEARNING

3 Developing Structural Similarities

Pacing: 37 Days

Topic 1: Relating Factors and Zeros

This topic presents opportunities for students to analyze, factor, solve, and expand polynomial functions. Relating Factors and Zeros begins with students expanding their knowledge of factoring quadratics to include polynomials. They use factors to determine zeros and sketch graphs of the functions. Students learn to divide polynomials using two methods and to expand on this knowledge to determine whether a divisor is a factor of the dividend. In addition, they determine that polynomial functions, just like the integers, are closed under addition, subtraction, and multiplication but not division. Finally, students solve polynomial inequalities graphically and algebraically.

Standards: AAPR.1; AAPR.2, AAPR.3, ACE.1, ACE.3, ASE.1, ASE.2, ASE.3.a, FIF.8.a, NCNS.8 **Pacing:** 10 Days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
1	**Satisfactory Factoring** Factoring Polynomials to Identify Zeros	AAPR.3 ASE.2 FIF.8.a NCNS.8	2	Students recall factors of whole numbers in preparation for determining factors of polynomials. Methods of factoring polynomials are introduced, such as factoring out the Greatest Common Factor (GCF), chunking, recognizing perfect square trinomials, factoring by grouping, and factoring in quadratic form. Worked examples are used throughout the lesson to show the steps involved using the methods. Students write polynomials in factored form over the set of real numbers and over the set of complex numbers. They also determine the most efficient method of factoring several polynomials and explain their reasoning.	• The graphs of all polynomials that have a monomial GCF that includes a variable will pass through the origin. • Analyzing the structure of a polynomial may help you determine which factoring method may be most helpful. • Chunking is a method of factoring a polynomial in quadratic form that does not have common factors in all terms. Using this method, the terms are rewritten as a product of 2 terms, the common term is substituted with a variable, and then it is factored as is any polynomial in quadratic form. • Factoring a perfect square trinomial can occur in two forms: $a^2 - 2ab + b^2 = (a - b)^2$ or $a^2 + 2ab + b^2 = (a + b)^2$ • Factoring by grouping is a method of factoring a polynomial that has four terms in which not all terms have a common factor. The terms can be first grouped together in pairs that have a common factor, and then factored again. • Factoring by using quadratic form is a method of factoring a polynomial of degree 4 of the form, $ax^4 + bx^2 + c$. • Factoring the difference of squares is in the form: $a^2 - b^2 = (a + b)(a - b)$.
2	**Divide and Conquer** Polynomial Division	ASE.1 ASE.2 ASE.3.a AAPR.1 AAPR.2	2	The algebraic representation of a cubic function is given and its graph is shown. Students determine the real factor of the function from the graph. They reason that the other factor must be a quadratic function with imaginary zeros, but they cannot represent it algebraically yet. A worked example of polynomial long division is provided and students determine the quadratic function that is the other factor. They distinguish between factoring over the real and the complex number system by determining the imaginary zeros of the quadratic function and rewriting the cubic function as a product of linear factors. Next, students investigate what the remainder means in terms of polynomial division. The Remainder Theorem is stated and students use the theorem to answer questions involving polynomial division with remainders. Finally, a worked example of synthetic division is provided. Students use the algorithm to determine the quotient in several problems.	• Factors of polynomials divide into a polynomial without a remainder. • A polynomial equation of degree n has n roots over the complex number system and can be written as the product of n factors of the form $(ax + b)$. • Polynomial long division is an algorithm for dividing one polynomial by another of equal or lesser degree. • Synthetic division is a shortcut method for dividing a polynomial by a linear expression of the form $(x - r)$. • The Factor Theorem states that a polynomial function $p(x)$ has $x - r$ as a factor if and only if the value of the function at r is 0, or $p(r) = 0$. • The Remainder Theorem states that when any polynomial equation or function $f(x)$ is divided by a linear expression of the form $(x - r)$, the remainder is $R = f(r)$ or the value of the function when $x = r$. • The difference of cubes can be written in factored form as: $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$. The sum of cubes can be written in factored form as: $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$.

*Pacing listed in 45-minute days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
3	**Closing Time** The Closure Property	AAPR.1	1	Students review the four basic operations over the set of natural numbers, whole numbers, integers, rational numbers, and irrational numbers and determine which operations are closed and not closed over which sets of numbers. They determine that integers and polynomials are not closed under division. The concept of polynomials closed under an operation is defined and students then prove that polynomials are closed under addition, subtraction, and multiplication. Students compare polynomials and use multiple representations to analyze and compare polynomial functions.	• When an operation is performed on any number or expression in a set and the result is in the same set, it is said to be closed under that operation. • Polynomials are closed under addition, subtraction, and multiplication. • Polynomials are not closed under division.
4	**Unequal Equals** Solving Polynomial Inequalities	ACE.1 ACE.3	1	Solving polynomial inequalities is very much like solving linear inequalities. Students solve polynomial inequalities both graphically and algebraically. Problem situations include profit models, vertical motion, glucose levels in the bloodstream, and volume. Graphing calculators are used in this lesson.	• Solving polynomial inequalities is similar to solving linear inequalities. • The solutions to a polynomial inequality are intervals of x-values that satisfy the inequality.
Learning Individually with MATHia or Skills Practice		AAPR.1 AAPR.2 AAPR.6 ACE.1 ASE.2 AREI.4.b	4	In the MATHia software, students multiply polynomials. They solve quadratic equations by factoring. Students use synthetic division to divide a polynomial by a linear divisor. They factor higher-order polynomials. Students solve polynomial equations and inequalities. **MATHia Unit:** Polynomial Operations **MATHia Workspaces:** Using a Factor Table to Multiply Polynomials / Multiplying Polynomials / Solving Quadratic Equations by Factoring / Synthetic Division **MATHia Unit:** Solving Polynomials **MATHia Workspaces:** Factoring Higher Order Polynomials / Solving Polynomial Functions	

*Pacing listed in 45-minute days

CARNEGIE LEARNING

Topic 2: Polynomial Models

In Polynomial Models, students use the concept of equality to express mathematical relationships using different representations. They begin by exploring polynomial identities, which are useful for showing the relationship between two seemingly unrelated expressions. Polynomial identities are used to perform calculations, verify Euclid's Formula, and generate Pythagorean triples. Students then explore patterns in Pascal's Triangle and use it to expand powers of binomials. They apply the Binomial Theorem and its combinatorics as an alternative method to expand powers of binomials. Finally, they move between function representations as they apply polynomial regressions to represent data in context.

Standards: AAPR.4, AAPR.5, ACE.3, FBF.1.a, FBF.1.b, FIF.4, FIF.5, SPID.6 **Pacing:** 6 Days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
1	**Not a Case of Mistaken Identity** Exploring Polynomial Identities	AAPR.4	1	Polynomial identities such as $(a + b)^2$, $(a - b)^2$, $a^2 - b^2$, $(a + b)^3$, $(a - b)^3$, $a^3 + b^3$, and $a^3 - b^3$ are used to perform calculations involving large numbers without a calculator. Euclid's Formula is stated and used to generate Pythagorean triples. In the last activity, students verify algebraic statements by transforming one side of the equation to show that it is equivalent to the other side of the equation.	• Polynomial identities such as $(a + b)^2 = a^2 + 2ab + b^2$ can be used to help perform calculations with large numbers. • Euclid's Formula can be used to generate Pythagorean triples given positive integers r and s, where $r > s$: $(r^2 + s^2)^2 = (r^2 - s^2)^2 + (2rs)^2$.
2	**Elegant Simplicity** Pascal's Triangle and the Binomial Theorem	AAPR.5 SPID.6	1	Students analyze and extend the patterns in the rows of Pascal's Triangle. They then explore a use of Pascal's Triangle when raising a binomial to a positive integer. Students expand several binomials using Pascal's Triangle. The combination formula is given and technology and Pascal's Triangle are used to calculate combinations. The Binomial Theorem is stated and students use it to expand $(a + b)^{15}$. Finally, students expand several binomials with coefficients other than 1.	• The Binomial Theorem states that it is possible to extend any power of $(a + b)$ into a sum of the form: $(a + b)^n =$ (n (over) 0) a^nb^0 + (n (over) 1) $a^{n-1}b^1$ + (n (over) 2) $a^{n-2}b^2$ + · · · + (n (over) (n – 1)) a^1b^{n-1} + (n (over) n) a^0b^n • The formula for a combination of k objects from a set of n objects for $n \geq k$ is: (n (over) k) = ${}_nC_k = n!/(k!(n - k)!)$
3	**Modeling Gig** Modeling with Polynomial Functions and Data	ACE.3 FBF.1.a FBF.1.b FIF.4 FIF.5 SPID.6	2	Traffic patterns in a downtown area, the federal minimum wage, monthly precipitation, and inflation are contexts modeled by polynomial functions. Data are organized in a table of values for each situation, and students use technology to create a scatter plot and determine polynomial regression equations. The coefficient of determination is used to determine which regression equation best describes the data. The regression equations are used to make predictions, and students then construct graphs to represent different periods of time.	• A regression equation is a function that models the relationship between two variables in a scatter plot. • The coefficient of determination, or R^2, measures the strength of the relationship between the original data and their regression equation. The value ranges from 0 to 1 with a value of 1 indicating a perfect fit between the regression equation and the original data. • Regression equations can be used to make predictions about future events.
Learning Individually with MATHia or Skills Practice		AAPR.5 SPID.6	2	In the MATHia software, students expand powers of binomials using Pascal's Triangle and the Binomial Theorem. They determine polynomial regression equations to model problem and situations and use them to answer questions. **MATHia Unit:** Polynomial Models **MATHia Workspaces:** Pascal's Triangle / Binomial Theorem / Exploring Polynomial Regression / Solving Polynomial Inequalities	

*Pacing listed in 45-minute days

CARNEGIE LEARNING

Topic 3: Rational Functions

Students analyze, graph, and transform rational functions. The topic begins with an analysis of key characteristics of rational functions and graphs. Lessons then expand on this knowledge to transform rational functions. Students determine whether graphs of rational functions have vertical asymptotes, removable discontinuities, both, or neither, and then sketch graphs of rational functions detailing any holes and/or asymptotes. They then explore problem situations modeled by rational functions and answer questions related to each scenario. Rational Functions provides opportunities for students to connect their knowledge of operations with rational numbers to operations with rational expressions. They conclude that rational expressions are similar to rational numbers and are closed under all the operations. Students then write and solve rational equations and list restrictions, considering efficient ways to operate with rational expressions and to solve rational equations based on the structure of the original equation. The topic closes with problems related to work, mixture, cost, and distance.

Standards: AAPR.6, AAPR.7, ACE.1, AREI.1, AREI.2, ASE.2, FBF.3, FIF.5, FIF.7.a, FIF.8.a, GM.2 **Pacing:** 21 Days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
1	**There's a Fine Line Between a Numerator and a Denominator** Introduction to Rational Functions	FIF.7.a	2	Students explore and compare the graphs, tables, and values of two basic functions, $f(x) = x$ and $q(x) = x^2$, and their reciprocal functions, $g(x) = 1 / x$ and $r(x) = 1 / x^2$. Technology is used to explore the key characteristics of the reciprocals of all power functions, including horizontal and vertical asymptotes. Students then construct a Venn diagram to show the similarities and differences between the groups of reciprocal power functions.	• A rational function is any function that can be written as the ratio of two polynomials. It can be written in the form $f(x) = P(x) / Q(x)$, where $P(x)$ and $Q(x)$ are polynomial functions, and $Q(x) \neq 0$. • The reciprocals of power functions are rational functions. • A vertical asymptote is a vertical line that a function gets closer and closer to, but never intersects. • The reciprocals of all power functions have a vertical asymptote at $x = 0$, a horizontal asymptote at $y = 0$, and a domain of all real numbers except $x \neq 0$. • The reciprocals of power functions with an exponent that is an even number lie in Quadrants I and II, and their range is $y > 0$. • The reciprocals of power functions with an exponent that is an odd number lie in Quadrants I and III, and their range is all real numbers except y (not equal to) 0.
2	**Approaching Infinity** Transformations of Rational Functions	FBF.3	2	Students explore transformations of rational functions. Without using technology, students sketch several rational functions and indicate the domain, range, vertical and horizontal asymptotes, and the y-intercept. They then match or sketch transformed rational functions with their graphs and vice versa.	• Translations of a rational function $f(x)$ are given in the form $g(x) = Af(B(x - C)) + D$, where a negative A-value reflects $f(x)$ vertically, the D–value translates $f(x)$ vertically, and a C-value translates $f(x)$ horizontally. • The C-value affects the vertical asymptote. The vertical asymptote affects the domain. • The D-value affects the horizontal asymptote. The horizontal asymptote affects the range. • Vertical asymptotes of a rational function can be determined by identifying values of x for which the denominator equals 0. • The reciprocal of a function of degree n can have at most n vertical asymptotes.

*Pacing listed in 45-minute days

CARNEGIE LEARNING

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
3	**There's a Hole in My Function!** Graphical Discontinuities	AAPR.6 FIF.7.a FIF.8.a	2	Students match rational functions with their graphs and recognize that functions with a common factor in the numerator and denominator have removable discontinuities in their graphs, whereas functions that have undefined values in the denominator have vertical asymptotes. They graph several rational functions containing holes or asymptotes. A table shows similarities between rational numbers and rational functions, and students list any restrictions in the domain for each example. They then analyze a worked example and explain why a hole and a vertical asymptote are both present in the graph of the function. Given the functions, students determine whether the graphs of rational functions have vertical asymptotes, removable discontinuity, both, or neither.	• A removable discontinuity is a single point at which the graph is not defined. • The graphs of rational functions have either a removable discontinuity or a vertical asymptote for all domain values that result in division by 0. • Holes are created in the graphs of rational functions when a common factor divides out of the numerator and denominator of the function.
4	**Must Be a Rational Explanation** Operations with Rational Expressions	ASE.2 AAPR.6 AAPR.7	3	The process for adding and subtracting rational expressions is compared to the process for adding and subtracting rational numbers. Students add and subtract several rational expressions by first determining common denominators and identifying restrictions on the domain of the function. The process for multiplying and dividing rational expressions is similar to the process for multiplying and dividing rational numbers. Students then multiply and divide several rational expressions and list restrictions on the variables, recognizing that the processes are similar to multiplying and dividing rational numbers. Students determine that the set of rational expressions is closed under addition, subtraction, multiplication, and division.	• The processes of adding, subtracting, multiplying, and dividing rational expressions are similar to the processes for rational numbers. • To determine the least common denominator of algebraic expressions, first factor the expressions and divide out common factors. • The domain restrictions for a rational expression must be based upon the original expressions. • Rational expressions are closed under the operations of addition, subtraction, multiplication, and division.
5	**Thunder. Thun- Thun- Thunder.** Solving Problems with Rational Equations	ACE.1 AREI.1 AREI.2 ASE.2 FIF.5	2	The average cost per month for cable television, grams of chocolate in trail mix, a thunderstorm, and the Golden Ratio are all situations students model using rational equations. They answer questions related to each scenario, create proportions, write rational expressions, describe the behavior of the ratios in the proportions, identify the domain and range, and calculate average costs. Students use multiple methods to solve rational equations, which are identified as proportions that students have solved in previous courses. A sorting activity is used to group and solve rational equations by different methods.	• Rational functions can be used to model real-world problems. • A rational equation is an equation that contains one or more rational expressions. Rational equations are proportions. • The structure of an equation often determines the most efficient method to solve the equation.

*Pacing listed in 45-minute days

CARNEGIE LEARNING

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
6	**16 Tons and What Do You Get?** Solving Work, Mixture, Distance, and Cost Problems	ACE.1 AREI.1 AREI.2 GM.2	2	Rational equations are used to model work problems, mixture problems, distance problems, and cost problems.	• Rational functions can be used to model real-world problems. • A work problem is a type of problem that involves the rates of several workers and the time it takes to complete a job. • A mixture problem is a type of problem that involves the combination of two or more liquids and the concentrations of those liquids. • A distance problem is a type of problem that involves distance, rate, and time. • A cost problem is a type of problem that involves the cost of ownership of an item over time.
Learning Individually with MATHia or Skills Practice		AAPR.6 ACE.1 AREI.2 FIF.7.a	8	In the MATHia software, students describe the asymptotes of a rational function given its graph or equation. They operate with and rewrite rational expressions. They solve rational equations in both mathematical and real-world contexts. **MATHia Unit:** Rational Functions **MATHia Workspaces:** Introduction to Rational Functions / Modeling Ratios as Rational Functions **MATHia Unit:** Rational Expressions and Equations **MATHia Workspaces:** Simplifying Rational Expressions / Adding and Subtracting Rational Expressions / Multiplying and Dividing Rational Expressions / Solving Rational Equations that Result in Linear Equations **MATHia Unit:** Rational Models **MATHia Workspaces:** Modeling Rational Functions / Using Rational Models / Solving Work, Mixture, and Distance Problems / Modeling and Solving with Rational Functions	

*Pacing listed in 45-minute days

CARNEGIE LEARNING

4 Inverting Functions

Pacing: 40 Days

Topic 1: Radical Functions

This topic presents opportunities for students to explore radical functions, simplify radical expressions, and solve radical equations. Radical Functions begins with an introduction to radical functions as inverses of power functions. Students graph radical functions, write their equations, and determine their key characteristics. Lessons then expand on this knowledge to explore transformations of radical functions. In the later part of the topic, students rewrite radicals using rational exponents and extract roots from radical expressions. Students also multiply, divide, add, and subtract radical expressions. Finally, students analyze solution strategies for radical equations, and use radical equations to solve real-world problem situations.

Standards: ACE.4, AREI.2, FBF.3, FBF.4.a, FBF.4.b, FIF.4, FIF.5, FIF.7.b, FIF.9, GM.2, NRNS.1, NRNS.2 **Pacing:** 13 Days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
1	**Strike That, Invert It** Inverses of Power Functions	FBF.4.b FIF.4 FIF.7.b	1	Students trace the graphs of several power functions on patty paper. They transpose the independent and dependent quantities for each situation and determine whether the graph of the new function is also a function. The Vertical Line Test is used to determine whether or not the inverse of a power function is also a function. Students conclude that the Horizontal Line Test can be used to determine whether the inverse of a function is also a function. The term invertible function is defined.	• A function is the set of all ordered pairs (x, y), or $(x, f(x))$, where for every value of x there is one and only one value of y, or $f(x)$. • The inverse of a function is the set of all ordered pairs (y, x), or $(f(x), x)$. • If the inverse of a function is also a function, the function is said to be an invertible function, and its inverse is written as $f^{-1}(x)$. • A Horizontal Line Test is a visual method to determine whether a function has an inverse that is also a function. • A power function is a polynomial function of the form $P(x) = ax^n$, where n is a non-negative integer. When n is an odd number, the function is invertible, and when n is an even number, the function is not invertible.
2	**Such a Rad Lesson** Radical Functions	FBF.4.a FIF.4 FIF.5 FIF.7b GM.2	3	Students determine and graph the inverse of the power function $f(x) = x^2$ by transposing the coordinates of the points in a table of values. They restrict the domain to create a square root function—the inverse of a power function—and identify the key characteristics of each. They also determine the cube root function—the inverse of the power function $f(x) = x^3$—and conclude that the domain need not be restricted to create this inverse function. The general term radical function is introduced and defined. Students learn to use the composition of functions to determine algebraically whether pairs of functions are inverse functions. Students answer questions related to radical functions in real-world and mathematical problems.	• The square root function is the inverse of the power function $f(x) = x^2$ when the domain of the power function is restricted to values greater than or equal to 0. • The cube root function is the inverse of the power function $f(x) = x^3$. • Radical functions are inverses of power functions with exponents greater than or equal to 2. • For two functions f and g, the composition of functions uses the output of one as the input of the other. It is expressed as $f(g(x))$ or $g(f(x))$. If $f(g(x)) = g(f(x)) = x$, then $f(x)$ and $g(x)$ are inverse functions. • Radical functions may be used to model and solve real-world problems.
3	**Making Waves** Transformations of Radical Functions	FBF.3 FIF.5 FIF.7.b FIF.9	1	Transformations of radical functions are used to create a graphic design that will serve as a logo for a surfing school. Students write equations and graph transformations of radical functions with restricted domains using the transformation function form. The effects of transformations on radical functions are identified, and students describe key characteristics and restrictions on the domains.	• Transformations of radical functions can be described by the transformation function form, $g(x) = Af(B(x - C)) + D$. • Transformations can be used to model graphic designs.

*Pacing listed in 45-minute days

CARNEGIE LEARNING

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
4	**Keepin' It Real** Rewriting Radical Expressions	NRNS.1 NRNS.2	2	Students analyze tables and graphs for different values of n in the expression $\sqrt[n]{x^n}$ and conclude that when extracting a variable from a radical expression, $\sqrt[n]{x^n}$ can be written as $\|x\|$ when n is even and as x when n is odd. Next, they use the rules of exponents to verify that a radical expression $\sqrt[n]{x^a}$ can be written as the exponential expression $x^{a/n}$. Students analyze examples of extracting roots and radicals with extracted roots. They multiply, divide, add, subtract, and simplify radical expressions. Finally, students complete a graphic organizer by providing examples of the sum, difference, product, and quotient of radical expressions.	• To extract a variable from a radical, the expression $\sqrt[n]{x^n}$ can be written as $\|x\|$, when n is even, or x, when n is odd. • A radical expression $\sqrt[n]{x^a}$ can be rewritten as an exponential expression $x^{a/n}$ for all real values of x if the index is n is odd, and for all real values of x greater than or equal to 0 if the index n is even. • The root of a product is equal to the product of its roots, $\sqrt[p]{a^m/b^n} = \sqrt[p]{a^m} \cdot \sqrt[p]{b^n}$. • The root of a quotient is equal to the quotient of its roots, $\sqrt[p]{a^m/b^n} = \sqrt[p]{a^m}/\sqrt[p]{b^n}$. • When multiplying or dividing radical expressions, multiply or divide numbers and variables separately and then extract roots. • Radical expressions with the same degree and same radicand are like terms. • When adding or subtracting radical expressions, add or subtract the coefficients of like terms.
5	**Into the Unknown** Solving Radical Equations	ACE.4 AREI.2	1	Students learn solution strategies to solve radical equations and check their answers to identify any extraneous solutions. They solve several radical equations both in and out of context.	• Strategies to solve equations, such as using the Properties of Equality and isolating the term containing the unknown, can be applied be solve radical equations. • Raising both sides of the equation to a power when solving a radical equation may introduce extraneous solutions. To identify extraneous solutions, you must substitute each solution into the original equation to determine whether it results in a true statement. • Radical equations can be used to model real-world problems.
Learning Individually with MATHia or Skills Practice		FBF.4 FBF.4.a FIF.7.b NRNS.1 NRNS.2	5	In the MATHia software, students describe the asymptotes of a rational function given its graph or equation. They operate with and rewrite rational expressions. Students solve rational equations in both mathematical and real-world contexts. **MATHia Unit:** Inverses of Functions **MATHia Workspaces:** Investigating Inverses of Functions / Graphing Square Root Functions / Sketching Graphs of Inverses / Calculating Inverses of Linear Functions **MATHia Unit:** Rewriting and Operating with Radicals **MATHia Workspaces:** Simplifying Radicals / Adding and Subtracting Radicals / Multiplying Radicals / Dividing Radicals **MATHia Unit:** Radical Expressions with Variables **MATHia Workspaces:** Simplifying Radicals with Variables / Adding and Subtracting Radicals with Variables	

*Pacing listed in 45-minute days

Topic 2: Exponential and Logarithmic Functions

Students analyze, graph, and transform exponential and logarithmic functions. Exponential and Logarithmic Functions begins with an exploration of exponential functions. Students analyze key characteristics of exponential functions and graphs. Lessons then expand on this knowledge for transformations of exponential functions. In the later part of the topic, lessons focus on logarithmic functions. Students determine key characteristics of logarithmic functions and graphs. They also transform logarithmic functions and make generalizations about the effect of a transformation on an inverse function.

Standards: AREI.11, ASE.3.c, FBF.3, FBF.4, FIF.4, FIF.5, FIF.7.c, FIF.8.b, FIF.9, FLQE.1, FLQE.2, FLQE.5 **Pacing:** 10 Days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
1	**Half-Life** Comparing Linear and Exponential Functions	AREI.11 FIF.4 FIF.8.b FLQE.1 FLQE.2 FLQE.5	2	Students use a real-world problem situation to explore exponential functions. They create exponential equations using their knowledge of geometric sequences and analyze the graphs. Different exponential functions are compared using tables of values, equations, and graphs. Graphing technology is used to locate points at which both functions are equal. The term half-life is defined and used within the context of a situation. Graphing technology is used to make predictions.	• A geometric sequence with a positive common ratio that is not 1 can be written as an exponential function using the properties of powers. • Over time, an exponential function with a *b*-value greater than one always exceeds a linear function with an *m*-value greater than zero. • A half-life refers to the amount of time it takes a substance to decay to half of its original amount.
2	**Pert and Nert** Properties of Exponential Graphs	AREI.11 FIF.4 FIF.7.c FIF.9 FLQE.2 FLQE.5	2	A sorting activity compares the equations, tables, and graphs of exponential growth and exponential decay functions. Students write exponential growth and decay functions given specified characteristics. They also summarize the characteristics for the basic exponential growth and decay functions using a table. The irrational number *e*, or natural base *e*, is developed through an exploration of continuously compounding interest. A worked example derives the formula for compound interest with continuous compounding, and students use a parallel formula to investigate population growth and decline.	• For basic exponential growth functions, $f(x) = b^x$, *b* is a value greater than 1. For basic exponential decay functions, $f(x) = b^x$, *b* is a value between 0 and 1. • The compound interest formula is $A = P \cdot (1 + (r/k))^{kt}$, where A represents the value, *P* represents the principal amount, *r* represents the interest rate, and *k* represents the frequency of compounding in time *t*. • The natural base $e \approx 2.7182818\ldots$ is an irrational number, also known as Euler's number. • The formula for compound interest with continuous compounding is $A = Pe^{rt}$, where *P* represents the principal, *r* represents the interest rate, and *t* represents time in years. • The formula for population growth is $N(t) = N_0e^{rt}$, where N_0 represents the initial population, *r* represents the rate of growth, *t* represents time in years.
3	**Return of the Inverse** Logarithmic Functions	FBF.4 FIF.4 FIF.5 FIF.7.c	2	Logarithmic functions are introduced as the inverse of exponential functions. Students explore the key characteristics of logarithmic functions and state restrictions on the variables for any logarithmic equation. Graphs of logarithmic functions are analyzed and compared. Students solve real-world scenarios using logarithmic functions, including the Richter scale and computing the intensity of earthquakes as well as converting a linear scale on a graph to a logarithmic scale. A graphic organizer summarizes the key characteristics of exponential functions and logarithmic functions.	• The exponential equation $y = b^x$ can be written as the logarithmic equation $x = \log_b y$. • All exponential functions are invertible. The inverse of an exponential function is a logarithmic function. • A common logarithm is a logarithm with base 10, and is usually written as log *x* without a base specified. • A natural logarithm is a logarithm with base e, and is usually written as ln *x*. • Logarithmic functions can be used to model real-world situations, such as the intensity of earthquakes.

*Pacing listed in 45-minute days

CARNEGIE LEARNING

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
4	**I Like to Move It** Transformations of Exponential and Logarithmic Functions	ASE.3.c FBF.3	2	The general transformation function form $g(x) = Af(B(x - C)) + D$ is applied to both exponential and logarithmic functions. Students sketch the graphs of single transformations and multiple transformations of both exponential and logarithmic functions given an equation. They identify any effects that the transformations have on the domain, range, and asymptotes of the functions. Next, students use graphs of functions to write equations for transformed exponential and logarithmic functions. Students describe the graphical transformation(s) performed on an original function that produce a transformed function, given the equations of each. They write a transformed logarithmic function in terms of the original function given the equation of the original function and a key characteristic of the transformed function. Finally, students generalize the effect that a transformation on a function will have on its inverse.	• In the transformation function form $g(x) = Af(B(x - C)) + D$, the D-value translates the function $f(x)$ vertically, the C-value translates $f(x)$ horizontally, the A-value vertically stretches or compresses $f(x)$, and the B-value horizontally stretches or compresses $f(x)$. • Reflections of a basic exponential function do not affect the domain or horizontal asymptote. Reflections of a basic logarithmic function do not affect the range or vertical asymptote. • Vertical translations affect the range and the horizontal asymptote of exponential functions, while horizontal translations affect the domain and the vertical asymptote of logarithmic functions. • Transformations can be described through graphs, tables, key characteristics, writing an equation in terms of the original function, or by using a transformation equation. • A horizontal translation on a function produces a vertical translation on its inverse, while a vertical translation on a function produces a horizontal translation on its inverse. • A vertical dilation on a function produces a horizontal dilation the same number of units on its inverse, and a horizontal dilation on a function produces a vertical dilation the same number of units on its inverse.
Learning Individually with MATHia or Skills Practice		FIF.7.c	2	In the MATHia software, students identify, graph, and determine the equation of the inverse of a function. They graph square root functions and write equations for simple graphed transformations of this function. Students operate with and rewrite numerical and algebraic radical expressions. **MATHia Unit:** Exponential and Logarithmic Functions **MATHia Workspaces:** Properties of Exponential Graphs / Introduction to Logarithmic Functions	

*Pacing listed in 45-minute days

CARNEGIE LEARNING

Topic 3: Exponential and Logarithmic Equations

In Exponential and Logarithmic Equations, students use their understanding of exponential and logarithmic functions to solve exponential and logarithmic equations. Students begin by building understanding and fluency with exponential and logarithmic expressions, including estimating the values of logarithms on a number line and then deriving the properties of logarithms. Students explore alternative methods for solving logarithmic equations and solve exponential and logarithmic equations in context.

Standards: AREI.11, FBF.5, FLQE.2, FLQE.4, SPID.6 **Pacing:** 10 Days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
1	**All the Pieces of the Puzzle** Logarithmic Expressions	FBF.5	2	Students convert between exponential and logarithmic forms of an equation. They then use this relationship to solve for an unknown base, exponent, or argument in a logarithmic equation. Students use a number line to estimate logarithms that are irrational numbers. An always, sometimes, never activity is used to summarize the lesson.	• The value of a logarithmic expression is equal to the value of the exponent in the corresponding exponential expression. • For an exponential equation $a^c = b$ and its corresponding logarithmic equation $\log_a b = c$, the variables have the same restrictions. The base, *a*, must be greater than 0 but not equal to 1, the argument, *b*, must be greater than zero, and the value of the exponent/logarithm, *c*, has no restrictions. • A simple logarithmic equation can be solved by converting it to and exponential equation. To solve for an argument in a logarithmic equation, calculate the resulting expression. To solve for an exponent in a logarithmic equation, use like bases. To solve for a base in a logarithmic equation, use common exponents. • You can estimate the value of a logarithm using the relationship that exists between logarithms and exponents. • For a fixed base greater than 1, as the value of the argument increases, the value of the logarithm increases. For a fixed argument, when the value of the base is greater than 1 and increasing, the value of the logarithm is decreasing.
2	**Mad Props** Properties of Logarithms	FBF.5	1	Students develop rules and properties of logarithms based on their knowledge of various exponent rules and properties. They summarize the different properties by completing a table that defines each exponential and logarithmic property verbally and symbolically, providing examples for each instance.	• Logarithms by definition are exponents, so they have properties that are similar to those of exponents and powers. • The Zero Property of Logarithms states: "The logarithm of 1, with any base, is always equal to 0." • The Logarithm with Same Base and Argument Rule states: "When the base and argument are equal, the logarithm is always equal to 1." • The Product Rule of Logarithms states: "The logarithm of a product is equal to the sum of the logarithms of the factors." • The Quotient Rule of Logarithms states: "The logarithm of a quotient is equal to the difference of the logarithms of the dividend and divisor." • The Power Rule of Logarithms states: "The logarithm of a power is equal to the product of the exponent and the logarithm of the base of the power."

*Pacing listed in 45-minute days

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Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
3	**More Than One Way to Crack an Egg** Solving Exponential Equations	AREI.11 FBF.5 FLQE.4	1	Students analyze a real-world context to solve exponential equations by using the Change of Base Formula. They then derive the Change of Base of Formula. Students explore solving an exponential equation by taking the log of both sides of the equation. They analyze alternative solution methods and common misconceptions. Student practice solving exponential equations using methods of their choosing.	• The Change of Base Formula allows you to calculate an exact value for a logarithm by rewriting it in terms of a different base. By writing the logarithm using base 10 or base *e*, technology can be used to evaluate the expressions. • The Change of Base Formula states: $\log_b c = (\log_a c)/(\log_a b)$, where a, b, $c > 0$ and $a, b \neq 1$. • One method to solve an exponential equation is to write the equation in logarithmic form and then apply the Change of Base Formula. • Another method to solve an exponential equation is to take the common logarithm or natural logarithm of both sides of the exponential equation and then use the rules of logarithms to solve for x.
4	**Logging On** Solving Logarithmic Equations	FBF.5 FLQE.4	2	Students solve logarithmic equations for the base, argument, or exponent by rewriting them as exponential equations or using the Change of Base Formula. Properties of logarithms are used to solve equations containing multiple logarithms, and students solve logarithmic equations in real-world contexts. A decision tree is created describing the first step used to solve each type of exponential and logarithmic equation.	• Logarithmic equations can be used to model real-world contexts. • When solving equations with more than one logarithmic expression, the rules of logarithms must first be applied to rewrite the equation as a single logarithm. • The structure of an exponential equation or logarithmic equation determines the most efficient solution strategy.
5	**What's the Use?** Applications of Exponential and Logarithmic Equations	FBF.5 FLQE.2 FLQE.4 SPID.6	2	Students use exponential and logarithmic equations that model real-world situations to solve problems related to the situations. They write a function to model exponential decay from the description of a situation using their knowledge of transformation function form and use the function to answer a related question. Students use technology to write a regression equation for an exponential model and use it to solve problems.	• Exponential and logarithmic equations are used to model situations in the real-world. • Rounding too early in a series of calculations involving exponential or logarithmic equations has a great effect on the level of accuracy of the solution.
Learning Individually with MATHia or Skills Practice		F.LE.4	2	In the MATHIa software, students solve logarithmic equations with base 2, 10, or e. **MATHia Unit:** Solve Equations with Base 2, 10, or e **MATHia Workspaces:** Solving Base 2 and Base 10 Equations / Solving Base e Equations / Solving Any Base Equations **MATHia Unit:** Finite Geometric Solutions **MATHia Workspaces:** Introduction to Finite Geometric Series / Problem Solving using Finite Geometric Series	

*Pacing listed in 45-minute days

CARNEGIE LEARNING

Topic 4: Applications of Growth Modeling

Students explore various real-world and mathematical situations that are modeled with exponential functions. Lessons provide opportunities for students to apply their understanding of geometric series to solve problems. They also use exponential functions to draw graphics, explore fractals, and study situations modeled by growth.

Standards: AAPR.3, ASE.1, ASE.4, FBF.1.a, FBF.2, FIF.3, FIF.7.a, FIF.7.b, FIF.7.c **Pacing:** 7 Days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
1	**Series Are Sums** Geometric Series	ASE.1 ASE.4 FBF.2	2	The term geometric series is defined, and students explore different methods to compute any geometric series. Worked examples are provided and Euclid's method is introduced. Next, students use the pattern generated from repeated polynomial long division to write a formula for the sum of any geometric series as $S_n = r^n - 1/ r - 1$. They learn that any series where $g1$ does not equal 1 can be rewritten by factoring out a greatest common factor. A second formula to compute any geometric series is derived, $S_n = g_1(r^n - 1)/ r - 1$; the equivalence of the two formulas is verified. Students then rewrite a geometric series using summation notation and compute geometric series. Finally, students write explicit formulas to determine payoff amounts for credit cards.	• A geometric series is the sum of the terms of a geometric sequence. • The formula to compute any geometric series is $S_n = g_n(r) - g_1/ r - 1$, where g_n is the last term, r is the common ratio, and g_1 is the first term. • Another formula to calculate the sum of a geometric series is $S_n = g_n(r) - g_1/ r - 1$ where n is the number of terms, r is the common ratio, and g_1 is the first term. • Geometric series can be use to model real-world situations.
2	**Paint By Numbers** Art and Transformations	AAPR.3 FIF.7.a FIF.7.b FIF.7.c	1	Students use their knowledge of function transformations to create graphics on the coordinate plane. Fifteen different equations/ relations are graphed to create artwork on the coordinate plane. Students then write the equations/relations associated with a different created artwork. Finally, they create their own artwork and list all associated equations/ relations and domain restrictions.	• Basic functions and equations, along with their transformations and restricted domains, can be used to create graphics on the coordinate plane.
3	**This Is the Title of This Lesson** Fractals	FBF.1.a FBF.2 FIF.3	2	The terms self-similar, fractal, and iterative process are defined. The Sierpinski Triangle, the Menger Sponge, and the Koch Snowflake are the self-similar objects that students explore. They construct different stages of the models, use the images to complete tables of values, use the tables of values to identify infinite geometric sequences and patterns, describe end behaviors, write formulas, and make predictions.	• A fractal is a complex geometric shape that is formed by an iterative process. Fractals are infinite and self-similar across different scales. • The Sierpinski Triangle, the Menger Sponge, the Koch Snowflake, and the Sierpinski Carpet are examples of fractals with characteristics that can be described by geometric sequences.
Learning Individually with MATHia or Skills Practice		ASE.4	2	In the MATHia software, students calculate sums of finite series both in mathematical and real-world contexts. **MATHia Unit:** Graphs of Trigonometric Functions **MATHia Workspaces:** Understanding the Unit Circle / Representing Periodic Behavior	

Total Days: 133

Learning Together: 87
Learning Individually: 46

*Pacing listed in 45-minute days