

# 1 Searching for Patterns

Pacing: 29 Days

## Topic 1: Quantities and Relationships

Students analyze scenarios and graphs representing the functions they will study throughout the course. They learn to write equations for functions in function notation. Students recognize that different function families have different key characteristics, and they use graphical behavior to classify functions according to their function families.

**Standards:** AREI.10, FIF.1.c, FIF.4, FIF.5, NQ.1, NQ.2 **Pacing:** 8 Days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
1	<b>A Picture Is Worth a Thousand Words</b> Understanding Quantities and Their Relationships	AREI.10 FIF.1.c FIF.4 NQ.1 NQ.2	1	Students identify the independent and dependent quantities for various real-world scenarios, match a graph to the scenario, and interpret the scale of the axes. They observe similarities and differences in the graphs, and then focus on key characteristics, such as intercepts, increasing and decreasing intervals, and relative maximum and minimum points.	<ul style="list-style-type: none"> <li>There are two quantities that change in problem situations.</li> <li>When one quantity depends on another, it is said to be the dependent quantity. The quantity that the dependent quantity depends upon is called the independent quantity.</li> <li>The independent quantity is used to label the x-axis. The dependent quantity is used to label the y-axis.</li> <li>The domain includes the values that make sense for the independent quantity. The range includes the values that make sense for the dependent quantity.</li> <li>Graphs can be used to model problem situations.</li> </ul>
2	<b>A Sort of Sorts</b> Analyzing and Sorting Graphs	F.IF.4	1	Students sort a variety of graphs based on their own rationale, compare their groupings with their classmates', and discuss the reasoning behind their choices. Next, four different groups of graphs are given and students analyze the groupings and explain possible rationales behind the choices made. Students explore different representations of relations.	<ul style="list-style-type: none"> <li>A relationship between two quantities can be graphed on the coordinate plane.</li> <li>Graphical behaviors can reveal important information about a relationship.</li> <li>A graph of a relationship can have a minimum or maximum or no minimum or maximum. A graph can pass through one or more quadrants. A graph can exhibit vertical or horizontal symmetry. And a graph can be increasing, decreasing, neither increasing nor decreasing, or both increasing and decreasing.</li> </ul>
3	<b>F of X</b> Recognizing Functions and Function Families	FIF.4 FIF.5 FIF.1.c	2	Function notation is introduced. The terms <i>increasing function</i> , <i>decreasing function</i> , and <i>constant function</i> are defined. Students sort the graphs from the previous lesson into groups using these terms and match each graph with its appropriate equation written in function notation. The terms <i>function family</i> , <i>linear function</i> , and <i>exponential function</i> are then defined. Next, the terms <i>absolute minimum</i> and <i>absolute maximum</i> are defined. Students sort the remaining graphs into groups using these terms and match each graph with its appropriate equation written in function notation. The terms <i>quadratic function</i> and <i>linear absolute value function</i> are then defined. Linear piecewise functions are defined, and students match the remaining graphs to their appropriate functions. In the final activity, students demonstrate how the families differ with respect to their intercepts.	<ul style="list-style-type: none"> <li>A function is a relation that assigns to each element of the domain exactly one element of the range.</li> <li>The family of linear functions includes functions of the form <math>f(x) = mx + b</math>, where <math>a</math> and <math>b</math> are real numbers.</li> <li>The family of exponential functions includes functions of the form <math>f(x) = abx + c</math>, where <math>a</math>, <math>b</math>, and <math>c</math> are real numbers, and <math>b</math> is greater than 0 but is not equal to 1.</li> <li>The family of quadratic functions includes functions of the form <math>f(x) = ax^2 + bx + c</math>, where <math>a</math>, <math>b</math>, and <math>c</math> are real numbers, and <math>a</math> is not equal to 0.</li> <li>The family of linear absolute value functions includes functions of the form <math>f(x) = a x + b  + c</math>, where <math>a</math>, <math>b</math>, and <math>c</math> are real numbers, and <math>a</math> is not equal to 0.</li> <li>Linear piecewise functions include functions that have equation changes for different parts, or pieces, of the domain.</li> </ul>

\*Pacing listed in 45-minute days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
4	<p><b>Function Families for 200, Alex</b></p> <p>Recognizing Functions by Characteristics</p>	F.IF.4	2	<p>Given characteristics describing the graphical behavior of specific functions, students name the possible function family/families that fit each description. Students revisit the scenarios and graphs from the first lesson, name the function family associated with each scenario, identify the domain, and describe the graph. Students then write equations and sketch graphs to satisfy a list of characteristics. They conclude by determining that a function or equation, not just a list of characteristics, is required to generate a unique graph.</p>	<ul style="list-style-type: none"> <li>• The graph of an exponential or quadratic function is a curve.</li> <li>• The graph of a linear or linear absolute value function is a line or pair of lines, respectively.</li> <li>• The graph of a linear or exponential function is either increasing or decreasing.</li> <li>• The graph of a quadratic function or a linear absolute value function has intervals where it is increasing and intervals where it is decreasing. Each function also has an absolute maximum or absolute minimum.</li> <li>• Key characteristics of graphs help to determine the function family to which it belongs.</li> </ul>
<p><b>Learning Individually with MATHia or Skills Practice</b></p>		<p>NQ.2 FIF.1 FIF.2 ASE.1</p>	2	<p>In the MATHia software, students answer questions about the relationship between quantities in different scenarios. They answer questions related to different function families, their graphs, equations, and general characteristics. Students determine input and output values for given functions. They identify parts of complex algebraic expressions.</p> <p><b>MATHia Unit:</b> Function Overview <b>MATHia Workspaces:</b> Identifying Quantities / Introduction to Function Families / Evaluating Linear Functions / Identifying Parts of Complex Algebraic Expressions</p>	

\*Pacing listed in 45-minute days

## Topic 2: Sequences

Students explore sequences represented as lists of numbers, as tables of values, as equations, and as graphs modeled on the coordinate plane. Students recognize that all sequences are functions. They also recognize the characteristics of arithmetic and geometric sequences and learn to write recursive and explicit formulas for both. They learn to use a modeling process as a structure for approaching real-world mathematical problems.

**Standards:** ACE.2, FBF.1.a, FBF.2, FBF.1.a, FIF.3, FIF.5, FLQE.2 **Pacing:** 10 Days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
1	<b>Is There a Pattern Here?</b> Recognizing Patterns and Sequences	FBF.1.a FIF.1.a FIF.3 FIF.5 FLQE.2	2	Given ten contexts or geometric patterns, students write a numeric sequence to represent each problem. They represent each sequence as a table of values, state whether each sequence is increasing or decreasing, and describe the sequence using a starting value and operation. They determine that all sequences are functions and have a domain that includes only positive integers. Infinite sequence and finite sequence are defined.	<ul style="list-style-type: none"> <li>A sequence is a pattern involving an ordered arrangement of numbers, geometric figures, letters, or other objects.</li> <li>A term of a sequence is an individual number, figure, or letter in the sequence.</li> <li>A sequence can be written as a function. The domain includes only positive integers.</li> <li>An infinite sequence is a sequence that continues forever, or never ends.</li> <li>A finite sequence is a sequence that terminates, or has an end term.</li> </ul>
2	<b>The Password Is ... Operations!</b> Arithmetic and Geometric Sequences	FBF.2 FIF.1.a FIF.5 FLQE.2	2	Given 16 numeric sequences, students generate additional terms and describe the rule they used for each sequence. They sort the sequences into groups based upon common characteristics and explain their rationale. The terms arithmetic sequence, common difference, geometric sequence, and common ratio are defined with examples. They then categorize the given sequences based on the definitions and identify the common difference or common ratio where appropriate. Students then practice writing sequences with given characteristics.	<ul style="list-style-type: none"> <li>An arithmetic sequence is a sequence of numbers in which the difference between any two consecutive terms is a positive or negative constant. This constant is called the common difference and is represented by the variable <math>d</math>.</li> <li>A geometric sequence is a sequence of numbers in which the ratio between any two consecutive terms is a constant. This constant is called the common ratio and is represented by the variable <math>r</math>.</li> <li>The graph of a sequence is a set of discrete points.</li> <li>The points of an arithmetic sequence lie on a line. When the common difference is a positive, the graph is increasing, and when the common difference is a negative, the graph is decreasing.</li> <li>The points of a geometric sequence do not lie on a line. When the common ratio is greater than 1, the graph is increasing; when the common ratio is between 0 and 1, the graph is decreasing; and when the common ratio is less than 0, the graph alternates between increasing and decreasing between consecutive points.</li> </ul>
3	<b>Did You Mean: Recursion?</b> Determining Recursive and Explicit Expressions from Contexts	ACE.2 FBF.1.a FLQE.2	1	Scenarios are presented that can be represented by arithmetic and geometric sequences. Students determine the value of different terms in each sequence. As the term number increases it becomes more time-consuming to generate the term value, which sets the stage for explicit formulas to be defined and used. Students practice using these formulas to determine the values of terms in both arithmetic and geometric sequences.	<ul style="list-style-type: none"> <li>A recursive formula expresses each new term of a sequence based on a preceding term of the sequence.</li> <li>An explicit formula for a sequence is a formula for calculating each term of the sequence using the term's position in the sequence.</li> <li>The explicit formula for determining the <math>n</math>th term of an arithmetic sequence is <math>a_n = a_1 + d(n - 1)</math>, where <math>n</math> is the term number, <math>a_1</math> is the first term in the sequence, <math>a_n</math> is the <math>n</math>th term in the sequence, and <math>d</math> is the common difference.</li> <li>The explicit formula for determining the <math>n</math>th term of a geometric sequence is <math>g_n = g_1 \cdot r^{n-1}</math>, where <math>n</math> is the term number, <math>g_1</math> is the first term in the sequence, <math>g_n</math> is the <math>n</math>th term in the sequence, and <math>r</math> is the common ratio.</li> </ul>

\*Pacing listed in 45-minute days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
4	<b>3 Pegs, N Discs</b> Modeling Using Sequences	ACE.2 FBF.2 FLQE.2	2	Students are introduced to the process of mathematical modeling, with each of the four activities representing a specific step in the process. Students are invited to play a puzzle game, observe patterns, and think about a mathematical question. Students then organize their information and pursue a given question by representing the patterns they noticed using mathematical notation. As a third step, students analyze their recursive and explicit formulas and use them to make predictions. Finally, students test their predictions and interpret their results.	<ul style="list-style-type: none"> <li>Mathematical modeling involves noticing patterns and formulating mathematical questions, organizing information and representing this information using appropriate mathematical notation, analyzing mathematical representations and using them to make predictions, and then testing these predictions and interpreting the results.</li> <li>Both recursive and explicit formulas can be used for sequences that model situations.</li> <li>Sequence formulas can be used to make predictions for real-world situations.</li> </ul>
<b>Learning Individually with MATHia or Skills Practice</b>		F.IF.3 F.BF.1a	3	<p>In the MATHia software, students determine and classify the patterns in sequences and identify the next terms. They match sequences to their graphs. Students determine the recursive and explicit formulas for given sequences. They write sequences from a context.</p> <p><b>MATHia Unit:</b> Sequences <b>MATHia Workspaces:</b> Describing Patterns in Sequences / Writing Recursive Formulas / Writing Explicit Formulas</p>	

### Topic 3: Linear Regressions

Students formalize their understanding of how to use lines of best fit to model bivariate data. Students use technology to generate linear regressions. To determine the appropriateness of a line to model a given data set, they analyze the shape of the scatterplot, calculate and assess the correlation coefficient, and build and evaluate residual plots. Students differentiate between correlation and causation, recognizing that a correlation between two quantities does not necessarily mean that there is also a causal relationship.

**Standards:** NQ.3, SPID.6, SPID.7, SPID.8, SPID.9 **Pacing:** 11 Days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
1	<b>Like a Glove</b> Least Squares Regressions	NQ.3 SPID.6 SPID.7	2	Students informally approximate a line of best fit for a given data set, write an equation for their line, and then use their function to make predictions, learning about interpolation and extrapolation. They are then introduced to a formal method to determine the linear regression line of a data set via graphing technology.	<ul style="list-style-type: none"> <li>Interpolation is the process of using a regression equation to make predictions within the data set.</li> <li>Extrapolation is the process of using a regression equation to make predictions beyond the data set.</li> <li>A least squares regression line is the line of best fit that minimizes the squares of the distances of the points from the line.</li> </ul>
2	<b>Gotta Keep It Correlatin'</b> Correlation	NQ.3 SPID.6 SPID.8 SPID.9	2	Students analyze graphs and estimate a reasonable correlation coefficient based on visual evidence. They then use technology to determine a linear regression and interpret the correlation coefficient. Next, students analyze several problem situations to determine whether correlation is always connected to causation.	<ul style="list-style-type: none"> <li>A correlation is a measure of how well a regression model fits a data set.</li> <li>The correlation coefficient, <math>r</math>, is a value between -1 and 1 that indicates the type (positive or negative) of association and the strength of the relationship. Values close to 1 or -1 demonstrate a strong association, while a value of 0 signifies no association.</li> <li>Causation is when one event causes a second event. A correlation is a necessary condition for causation, but not a sufficient condition for causation.</li> <li>Two relationships that are often mistaken for causation are a common response, when some other reason may cause the same result, and a confounding variable, when there are other variables that are unknown or unobserved.</li> </ul>
3	<b>The Residual Effect</b> Creating Residual Plots	SPID.6	2	Students calculate a linear regression for a real-world problem and analyze the correlation coefficient to conclude whether the linear model is a good fit. The terms <i>residual</i> and <i>residual plot</i> are defined. Students calculate the residuals, construct a residual scatter plot, and conclude by its shape that there may be a more appropriate model. They are given a second data plot. Students create a scatter plot and determine the equation for the least squares regression line and the correlation coefficient with respect to the problem situation. They then calculate residuals and create a residual plot to conclude how the data are related.	<ul style="list-style-type: none"> <li>A residual is the distance between an observed data value and its predicted value using the regression equation.</li> <li>Analyzing residuals is a method to determine whether a linear model is appropriate for a data set.</li> <li>A residual plot is a scatter plot of the independent variable on the x-axis and the residuals on the y-axis.</li> <li>The shape of a residual plot can be useful to determine whether there may be a more appropriate model than a line of best fit for a data set.</li> </ul>

\*Pacing listed in 45-minute days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
4	<b>To Fit or Not to Fit? That Is the Question!</b> Using Residual Plots	NQ.3 SPID.6	2	Students construct a scatter plot, determine a linear regression equation, compute the correlation coefficient, determine the residuals, and create a residual plot for a data set with one variable. They use all of the given information to decide whether a linear model is appropriate. A quadratic function is given and students conclude that this type of function appears to be a better fit. Finally, students summarize how the shape of a scatter plot, the correlation coefficient, and the residual plot help determine whether a linear model is an appropriate fit for the data set. The lesson emphasizes the importance of using more than one measure to determine if a linear model is a good fit.	<ul style="list-style-type: none"> <li>Scatter plots, regression functions, correlation coefficients, residuals, and residual plots are used to determine the appropriate model of best fit.</li> <li>It is important to use several measures to determine the appropriate model of best fit.</li> </ul>
<b>Learning Individually with MATHia or Skills Practice</b>		SPID.6 SPID.7 SPID.8	3	<p>In the MATHia software, students explore and use linear regression equations to answer questions about data sets. They interpret lines of best fit and estimate correlation coefficients. Students differentiate between correlation and causation. They analyze residuals of lines of best fit.</p> <p><b>MATHia Unit:</b> Linear Regression <b>MATHia Workspaces:</b> Exploring Linear Regression / Using Linear Regression / Interpreting Lines of Best Fit / Analyzing Residuals of Lines of Best Fit</p>	

2

Exploring Constant Change  
Pacing: 43 Days

Topic 1: Linear Functions

Students connect arithmetic sequences to linear functions, proving that the constant difference,  $d$ , is always equal to the slope,  $m$ , of the corresponding linear function. Students examine the structure of equations representing functions and compare the graphs to determine what their differences indicate about the functions and the scenarios they model. Students are introduced to transformation notation,  $y = A \cdot f(B(x - C)) + D$ , although only the A-value (vertical dilations and/or reflections across the  $x$ -axis) and the D-value (vertical translations) are explicitly addressed. The topic concludes with students comparing key characteristics of linear functions presented in different forms.

**Standards:** ACE.1, AREI.10, ASE.1, FBF.3, FIF.1, FIF.2, FIF.4, FIF.6, FIF.7, FIF.9, FLQE.1.a, FLQE.2, FLQE.3, NQ.1 **Pacing:** 10 Days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
1	<p><b>Connecting the Dots</b></p> <p>Making Connections Between Arithmetic Sequences and Linear Functions</p>	<p>FIF.1 FIF.6 FLQE.1.a FLQE.2 FLQE.3</p>	2	<p>The lesson builds from what students know about arithmetic sequences to a general understanding of linear functions. Students connect an arithmetic sequence written in explicit form to a linear function in slope-intercept form. They compare the terms of each equation and prove that the common difference and the slope are always constant and equal. First differences is defined as a strategy to determine whether a table represents a linear relationship. Average rate of change is defined and presented graphically. Finally, students use what they know about arithmetic sequences to complete a graphic organizer to summarize the characteristics and representations of linear functions.</p>	<ul style="list-style-type: none"> <li>The explicit formula of an arithmetic sequence can be rewritten as the slope-intercept form of a linear function using algebraic properties.</li> <li>The explicit formula of an arithmetic sequence, <math>a_n = a_1 + d(n - 1)</math>, includes the first term of the sequence, <math>f(1)</math>, and the common difference. The slope-intercept form of a linear function, <math>f(x) = mx + b</math>, includes <math>f(0)</math> and the slope.</li> <li>Both the average rate of change formula and slope formula calculate the unit rate over a given interval. The average rate of change formula refers to the dependent variable as <math>f(x)</math>, while the slope formula uses <math>y</math>.</li> <li>First differences is a strategy to determine whether a table of values can be modeled by a linear function. First differences are the values determined by subtracting consecutive output values when the input values have an interval of 1.</li> <li>The domain of an arithmetic sequence is consecutive integers beginning with 1, while the domain of a linear function may include all real numbers.</li> </ul>
2	<p><b>Fun with Functions, Linear Ones</b></p> <p>Making Sense of Different Representations of a Linear Function</p>	<p>ACE.1 AREI.10 ASE.1 FBF.3 FIF.2 FIF.4 NQ.1</p>	2	<p>Students determine whether functions represented as scenarios, equations, or graphs are linear functions. They extend what they learned about first differences to analyze tables with input values that are not consecutive integers. Students then analyze a scenario and graph that can be represented by a function in the form <math>f(x) = ax</math>. A new scenario requires an equation in the form <math>f(x) = a(x - c)</math>. They analyze the meaning of this shift in the graph in terms of the context and compare the structure to that of <math>f(x) = ax + b</math>. The scenario changes a second time, and the students explore an equation in the form <math>f(x) = a(x - c) + d</math>.</p>	<ul style="list-style-type: none"> <li>If a table represents a linear function, the slope, or average rate of change, is constant between all given points.</li> <li>Using an equation to solve for the independent value given the dependent value always results in an exact answer. Using a graph or a table to determine the independent value sometimes results in an exact answer.</li> <li>The graph of an equation plotted on the coordinate plane represents the set of all its solutions.</li> <li>The general form of a linear function is <math>f(x) = ax + b</math>, where <math>a</math> and <math>b</math> are real numbers and <math>a \neq 0</math>. In this form, the <math>a</math>-value is the leading coefficient, which describes the steepness and direction of the line. The <math>b</math>-value describes the <math>y</math>-intercept.</li> <li>The factored form of a linear function is <math>f(x) = a(x - c)</math>, where <math>a</math> and <math>c</math> are real numbers and <math>a \neq 0</math>. When a polynomial is in factored form, the value of <math>x</math> that makes each factor equal to zero is the <math>x</math>-intercept. This value is called the zero of the function.</li> <li>A linear function is a polynomial with a degree of one.</li> </ul>

\*Pacing listed in 45-minute days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
3	<b>Get Your Move On</b> Transforming Linear Functions	FBF.3 FIF.4 FIF.7	2	Students identify key characteristics of several linear functions. A graph and a table of values for the basic linear function $f(x) = x$ are given, and students investigate $f(x) + D$ and $A \cdot f(x)$ . Given a function $g(x)$ in terms of $f(x)$ , students graph $g(x)$ and describe each transformation on $f(x)$ to produce $g(x)$ . Finally, students use their knowledge of linear function transformations to test a video game that uses linear functions to hit targets. Students write the function transformations several ways and identify the domains, ranges, slopes, and $y$ -intercepts of the new functions.	<ul style="list-style-type: none"> <li>For the basic function <math>f(x) = x</math>, the transformed function <math>y = f(x) + D</math> affects the output values of the function. For <math>D &gt; 0</math>, the graph vertically shifts up. For <math>D &lt; 0</math>, the graph vertically shifts down. The amount of shift is given by <math> D </math>.</li> <li>For the basic function <math>f(x) = x</math>, the transformed function <math>y = Af(x)</math> affects the output values of the function. For <math> A  &gt; 1</math>, the graph stretches vertically by a factor of <math>A</math> units. For <math>0 &lt;  A  &lt; 1</math>, the graph compresses vertically by a factor of <math>A</math> units. For <math>A &lt; 0</math>, the graph reflects across the <math>x</math>-axis.</li> </ul>
4	<b>Connect Four</b> Comparing Linear Functions in Different Forms	NQ.1 FIF.9	1	Students compare linear functions represented in different forms to answer questions about real-world scenarios. They also identify the scale and origin on the graph of a function given a situation description. Finally, students generate and compare their own linear functions using tables, graphs, and equations.	<ul style="list-style-type: none"> <li>Functions can be represented using tables, equations, graphs, and with verbal descriptions.</li> <li>Features of linear functions such as <math>y</math>-intercepts, slope, independent quantities, and dependent quantities can be determined from different representations of functions.</li> </ul>
<b>Learning Individually with MATHia or Skills Practice</b>		ACE.1 AREI.10 FIF.1 FIF.6 FIF.9 FBF.3 NQ.2	3	<p>In the MATHia software, students translate between arithmetic sequences and linear functions. They model and solve problems involving linear functions in context. Students identify transformations of linear functions. They identify key characteristics from the graph of a function. Students analyze, model, and compare linear functions in different forms.</p> <p><b>MATHia Unit:</b> Linear Function Overview <b>MATHia Workspaces:</b> Writing Sequences as Linear Functions / Understanding Linear Functions</p> <p><b>MATHia Unit:</b> Graphs of Linear Functions <b>MATHia Workspaces:</b> Exploring Graphs of Linear Functions / Identifying Key Characteristics of Graphs of Functions</p> <p><b>MATHia Unit:</b> Modeling with Linear Functions <b>MATHia Workspaces:</b> Multiple Representations of Linear Functions / Modeling Linear Functions Using Multiple Representations / Comparing Linear Functions in Different Forms</p>	

## Topic 2: Solving Linear Equations and Inequalities

Students use the Properties of Equality to justify the steps to solve one-variable equations. They examine and compare the structure of equations that have one, zero, and infinitely many solutions. Students use what they know about solving equations to solve literal equations for variables of interest and to convert between common formulas. When constraints are put on a scenario, students connect what they know about equations to solve linear inequalities. They learn how a negative coefficient on the variable affects the inequality sign and solve more complex inequalities. Finally, students solve compound inequalities and represent their solutions on number lines.

**Standard:** ACE.1, AREI.10, ASE.1, FBF.3, FIF.1, FIF.2, FIF.4, FIF.6, FIF.7, FIF.9, FLQE.1.a, FLQE.2, FLQE.3, NQ.1 **Pacing:** 9 Days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
1	<b>Strike a Balance</b> Solving Linear Equations	ACE.1 AREI.1 AREI.3	1	Students are given a mathematical sentence that is always true and one that is always false. They choose any variable or constant and use the Properties of Equality to investigate ways to change the outcome of the given number sentences. Students reason that the mathematical sentence that is always true is still always true and that one that is false is still false. The terms <i>no solution</i> and <i>infinite solutions</i> are defined. Finally, students play Tic-Tac-Bingo as they work together to create equations with given solution types from assigned expressions.	<ul style="list-style-type: none"> <li>A solution to an equation is any variable value that makes that equation true.</li> <li>Solving equations requires the use of number properties and the Properties of Equality.</li> <li>The Properties of Equality state that if an operation is performed on both sides of the equation, to all terms of the equation, the equation maintains its equality.</li> <li>When the Properties of Equality are applied to an equation, the transformed equation will have the same solution as the original equation.</li> <li>Equations with infinite solutions are created by equating two equivalent expressions.</li> <li>Equations with no solution are created by equating expressions of the form <math>ax + b</math> with the same value for <math>a</math> and different values for <math>b</math>.</li> <li>Equations with a solution <math>x = 0</math> are created by equating expressions of the form <math>ax + b</math> with different values for <math>a</math> and the same value for <math>b</math>.</li> </ul>
2	<b>It's Literally About Literal Equations</b> Literal Equations	ACE.4 NQ.1	1	Students identify the slope and intercepts of functions in general, factored, and standard form. They determine the same characteristics for the equation $Ax + By = C$ . They then explain which form is more efficient in determining the slope and the $x$ - and $y$ -intercepts. Next, the term <i>literal equation</i> is defined. Students rewrite different literal equations to solve for given variables.	<ul style="list-style-type: none"> <li>The general form of a linear equation is <math>y = ax + b</math>, where <math>a</math> and <math>b</math> are real numbers; <math>a</math> represents the slope, and <math>b</math> represents the <math>y</math>-intercept.</li> <li>The factored form of a linear equation is <math>y = a(x - c)</math>, where <math>a</math> and <math>b</math> are real numbers; <math>a</math> represents the slope, and <math>c</math> represents the <math>x</math>-intercept.</li> <li>The standard form of a linear equation is <math>Ax + By = C</math> where <math>A</math> is a positive integer, <math>B</math> and <math>C</math> are integers and both <math>A</math> and <math>B \neq 0</math>. It can be rewritten in general form as <math>y = (-A/B)x + C/B</math>; <math>-A/B</math> represents the slope, <math>C/B</math> represents the <math>y</math>-intercept, and <math>C/A</math> represents the <math>x</math>-intercept.</li> <li>General form of a linear equation is most useful form to identify the slope and <math>y</math>-intercept. Factored form of a linear equation is the most useful form to identify the slope and <math>x</math>-intercept.</li> <li>Literal equations can be rewritten to highlight a specific variable.</li> </ul>

\*Pacing listed in 45-minute days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
3	<b>Not All Statements Are Made Equal</b> Modeling Linear Inequalities	ACE.1 ACE.3 AREI.3 NQ.3	2	Students use the graph of a function modeling a scenario with a positive rate of change to determine solutions to linear inequalities. The term solve an inequality is defined. Students write and solve two-step inequalities algebraically, choosing the most accurate solution in the context of the problem situation. Students solve linear inequalities for a scenario with a negative rate of change that affects the sign of the inequality. Finally, they solve linear inequalities that require more than two steps to solve.	<ul style="list-style-type: none"> <li>A linear inequality context can be modeled with a table of values, a graph on a coordinate plane, a graph on a number line, and with an inequality statement.</li> <li>Solutions to linear inequalities can be determined both graphically and algebraically; they can be expressed using a number line or inequality statement.</li> <li>The steps to solving a linear inequality algebraically are the same steps to solve a linear equation, except that when solving a linear inequality with a negative rate of change, the inequality sign of the solution must be reversed to accurately reflect the relationship.</li> </ul>
4	<b>Don't Confound Your Compounds</b> Solving and Graphing Compound Inequalities	ACE.1 AREI.3 NQ.3	2	The term compound inequality is defined. Students determine the inequality symbols that complete statements about a scenario represented by compound inequalities and express them in compact form. Given a scenario, they express the inequalities using symbols, then solve and graph the inequalities. The terms <i>solution of a compound inequality</i> , <i>conjunction</i> , and <i>disjunction</i> are defined. Students solve and graph compound inequalities, including those written in compact form.	<ul style="list-style-type: none"> <li>A compound inequality is an inequality that is formed by the union, "or," or the intersection, "and" of two simple inequalities.</li> <li>Certain compound inequalities can be written in compact form.</li> <li>The solution of a compound inequality (conjunction) written in the form <math>a &lt; x &lt; b</math>, where <math>a</math> and <math>b</math> are any real numbers, is the part or parts of the solutions that satisfy both of the inequalities.</li> <li>The solution of a compound inequality (disjunction) written in the form <math>x &lt; a</math> or <math>x &gt; b</math>, where <math>a</math> and <math>b</math> are any real numbers, is the part or parts of the solutions that satisfy either of the inequalities.</li> </ul>
<b>Learning Individually with MATHia or Skills Practice</b>		ACE.4 AREI.3	3	In the MATHia software, students solve linear equations in different forms. They then solve literal equations in the form of linear equations. Students graph and solve two-step linear inequalities. They write and solve compound inequalities, using graphs to interpret solutions.  <b>MATHia Unit:</b> Linear Equations <b>MATHia Workspaces:</b> Extending Equations to Literal Equations / Solving Literal Equations  <b>MATHia Unit:</b> Linear Inequalities <b>MATHia Workspaces:</b> Graphing Inequalities / Solving Two-Step Linear Inequalities / Representing Compound Inequalities	

\*Pacing listed in 45-minute days

### Topic 3: Systems of Equations and Inequalities

Students build on their current tools for solving systems of equations. They first use equations in standard form to solve systems of equations using linear combinations. Students analyze the structure of equations to select an efficient method to solve: inspection, graphing, substitution, or linear combinations. Students are introduced to two-variable inequalities. They recognize that just as the solutions to a one-variable inequality are a set of numbers, the solutions to a two-variable inequality are a set of ordered pairs. Students solve systems of linear inequalities graphically. They write systems of equations and inequalities for real-world situations and use function notation to solve linear programming problems.

**Standards:** ACE.2, ACE.3, AREI.5, AREI.6, AREI.10, AREI.11, AREI.12, NQ.3 **Pacing:** 12 Days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
1	<b>Double the Fun</b> Introduction to Systems of Equations	ACE.2 AREI.6a AREI.10 AREI.11	1	Students explore a scenario that can be modeled with a system of linear equations in standard form. They graph the equations using the intercepts. They determine the intersection of the lines graphically and algebraically using substitution. Finally, students write a system of equations for given scenarios and analyze the slopes and $y$ -intercepts and their relevance to the problem situation. They solve each system of equations graphically and algebraically, concluding that for any system there is no solution, one solution, or an infinite number of solutions.	<ul style="list-style-type: none"> <li>The standard form of a linear equation is <math>Ax + By = C</math> where <math>A</math>, <math>B</math>, and <math>C</math> are constants and <math>A</math> and <math>B</math> are not both zero. Linear functions written in standard form can be graphed using the <math>x</math>- and <math>y</math>-intercepts.</li> <li>Understand that the graph of an equation in two variables is the set of all its solutions plotted on the coordinate plane.</li> <li>A linear system of equations is two or more linear equations that define a relationship between quantities. The solution of a linear system is an ordered pair that makes both equations in the system true.</li> <li>Lines that do not intersect describe a system of equations in which each linear equation has the same slope and there is no solution.</li> <li>Lines intersecting at a single point describe a system of equations in which each linear equation has a different slope and there is one solution.</li> <li>Lines intersecting at an infinite number of points describe a system of equations in which each linear equation is the same equation and there are an infinite number of solutions.</li> <li>Consistent systems of equations are systems that have one or many solutions. Inconsistent systems of equations are systems that have no solutions.</li> </ul>
2	<b>The Elimination Round</b> Using Linear Combinations to Solve a System of Linear Equations	ACE.2 AREI.5 AREI.6b	2	Students explore a system of equations with opposite $y$ -coefficients that is solved for $x$ by adding the equations together. The term <i>linear combinations method</i> is defined, and students analyze systems that are solved by multiplying either one or both equations by a constant to rewrite the system with a single variable. Students analyze different systems of equations to determine how they would rewrite the equations to solve for one variable. Next, they apply the linear combinations method to two real-world problems, one with fractional coefficients.	<ul style="list-style-type: none"> <li>The linear combination method is a process to solve a system of linear equations by adding two equations together, resulting in an equation with one variable.</li> <li>When using the linear combination method, it is often necessary to multiply one or both equations by a constant to create two equations in which the coefficients of one of the variables are additive inverses.</li> </ul>
3	<b>Throwing Shade</b> Graphing Inequalities in Two Variables	ACE.2 ACE.3 AREI.12 NQ.3	2	Scenarios are used that are represented by two-variable inequalities. Students write the inequality, complete a table of values, and use the table of values to graph the situation. The terms <i>half-plane</i> and <i>boundary line</i> are defined. Students use shading and solid or dashed lines to indicate which regions on the coordinate plane represent solution sets to the problem situation. Multiple representations such as equations, tables, and graphs are used to represent inequalities and their solutions.	<ul style="list-style-type: none"> <li>The graph of a linear inequality is a half-plane, or half of a coordinate plane.</li> <li>Shading is used to indicate which half-plane describes the solution to the inequality.</li> <li>Dashed and solid lines are used to indicate if the line itself is included in the solution set of an inequality.</li> <li>Linear inequalities and their graphs can be used to represent and solve problems in context.</li> </ul>

\*Pacing listed in 45-minute days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
4	<b>Working with Constraints</b> System of Linear Inequalities	ACE.3 AREI.12 NQ.3	2	The term <i>constraints</i> is defined. Students write a system of linear inequalities to model a scenario, and graph the system, determining that overlapping shaded regions identify the possible solutions to the system. They practice graphing several systems of inequalities and determining the solution set. Finally, students match systems, graphs, and possible solutions of systems.	<ul style="list-style-type: none"> <li>In a system of linear inequalities, the inequalities are known as constraints because the values of the expressions are constrained to lie within a certain region.</li> <li>The solution of a system of linear inequalities is the intersection of the solutions to each inequality. Every point in the intersecting region satisfies all inequalities in the system.</li> </ul>
5	<b>Working the System</b> Solving Systems of Equations and Inequalities	ACE.3 AREI.6 AREI.12	1	Students write a system of linear equations for each of three different scenarios: one in the form $y = ax + b$ , one in the form $y = a(x - c) + b$ , and one in the form $y = a(cx) + b$ . They use any method to solve the system before reasoning about the solution in terms of the problem context. Students write a system composed of four linear inequalities to model a scenario and graph the system. Students determine the correct region that contains the solution set that satisfies all of the inequalities in the system.	<ul style="list-style-type: none"> <li>Contexts about choosing between two options can sometimes be modeled by a system of linear equations or inequalities.</li> <li>The point of intersection of two lines separates the input values, with <math>x</math>-values less than and <math>x</math>-values greater than the <math>x</math>-value of the point of intersection. The solution to a problem in context may be dependent upon where the input values lie relative to the point of intersection.</li> <li>Based upon a context, the solution of a system may be represented by inequalities rather than a single coordinate pair.</li> </ul>
<b>Learning Individually with MATHia or Skills Practice</b>		AREI.6 AREI.11 AREI.12	4	<p>In the MATHia software, students solve systems of linear equations using graphs, substitution, and linear combinations in mathematical and real-world contexts. They graph and solve linear inequalities in two variables. Students determine a region of a graph that represents the solution to a system of linear inequalities and interpret the solutions to these systems.</p> <p><b>MATHia Unit:</b> Systems of Linear Equations <b>MATHia Workspaces:</b> Representing Systems of Linear Functions / Solving Linear Systems Using Linear Combinations / Solving Linear Systems Using Any Method</p> <p><b>MATHia Unit:</b> Linear Inequalities in Two Variables <b>MATHia Workspaces:</b> Exploring Linear Inequalities / Graphing Linear Inequalities in Two Variables / Systems of Linear Inequalities</p>	

\*Pacing listed in 45-minute days

### Topic 4: Functions Derived from Linear Relationships

Students connect the absolute value of a number to the absolute value of a function. They examine the graph of absolute value functions, noting the reflection of the negative  $y$ -values across the  $x$ -axis. The visual representation allows students to see that each output corresponds to two input values. From this intuition, students solve absolute value equations and inequalities. Students extend their understanding of transforming linear functions to transform absolute value functions, now considering the effect of adding a constant to the argument. Linear piecewise functions are introduced, and students analyze the restricted domains of each piece. They then write absolute value functions as piecewise functions to connect the two concepts. Step functions are introduced as a special case of piecewise functions. Finally, students reflect functions across  $y = x$  to generate the inverse of a linear function.

**Standards:** ACE.2, ACE.3, AREI.3, AREI.11, FBF.3, FBF.4.b, FIF.1, FIF.1b, FIF.2, FIF.4, FIF.7 **Pacing:** 12 Days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
1	<b>Putting the V in Absolute Value</b> Defining Absolute Value Functions and Transformations	FBF.3 FIF.7	2	Students model absolute value functions and their transformations on a human coordinate plane. They explore and analyze different transformations of absolute value functions, their graphs, and equations, and summarize the effects of these transformations. By transforming absolute value functions, students distinguish between the effects of changing values inside the argument of the function vs. changing values outside the function.	<ul style="list-style-type: none"> <li>An absolute value function is a function of the form <math>f(x) =  x </math>.</li> <li>A function <math>g(x)</math> of the form <math>g(x) = f(x) + D</math> is a vertical translation of the function <math>f(x)</math>. The value <math> D </math> describes the number of units the graph of <math>f(x)</math> is translated up or down. If <math>D &gt; 0</math>, the graph is translated up; if <math>D &lt; 0</math>, the graph is translated down.</li> <li>A function <math>g(x)</math> of the form <math>g(x) = Af(x)</math> is a vertical dilation of the function <math>f(x)</math>. For <math> A  &gt; 1</math>, the graph is vertically stretched by a factor of <math>A</math> units; for <math>0 &lt;  A  &lt; 1</math>, the graph vertically compresses by a factor of <math>A</math> units. For <math>A &lt; 0</math>, the graph also reflects across the <math>x</math>-axis.</li> <li>A function <math>g(x)</math> of the form <math>g(x) = f(x - C)</math> is a horizontal translation of the function <math>f(x)</math>. The value <math> C </math> describes the number of units the graph of <math>f(x)</math> is translated right or left. If <math>C &gt; 0</math>, the graph is translated to the right; if <math>C &lt; 0</math>, the graph is translated to the left.</li> <li>Transforming a function by changing the <math>A</math>- or <math>D</math>-values affects the output of the function, <math>y</math>. Transforming a function by changing the <math>C</math>-value affects the input of the function, <math>x</math>.</li> </ul>
2	<b>Play Ball</b> Absolute Value Equations and Inequalities	ACE.2 ACE.3 AREI.3 AREI.11 FIF.7	2	Students create a linear absolute value function to model a scenario and use the graph of the function to estimate solutions in the context of the problem situation. They then solve a linear absolute value equation algebraically by first rewriting it as two separate linear equations. One equation represents the case where the value of the expression inside the absolute value is positive, and the second represents the case where it is negative. Students model a scenario with a linear absolute value inequality and its corresponding graph. Finally, they solve linear absolute value inequalities algebraically by first rewriting them as equivalent compound inequalities.	<ul style="list-style-type: none"> <li>Linear absolute value equations have 0, 1, or 2 solutions. The solution set for linear absolute value inequalities may contain all real numbers, a subset of the real numbers represented by a compound inequality, or no solutions.</li> <li>Linear absolute value inequalities can be rewritten as equivalent compound inequalities.</li> <li>Linear absolute value equations and inequalities can be used to represent real-world situations.</li> </ul>

\*Pacing listed in 45-minute days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
3	<b>I Graph in Pieces</b> Linear Piecewise Functions	FBF.3 FIF.1b FIF.2 FIF.4 FIF.7	2	Students develop a piecewise function from a scenario. The terms <i>piecewise function</i> and <i>linear piecewise function</i> are defined. They analyze a piecewise graph and write a scenario and piecewise function to represent the graph. Students analyze statements that correspond to different pieces of the graphed function. They then write a scenario that can be modeled with a piecewise function, graph a partner's scenario, and work with their partner to write a piecewise function for each.	<ul style="list-style-type: none"> <li>A piecewise function is a function that can be represented by more than one function, where each function corresponds to a specific part of the domain.</li> <li>To write a piecewise function, you must write the equation and domain for each piece of the function.</li> <li>Piecewise functions can be represented by scenarios, equations, tables, and graphs.</li> <li>A linear absolute value function can be constructed using a piecewise function.</li> <li>Graphing technology can be used to model piecewise functions.</li> </ul>
4	<b>Step by Step</b> Step Functions	FIF.1b FIF.2 FIF.7	2	Students analyze the graph of a special piecewise function. The terms <i>discontinuous graph</i> and <i>step function</i> are defined, and students interpret those definitions through examination of graphs. Students are then given a context and must provide both the piecewise function and the graph that models it, explaining why the function is a step function. Next, they are introduced to the greatest integer function (floor function) and least integer function (ceiling function) through their definitions, notation, meaning of individual values, graphs and real-world examples.	<ul style="list-style-type: none"> <li>A discontinuous graph is a graph that is continuous for some values of the domain with at least one disjoint area between consecutive <math>x</math>-values.</li> <li>A step function is a piecewise function on a given interval whose pieces are discontinuous constant functions.</li> <li>The greatest integer function, also known as the floor function, is a special type of step function. The greatest integer function <math>G(x) = \lfloor x \rfloor</math> is defined as the greatest integer less than or equal to <math>x</math>.</li> <li>The least integer function, also known as the ceiling function, is a special type of step function. The least integer function <math>L(x) = \lceil x \rceil</math> is defined as the least integer greater than or equal to <math>x</math>.</li> </ul>
5	<b>A Riddle Wrapped in a Mystery</b> Inverses of Linear Functions	FIF.1 FIF.7 FBF.4.b	1	Students use a table of values to determine the inverse of a given problem situation. The term <i>inverse of a function</i> is defined and students are shown how to algebraically determine the inverse of a function. They then create the graph of the inverse of a linear function by reflecting the original function across the line $y = x$ using patty paper. The term <i>one-to-one function</i> is defined, and students determine whether given functions are one-to-one functions.	<ul style="list-style-type: none"> <li>Inverses of functions can be determined algebraically and graphically.</li> <li>The inverse of a function is determined by replacing <math>f(x)</math> with <math>y</math>, switching the <math>x</math> and <math>y</math> variables, and solving for <math>y</math>.</li> <li>The graph of the inverse of a function is a reflection of that function across the line <math>y = x</math>.</li> <li>A one-to-one function is a function in which its inverse is also a function.</li> <li>For a one-to-one function <math>f(x)</math>, the notation for its inverse is <math>f^{-1}(x)</math>.</li> </ul>
<b>Learning Individually with MATHia or Skills Practice</b>		ACE.3 FIF.7.b	3	<p>In the MATHia software, students build absolute value functions using transformations. They use graphs of absolute value functions to identify transformations and determine solutions. Students graph and solve absolute value equations and inequalities on a number line. They graph and interpret graphs of linear piecewise functions and step functions to solve problems in mathematical and real-world contexts.</p> <p><b>MATHia Unit:</b> Absolute Value Equations  <b>MATHia Workspaces:</b> Building Absolute Value Functions / Graphing Simple Absolute Value Equations Using Number Lines / Solving Absolute Value Equations / Reasoning About Absolute Value Inequalities</p> <p><b>MATHia Unit:</b> Graphs of Piecewise Functions  <b>MATHia Workspaces:</b> Introduction to Piecewise Functions / Graphing Linear Piecewise Functions / Interpreting Piecewise Functions / Using Linear Piecewise Functions / Analyzing Step Functions</p>	

\*Pacing listed in 45-minute days

# 3

## Investigating Growth and Decay

Pacing: 20 Days

### Topic 1: Introduction to Exponential Functions

Students recall geometric sequences and explore their graphs. They learn that some geometric sequences belong to the exponential function family. They learn to rewrite geometric sequences as exponential functions in the form  $f(x) = a \cdot b^x$ . They examine the structure of exponential expressions, concluding that  $2^{(1/2)} = \sqrt{2}$  and  $2^{(1/3)} = \sqrt[3]{2}$ . Finally, students expand their experiences with function transformations, now including horizontal transformations and dilations in their repertoire. They generalize the effect of these translations on points, sketch graphs of transformed functions, and write equations based on a given sequence of transformations.

**Standards:** ACE.1, AREI.3, AREI.10, FBF.3, FIF.1, FIF.7, FIF.8b, FLQE.1.a, FLQE.2, FLQE.5, NRNS.1, NRNS.2, NRNS.3 **Pacing:** 12 Days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
1	<b>A Constant Ratio</b> Geometric Sequences and Exponential Functions	AREI.10 FIF.1 FLQE.1.a FLQE.2 FLQE.5	2	Students learn through investigation that while all geometric sequences are functions, only some geometric sequences can be represented as exponential functions. They identify the constant ratio in different representations of exponential functions and then show algebraically that the constant ratio between output values of an exponential function is represented by the variable $b$ in the function form $f(x) = ab^x$ . Students also identify the $a$ -value of that form as the $y$ -intercept of the graph of the function. They learn to write an exponential equation from two given points. The lesson concludes with a comparison of the base of the power in the equation $f(x) = ab^x$ , the expression $(f(x+1))/f(x)$ , and the common ratio of the corresponding geometric sequence.	<ul style="list-style-type: none"> <li>All geometric sequences are functions; however, only geometric sequences with a positive constant ratio are exponential functions.</li> <li>If a geometric sequence represents an exponential function, then the Product of Powers Rule and the definition of negative exponents can be used to rewrite the explicit formula for the sequence as an exponential function.</li> <li>The form of an exponential function is <math>f(x) = a \cdot b^x</math>, where <math>b</math> represents the constant ratio and <math>(0, a)</math> represents the <math>y</math>-intercept.</li> <li>For an exponential function in the form <math>f(x) = a \cdot b^x</math>, the ratio <math>(f(x+1))/f(x)</math> is constant and equal to <math>b</math>.</li> </ul>
2	<b>The Power Within</b> Rational Exponents and Graphs of Exponential Functions	ACE.1 AREI.3 FLQE.2 NRNS.1 NRNS.2 NRNS.3	2	Students explore a scenario that can be represented by the function $f(x) = 2^x$ . They use the rules of exponents and their understanding of a constant ratio to determine output values for the exponential function when the input values are non-integers. Students use this exploration to connect expressions with rational exponents to those in radical notation. Students learn the term <i>horizontal asymptote</i> and explore this concept on different graphs, analyzing end behavior, particularly as the $x$ -values approach negative infinity. Finally, students practice converting between expressions with rational exponents and those in radical notation and make generalizations about the constant ratio for exponential functions.	<ul style="list-style-type: none"> <li>If the difference in the input values is the same, an exponential function shows a constant ratio between output values, no matter how large or how small the gap between input values.</li> <li>Multiple representations such as tables, equations, and graphs are used to represent exponential problem situations.</li> <li>Properties of powers can be used to rewrite numeric and algebraic expressions involving integer and rational exponents.</li> <li>Because rational exponents can be rewritten as radicals, the properties of powers apply to radical expressions, as well.</li> <li>A rational expression of the form <math>a^{m/n}</math> can be written as a radical expression <math>\sqrt[n]{a^m}</math> or <math>(\sqrt[n]{a})^m</math>.</li> <li>Common bases and properties of exponents are used to solve simple exponential equations.</li> </ul>

\*Pacing listed in 45-minute days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
3	<b>Now I Know My A, B, C, Ds</b> Transformations of Exponential Functions	FBF.3 FIF.7 FIF.8b NRNS.1	2	Students explore a variety of different transformations of exponential functions, including vertical translations, horizontal translations, vertical reflections and dilations, and horizontal reflections and dilations. For each transformation, students sketch graphs of the transformation, compare characteristics of the transformed graphs with the graph of the parent function, including the horizontal asymptote when appropriate, and write the transformations using coordinate notation. They also consider different ways to rewrite and interpret equations of function transformations. Finally, students summarize the effects of the different transformations at the end of the lesson.	<ul style="list-style-type: none"> <li>A reflection of a graph is a mirror image of the graph about a line. The line that the graph is reflected across is called the line of reflection.</li> <li>Reflections across the x-axis can be expressed using the notation <math>(x, y) \rightarrow (x, -y)</math>. It affects the y-coordinate of each point on the graph.</li> <li>Reflections across the y-axis can be expressed using the notation <math>(x, y) \rightarrow (-x, y)</math>. It affects the x-coordinate of each point on the graph.</li> </ul>
<b>Learning Individually with MATHia or Skills Practice</b>		ACE.1 FBF.3 FIF.3 FIF.4 FIF.5 FIF.8.b NRNS.1 NRNS.2	6	<p>In the MATHia software, students identify key characteristics of exponential functions from a sequence, function, table, graph, or scenario. They rewrite expressions with radical notation using rational exponents and vice versa. Students use common bases to solve real-world problems modeled by exponential equations. They transform exponential functions.</p> <p><b>MATHia Unit:</b> Exponential Functions <b>MATHia Workspaces:</b> Writing Sequences as Exponential Functions / Introduction to Exponential Functions / Relating the Domain to Exponential Functions</p> <p><b>MATHia Unit:</b> Rational Exponents <b>MATHia Workspaces:</b> Using the Properties of Exponents / Properties of Rational Exponents / Rewriting Expressions with Radical and Rational Exponents / Solving Contextual Exponential Equations Using Common Bases</p> <p><b>MATHia Unit:</b> Linear and Exponential Transformations <b>MATHia Workspaces:</b> Introduction to Transforming Exponential Functions / Shifting Vertically / Shifting Horizontally / Reflecting and Dilating Using Graphs / Transforming Using Tables of Values / Using Multiple Transformations</p>	

## Topic 2: Using Exponential Equations

In this topic, students use exponential equations to solve problems. They solve equations of the form  $a \cdot b^x = d$  by graphing  $y = a \cdot b^x$  and  $y = d$  and locating the point of intersection. They examine the structure of two equivalent exponential equations, revealing how rewriting the B-value of an exponential function affects the b-value of  $f(x) = a \cdot b^x$ . Students then solve real-world problems that can be modeled by exponential functions. They use technology to calculate regression equations and use them to make predictions. Students are reminded of the modeling process for problem solving.

**Standards:** ACE.1, ACE.2, AREI.10, AREI.11, ASE.1, FBF.1b, FIF.6, FIF.8b, FLQE.1.b, FLQE.3.FLQE.5, NQ.2, NQ.3, SPID.6 **Pacing:** 8 Days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
1	<b>Uptown and Downtown</b> Exponential Equations for Growth and Decay	ACE.1 ACE.2 ASE.1 FIF.6 FLQE.1.b FLQE.3 FLQE.5 NQ.3	2	Students compare linear and exponential functions in the context of simple interest and compound interest situations. They identify the values in the exponential function equation that indicate whether an exponential function is a growth or decay function, and they apply this reasoning in context.	<ul style="list-style-type: none"> <li>Simple interest can be represented by a linear function. Compound interest can be represented by an exponential function.</li> <li>An exponential growth function is of the form <math>y = a \cdot (1 + r)^x</math>, where <math>r</math> is the rate of growth.</li> <li>An exponential decay function is of the form <math>y = a \cdot (1 - r)^x</math>, where <math>r</math> is the rate of decay.</li> </ul>
2	<b>Powers and the Horizontal Line</b> Interpreting Parameters in Context	ACE.1 ACE.2 AREI.10 AREI.11 FIF.8b FLQE.5	1	Students match exponential equations to their graphs to discern that the horizontal asymptote is always represented by $y = D$ . For exponential growth and decay scenarios, students complete tables of values, graph the functions, and write exponential equations using function notation. Students use graphs to estimate the solutions to equations by graphing both sides of the equation and locating the point of intersection. They use the properties of exponents to rewrite the band $B$ -values of exponential functions in equivalent forms to reveal properties of the quantity represented in the function. This allows them to re-interpret an equation showing annual rates of increase for a mutual fund to show monthly and quarterly rates of increase for the same fund.	<ul style="list-style-type: none"> <li>Given an exponential function, <math>f(x) = A \cdot b^{B(x-C)} + D</math>, the horizontal asymptote is always represented by the equation <math>y = D</math>, and the <math>y</math>-intercept is the <math>A</math>-value plus any vertical translation.</li> <li>Multiple representations such as tables, equations, and graphs can be used to represent and compare exponential problem situations.</li> <li>Graphs can be used to solve exponential equations by graphing both sides of the equation and estimating the point of intersection.</li> <li>A quantity increasing exponentially eventually exceeds a quantity increasing linearly.</li> <li>Properties of exponential functions can be compared using different representations.</li> <li>Transforming exponential functions into equivalent forms can reveal different properties of the quantities represented.</li> </ul>
3	<b>Savings, Tea, and Carbon Dioxide</b> Modeling Using Exponential Functions	FBF.1b NQ.2 SPID.6	2	Students model two savings scenarios, one by means of an exponential function $f(x)$ and one by a constant function $g(x)$ . They then create a third function, $h(x) = f(x) + g(x)$ , graph all three functions on the same graph, and explain how they are related. Given a data set, students create a scatter plot, write a regression equation, use the function to make predictions, and interpret the reasonableness of a prediction. The lesson concludes with students generalizing about the common features of scenarios that are modeled by exponential functions. They also describe the shape of a scatter plot representing an exponential function and sketch possible graphs of exponential functions.	<ul style="list-style-type: none"> <li>An exponential function and a constant function can be added to create a third function that is the sum of the two functions, resulting in a graph that is a vertical translation of the original exponential function.</li> <li>Technology can be used to determine exponential regression equations to model real-world situations. The regression equation can then be used to make predictions.</li> <li>Sometimes referring to the scenario or obtaining further information may be required to determine whether a scatter plot is best modeled by a linear or exponential function.</li> </ul>

\*Pacing listed in 45-minute days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
4	<b>BAC Is BAD News</b> Choosing a Function to Model Data	FBF.1b NQ.2 SPID.6	1	Students are given a context involving the blood alcohol content (BAC) of a driver and the driver's likelihood of causing an accident. Students are then given data from a study connecting BAC and the relative probability of causing an accident. They apply the relationship from the data to create a model predicting the likelihood of a person causing an accident based on their BAC. They summarize their learning by writing an article for a newsletter about the seriousness of drinking and driving. The lesson concludes with students connecting their process for solving the problem to the steps in the mathematical modeling process.	<ul style="list-style-type: none"> <li>Determining and using a regression equation is sometimes a step in the process of solving a more complex mathematical problem, rather than the final solution.</li> <li>The mathematical modeling process is an effective structure to solve complex mathematical problems.</li> </ul>
<b>Learning Individually with MATHia or Skills Practice</b>		FBF.1b NQ.2 SPID.6	2	<p>In the MATHia software, students compare linear and exponential functions. They use exponential functions in different forms to recognize exponential growth or decay, calculate and interpret average rate of change, and make comparisons. Students model scenarios with exponential equations. They solve exponential equations in context using a graph.</p> <p><b>MATHia Unit:</b> Compare Linear and Exponential Models  <b>MATHia Workspaces:</b> Recognizing Linear and Exponential Models / Recognizing Growth and Decay / Calculating and Interpreting Average Rate of Change / Modeling Equations with a Starting Point of 1 / Modeling Equations with a Starting Point Other Than 1 / Comparing Exponential Functions in Different Forms</p> <p><b>MATHia Unit:</b> Solving Exponential Equations  <b>MATHia Workspaces:</b> Solving Exponential Equations Using a Graph</p>	

\*Pacing listed in 45-minute days

# 4

## Maximizing and Minimizing

Pacing: 39 Days

### Topic 1: Introduction to Quadratic Functions

Students explore the structure of quadratic functions through four real-world situations. They learn key characteristics of quadratic functions through explorations with their graphs and equations: intercepts, absolute maximum/minimum, vertex, axis of symmetry, domain, range, and intervals of increasing and decreasing. Students transform functions using function transformation form  $y = A \cdot f(B(x - C)) + D$  and learn how coordinates are affected; any point  $(x, y)$  on the graph  $y = f(x)$  maps to the point  $((1/B)x - C, Ay + D)$  on the graph  $y = A \cdot f(B(x - C)) + D$ . Given quadratic functions represented in different forms (table, graph, equation, or scenario), students compare key characteristics.

**Standards:** AAPR.3, AREI.10, AREI.11, ASE.1, ASE.3a, FBF.3, FIF.4, FIF.5, FIF.6, FIF.7, FIF.8a, FIF.9, FLQE.3 **Pacing:** 16 Days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
1	<b>Up and Down or Down and Up</b> Exploring Quadratic Functions	AREI.10 AREI.11 FIF.4 FIF.5 FIF.7	2	Students are introduced to quadratic functions and their growth pattern through a sequence of pennies. They are then provided four different contexts that can be modeled by quadratic functions. For each function, students address the key characteristics of the graphs and interpret them in terms of the context. They also compare the domain and range of the functions and the context they represent. The first context involves area and is used to compare and contrast linear and quadratic relationships. The second context involves handshakes and has the student write the function. The third context involves catapulting a pumpkin. Students analyze this function written in general form. The final context involves revenue and demonstrates how a quadratic function can be written as the product of two linear functions.	<ul style="list-style-type: none"> <li>Quadratic functions can be used to model certain real-world situations.</li> <li>The graph of a quadratic function is called a parabola.</li> <li>A parabola is a smooth curve with reflectional symmetry.</li> <li>A parabola has an absolute maximum or absolute minimum point and an interval where it is increasing and an interval where it is decreasing.</li> <li>A parabola has one y-intercept and at most two x-intercepts.</li> <li>The domain of a quadratic function is the set of all real numbers. The range is a subset of the real numbers that is limited based upon the y-coordinate of the absolute maximum or absolute minimum point.</li> </ul>
2	<b>Endless Forms Most Beautiful</b> Key Characteristics of Quadratic Functions	ASE.1 ASE.3a FIF.4 FIF.6 FIF.7 FIF.8a	3	Students revisit the four scenarios from the previous lesson as a way to introduce equivalent quadratic equations with different structures to reveal different characteristics of their graphs. They learn that a table of values represents a quadratic function if its second differences are constant. Students analyze the effect of the leading coefficient on whether the parabola opens up or down. They identify the axis of symmetry and vertex for graphs using the equations in each form. Finally, students determine the x- and y-intercepts along with intervals of increase and decrease, using a combination of technology, symmetry, and equations.	<ul style="list-style-type: none"> <li>A table of values representing a quadratic function has constant second differences.</li> <li>A quadratic function may be written in general form, <math>f(x) = ax^2 + bx + c</math>, where <math>a \neq 0</math>, and in factored form, <math>f(x) = a(x - r_1)(x - r_2)</math>, where <math>a \neq 0</math>.</li> <li>When the a-value of a quadratic function in general form or factored form is positive, the graph opens upward and has an absolute minimum; when the leading coefficient is negative, the graph opens downward and has an absolute maximum.</li> <li>The vertex (or maximum/minimum) of a parabola lies on its axis of symmetry. The axis of symmetry can be determined by the formula <math>x = (r_1 + r_2) / 2</math> from the factored form or by <math>x = -b / 2a</math> from the general form of the quadratic equation.</li> <li>In factored form, <math>f(x) = a(x - r_1)(x - r_2)</math>, the values of <math>(r_1, 0)</math> and <math>(r_2, 0)</math> are the x-intercepts of the quadratic function. In general form, <math>f(x) = ax^2 + bx + c</math>, <math>(0, c)</math> is the y-intercept of the quadratic function.</li> </ul>

\*Pacing listed in 45-minute days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
3	<b>More Than Meets the Eye</b> Transformations of Quadratic Functions	ASE.3a FBF.3	2	Students explore a variety of different transformations of quadratic functions, including vertical translations, horizontal translations, vertical dilations and reflections, and horizontal dilations and reflections. For each transformation, students sketch graphs of the transformation, compare characteristics of the transformed graphs with those of the graph of the basic function, and write the transformations using coordinate notation. Students write quadratic equations in vertex form using the coordinates of the vertex and another point on the graph and in factored form using the zeros and another point on the graph.	<ul style="list-style-type: none"> <li>The general transformation equation is <math>y = Af(B(x - C)) + D</math>, where the <math>A</math>-value describes a vertical dilation or reflection, the <math>B</math>-value describes a horizontal dilation or reflection, the <math>C</math>-value describes a horizontal translation, and the <math>D</math>-value describes a vertical translation.</li> <li>Given <math>f(x) = x^2</math> as the basic quadratic function, reference points can be used to graph <math>y = Af(B(x - C)) + D</math> such that any point <math>(x, y)</math> on <math>f(x)</math> maps to the point <math>((1/B)x + C, Ay + D)</math>.</li> <li>The vertex form for a quadratic function is <math>f(x) = a(x - h)^2 + k</math>, where <math>(h, k)</math> is the location of the vertex, and the sign of <math>a</math> indicates whether the parabola opens upward or downward.</li> <li>A function written in equivalent forms can reveal different characteristics of the function it defines.</li> <li>You can write a quadratic function in vertex form if you know the coordinates of the vertex and another point on the graph.</li> <li>You can write a quadratic function in factored form if you know the zeros and another point on the graph.</li> </ul>
4	<b>You Lose Some, You Lose Some</b> Comparing Functions Using Key Characteristics and Average Rate of Change	ASE.1 FIF.6 FIF.9 FLQE.3	1	Students compare quadratic functions in standard form, factored form, and vertex form, then analyze the properties of each form. Students then answer questions to compare linear, quadratic, and exponential functions. They compute average rates of change for the functions across different intervals and then compare the change in the average rates of change across the different intervals. Quadratic equations in different forms are compared by identifying key characteristics of their representations.	<ul style="list-style-type: none"> <li>The average rate of change of any function over an interval is the slope of a linear function passing through the beginning and end points of the interval.</li> <li>A quantity increasing exponentially eventually exceeds a quantity increasing linearly or quadratically.</li> <li>The average rate of change of an increasing exponential function will eventually exceed the average rate of change for an increasing linear and quadratic function.</li> <li>A function written in equivalent forms can reveal different characteristics of the function it defines.</li> <li>Quadratic equations in different forms are compared by identifying key characteristics of their representations or by changing one representation to match the other for comparison purposes.</li> </ul>
<b>Learning Individually with MATHia or Skills Practice</b>		ACE.1 FBF.3 FIF.4 FIF.7 FIF.8.a FIF.9	8	<p>In the MATHia software, students construct quadratic functions to model scenarios and answer questions. They identify key characteristics of quadratic functions from graphs and tables. Students determine transformations performed on quadratic functions. They identify properties of, sketch, and compare quadratic functions in general, factored, or vertex form.</p> <p><b>MATHia Unit:</b> Modeling Quadratic Functions <b>MATHia Workspaces:</b> Modeling Area as Product of Monomial and Binomial</p> <p><b>MATHia Unit:</b> Linear and Quadratic Transformations <b>MATHia Workspaces:</b> Shifting Vertically / Shifting Horizontally / Reflecting and Dilating Using Graphs / Transforming Using Tables of Values / Using Multiple Transformations</p> <p><b>MATHia Unit:</b> Properties of Quadratic Functions <b>MATHia Workspaces:</b> Identifying Properties of Quadratic Functions / Sketching Quadratic Functions / Comparing Quadratic Functions in Difference Forms</p>	

\*Pacing listed in 45-minute days

## Topic 2: Solving Quadratic Equations

Students begin by solving simple quadratic equations in the form  $x^2 = n$  using horizontal lines and the Properties of Equality, skills they learned in middle school. They recognize that the solutions to a quadratic equation are equidistant to the axis of symmetry on a graph of the function. They use these tools to solve increasingly complex quadratic equations, each time making a connection to the location of the solutions on a graphical representation. When they encounter an equation in general form, students learn to use the Zero Product Property to solve via factoring. For trinomials that cannot be factored, students learn to complete the square and to use the Quadratic Formula. In deriving the Quadratic Formula, students recognize that its structure supports the fact that all solutions are equidistant from the axis of symmetry for any quadratic equation. Students are provided multiple opportunities to solve quadratic equations using efficient methods.

**Standard:** AAPR.1, AREI.4, AREI.10, ASE.1, ASE.2, ASE.3.a, ASE.3.b, FIF.8, NRNS.1, NRNS.3, NCNS.1, NCNS.2, NCNS.7, NCNS.9 **Pacing:** 23 Days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
1	<b>This Time, with Polynomials</b> Adding, Subtracting, and Multiplying Polynomials	AAPR.1 ASE.1	2	Students are introduced to polynomials and identify the terms and coefficients of polynomials. Students sort polynomials by the number of terms, rewrite in general form if possible, and identify the degree. Students add and subtract polynomial functions algebraically and graphically and then determine that polynomials are closed under addition and subtraction. Students use area models and the Distributive Property to determine the product of binomials. They explore special products and are introduced to the terms difference of two squares and perfect square trinomial.	<ul style="list-style-type: none"> <li>A polynomial is a mathematical expression involving the sum of powers in one or more variables multiplied by coefficients. A polynomial in one variable is the sum of terms of the form <math>ax^k</math>, where <math>a</math> is any real number and <math>k</math> is a non-negative integer. In general, a polynomial is of the form <math>a_1x^k + a_2x^{k-1} + \dots + a_nx^0</math>.</li> <li>Polynomials with only one term are monomials. Polynomials with exactly two terms are binomials. Polynomials with exactly three terms are trinomials.</li> <li>Polynomials can be added, subtracted, and multiplied using algebraic operations.</li> <li>Polynomials are closed under addition, subtraction, and multiplication.</li> <li>The difference of two squares and perfect square trinomials are special products that can be recognized when multiplying binomials.</li> </ul>
2	<b>Solutions, More or Less</b> Representing Solutions to Quadratic Equations	AREI.4.b AREI.10 ASE.2 ASE.3.a NRNS.1	2	Students use the Properties of Equality and square roots to solve simple quadratic equations. They express solutions in terms of the distance from the axis of symmetry to the parabola. Students identify double roots, estimate square roots, and extract perfect roots from the square roots of products. They show graphically that a quadratic function is the product of two linear functions with the same zeros. Students then use the Zero Product Property to explain that the zeros of a quadratic function are the same as the zeros of its linear factors. Finally, they rewrite any quadratic in the form $f(x) = ax^2 - c$ as the product of two linear factors.	<ul style="list-style-type: none"> <li>Every whole number has two square roots, a positive principal square root and a negative square root.</li> <li>A quadratic function is a polynomial of degree 2. Thus, a quadratic function has two zeros or two solutions at <math>f(x) = 0</math>. If both solutions are the same, the quadratic function is said to have a double zero.</li> <li>The <math>x</math>-coordinates of the <math>x</math>-intercepts of a graph of a quadratic function are called the zeros of the quadratic function. The zeros are called the roots of the quadratic equation.</li> <li>The real solutions to a quadratic equation can be represented as the <math>x</math>-value of the axis of symmetry plus or minus a constant.</li> <li>The Zero Product Property states that if the product of two or more factors is equal to zero, then at least one factor must be equal to zero. Therefore, the zeros of a quadratic function are the same as the zeros of its linear factors.</li> <li>Any quadratic function in the form <math>f(x) = ax^2 - c</math> can be rewritten as the product of two linear factors, <math>(\sqrt{ax} + \sqrt{c})(\sqrt{ax} - \sqrt{c})</math>.</li> </ul>

\*Pacing listed in 45-minute days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
3	<b>Transforming Solutions</b> Solutions to Quadratic Equations in Vertex Form		2	Students solve quadratic equations that are squares of binomials, recognizing them as horizontal translations of the function. They use the Properties of Equality and square roots to solve quadratic equations of the form $y = a(x - c)^2$ and determine how the dilation affects the solutions. Finally, students solve quadratic functions of the form $y = a(x - c)^2 + d$ and determine how the translation affects the solutions. Students learn that a quadratic function can have one unique real zero, two real zeros, or no real zeros, and how the number of real zeros relates to the graph of the function.	<ul style="list-style-type: none"> <li>The solutions to a quadratic equation can be represented as the <math>x</math>-value of the axis of symmetry plus or minus a constant.</li> <li>A quadratic function written in the form <math>f(x) = a(x - h)^2 + k</math>, where <math>a \neq 0</math>, is in vertex form.</li> <li>The solutions for a quadratic equation of the form <math>y = (x - c)^2</math> are <math>x = c \pm \sqrt{y}</math>.</li> <li>The solutions for a quadratic equation of the form <math>y = a(x - c)^2</math> are <math>x = c \pm \sqrt{y/a}</math>.</li> <li>The solutions for a quadratic equation of the form <math>y = a(x - c)^2 + d</math> are <math>x = c \pm \sqrt{(y-d)/a}</math>.</li> </ul>
4	<b>The Missing Link</b> Factoring and Completing the Square	ASE.3.a ASE.3.b AREI.4 FIF.8	2	Students recall how to factor out the GCF from different polynomials. They follow examples to factor quadratic trinomials, first using area models and then recognizing patterns in the coefficients. Students use the Zero Product Property to solve quadratic equations by factoring. They are then introduced to completing the square, a method they can use to convert a quadratic equation given in general form to vertex form. Students complete the square to solve quadratic equations that cannot be solved using other methods.	<ul style="list-style-type: none"> <li>One method of solving quadratic equations in the form <math>y = ax^2 + bx + c</math> is to set the equation equal to zero, factor the trinomial expression, and use the Zero Product Property to determine the roots.</li> <li>Completing the square is a method for rewriting a quadratic equation in the form <math>y = ax^2 + bx + c</math> as a quadratic equation in vertex form.</li> <li>When a quadratic equation in the form <math>y = ax^2 + bx + c</math> is not factorable, completing the square is an alternative method of determining the roots of the equation.</li> <li>Completing the square is a useful method for converting a quadratic function written as <math>f(x) = ax^2 + bx + c</math> to vertex form for graphing purposes and determining the maximum or minimum in problem situations.</li> <li>Given a quadratic equation in the form <math>y = ax^2 + bx + c</math>, the vertex of the function is located at <math>(x, y)</math> such that <math>x = -b/2a</math> and <math>y = c - (b^2/4a)</math>.</li> </ul>

\*Pacing listed in 45-minute days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
5	<p><b>Ladies and Gentlemen, Please Welcome the Quadratic Formula!</b></p> <p>The Quadratic Formula</p>	<p>AREI.4 ASE.1 NRNS.1 NRNS.3 NCNS.1</p>	3	<p>The first activity focuses on the graphical interpretation of <math>x = -b/2a \pm \sqrt{(b^2-4ac)/2a}</math> as the distance from <math>(-b/2a, 0)</math> to each root. Students are introduced to the Quadratic Formula as a method to calculate the solutions to any quadratic equation written in general form. Students use the discriminant to determine the number and type of roots for a given function. Students learn why rational numbers are closed under addition and that the sum or product of a rational number and an irrational number is an irrational number. Students reason about the solution to a function with no x-intercepts. Students practice simplifying expressions with negative roots.</p>	<ul style="list-style-type: none"> <li>The Quadratic Formula, <math>x = (-b \pm \sqrt{b^2 - 4ac})/2a</math>, can be used to calculate the solutions to any quadratic equation written in general form, <math>f(x) = ax^2 + bx + c</math>, where <math>a</math>, <math>b</math>, and <math>c</math> represent real numbers and <math>a \neq 0</math>.</li> <li>On the graph of a quadratic function, <math>\pm \sqrt{b^2 - 4ac}/2a</math> is the distance from <math>(-b/2a, 0)</math> to each root.</li> <li>When a quadratic equation is in the form, <math>ax^2 + bx + c = 0</math>, where <math>a</math>, <math>b</math>, and <math>c</math> represent real numbers and <math>a \neq 0</math>, the discriminant is <math>b^2 - 4ac</math>.</li> <li>When <math>b^2 - 4ac &lt; 0</math>, the quadratic equation has no real roots.</li> <li>When <math>b^2 - 4ac = 0</math>, the quadratic equation has one real root. When <math>b^2 - 4ac &gt; 0</math>, the quadratic equation has two real roots.</li> <li>When <math>b^2 - 4ac</math> is a positive perfect square number, the roots are rational numbers.</li> <li>When <math>b^2 - 4ac</math> is positive, but not a perfect square the roots are irrational numbers.</li> <li>Understand that equations in one number system may have solutions in a larger number system.</li> <li>A rational number plus a rational number equals a rational number; rational numbers are closed under addition. A rational number plus an irrational number equals an irrational number.</li> <li>The number <math>i</math> is a number such that <math>i^2 = -1</math>. The set of imaginary numbers is the set of all numbers written in the form <math>a + bi</math>, where <math>a</math> and <math>b</math> are real numbers and <math>b</math> is not equal to 0.</li> </ul>
6	<p><b><math>i</math> Want to Believe</b></p> <p>Imaginary and Complex Numbers</p>	<p>NCNS.1 NCNS.2 NCNS.7 NCNS.9 AREI.4.b</p>	2	<p>Students begin by analyzing a quadratic function that does not cross the x-axis and use the Properties of Equality and square roots to solve the corresponding equation for its roots. Students are introduced to the number <math>i</math>, imaginary roots, imaginary zeros, and the complex number system. They use a complex coordinate plane to plot complex numbers and then use the graphical representation to understand how to add, subtract, and multiply complex numbers. Students solve quadratic equations with complex solutions using any method of their choosing. Finally, the Fundamental Theorem of Algebra is introduced, and students analyze graphs to determine the number of real and imaginary roots each corresponding quadratic equation has.</p>	<ul style="list-style-type: none"> <li>Equations with no solution in one number system may have solutions in a larger number system.</li> <li>The number <math>i</math> is a number such that <math>i^2 = -1</math>.</li> <li>The set of complex numbers is the set of all numbers written in the form <math>a + bi</math>, where <math>a</math> and <math>b</math> are real numbers and <math>b</math> is not equal to 0.</li> <li>The Commutative Property, the Associative Property, and Distributive Properties apply to complex numbers.</li> <li>Functions that do not intersect the x-axis have imaginary zeros.</li> <li>When the discriminant of a quadratic equation is a negative number, the equation has two imaginary roots.</li> <li>The Fundamental Theorem of Algebra states that any polynomial equation of degree <math>n</math> must have <math>n</math> complex roots or solutions.</li> </ul>

\*Pacing listed in 45-minute days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
	<b>Learning Individually with MATHia or Skills Practice</b>	AAPR.1 ASE.2 ASE.3a ASE.3b AREI.4.a AREI.4.b AREI.11 FBF.3 FIF.8.a NCNS.1 NCNS.2 NCNS.7	10	<p>In the MATHia software, students operate with polynomials. They use different strategies to factor quadratic expressions and determine the zeros of a quadratic function. Students complete the square to solve problems with quadratics. They solve quadratic equations using any method, including the Quadratic Formula. Students convert quadratics from one form to another. They watch a video introducing them to the imaginary number line and the complex number system. Students use addition, subtraction, and multiplication to rewrite complex expressions and solve quadratic equations that have complex solutions.</p> <p><b>MATHia Unit:</b> Polynomial Operations <b>MATHia Workspaces:</b> Introduction to Polynomial Arithmetic / Operating with Functions on the Coordinate Plane / Adding Polynomials / Subtracting Polynomials / Using a Factor Table to Multiply Binomials / Multiplying Binomials</p> <p><b>MATHia Unit:</b> Quadratic Equation Solving <b>MATHia Workspaces:</b> Making Sense of Roots and Zeros / Factoring Trinomials with Coefficients of One / Factoring Trinomials with Coefficients Other than One / Factoring using Difference of Squares / Factoring Quadratic Expressions / Solving Quadratic Equations by Factoring / Problem Solving Using Factoring / Completing the Square/ Problem Solving Using Completing the Square / Deriving the Quadratic Formula / Solving Quadratic Equations</p> <p><b>MATHia Unit:</b> Forms of Quadratics <b>MATHia Workspaces:</b> Converting Quadratics to General Form / Converting Quadratics to Factored Form / Converting Quadratics to Vertex Form</p> <p><b>MATHia Unit:</b> Operations with Complex Numbers <b>MATHia Workspaces:</b> Introduction to Complex Numbers / Simplifying Radicals with Negative Radicands / Simplifying Powers of <math>i</math> / Adding and Subtracting Complex Numbers / Multiplying Complex Numbers / Solving Quadratic Equations with Complex Roots</p>	

**Total Days: 131**

Learning Together: 84  
Learning Individually: 47

\*Pacing listed in 45-minute days