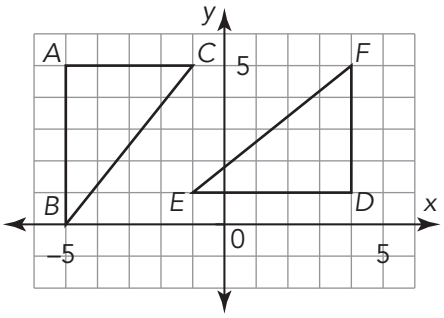
3

Consider Every Side

Constructing Triangles Given Sides

WARM UP

Use the coordinate plane to determine each distance. Show your work.



1. What is the distance from point *F* to point *D*?
2. What is the distance from point *A* to point *B*?
3. What is the distance from point *C* to point *A*?
4. What is the distance from point *E* to point *D*?

LEARNING GOALS

- Use patty paper to investigate triangles.
- Construct triangles from three angle measures or side lengths, identifying when the conditions determine a unique triangle, more than one triangle, or no triangle.

KEY TERM

- Triangle Inequality Theorem

You know how to draw a triangle. You know how to duplicate line segments and angles. Can you construct a specific triangle if you are given only two or three possible side lengths? Is there more than one possible triangle that you can construct?

Tri-, Tri-, and Tri- Again

Classify each statements as *always or sometimes* true about triangles.

- For each *always* true statement, explain your reasoning.
- For each *sometimes* true statement, provide an example and a counterexample.

1. The angles of a triangle have the same measure.

2. A triangle has three angles.

3. Two sides of a triangle have the same measure.

4. One angle of a triangle measures 90 degrees.

ACTIVITY
3.1

A Triangle Given Two Line Segments

NOTES

If you know the lengths of two sides of a triangle, can you construct the triangle? Can you construct multiple triangles from this information?

Let's first investigate with patty paper.

1. **Trace the two line segments onto a sheet of patty paper to create an angle. Can you draw a new line segment to create a triangle?**

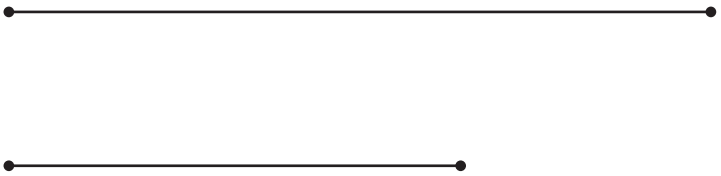
2. **Experiment with the two line segments and form different angles. Can you create different triangles? Use a different sheet of patty paper for each triangle attempt.**

3. **What do you notice? How many different triangles were you able to create?**

Label your sheets of patty paper "Given Two Segments" and store them in a safe place for use in the next lesson.

Now, let's use your construction tools: your compass and your straightedge.

4. Construct a triangle using the two line segments shown. Write the steps.



5. Use a protractor to measure each angle and a ruler to measure each side of your constructed triangle.

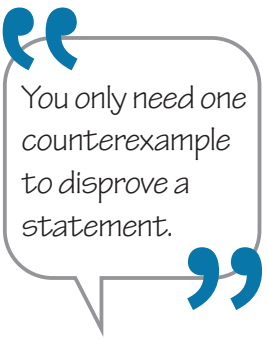
6. Classify your constructed triangle based on the measures of the angles and the lengths of the sides.

7. Compare the triangle that you constructed with the triangles that your classmates constructed. What do you observe? How does this compare to your patty paper triangles?

ACTIVITY
3.2 Pasta Triangles

Let's investigate the conditions necessary for forming a triangle with different side lengths.

1. **Sarah claims that even though 2 segment lengths would form many different triangles, she could use any 3 segment lengths as the three sides of a triangle. Sam does not agree. He thinks some combinations will not work. Who is correct? Remember, you need one counterexample to disprove a statement.**



Sam then claims that he can just look at the three lengths and know immediately if they will work. Sarah is unsure. She decides to explore this for herself.

Help Sarah by working through the following investigation.

To begin, you will need a piece of strand pasta. Break the pasta in two random points so the strand is divided into three pieces.

- Try to form a triangle from your three pieces of pasta.
- Measure each of your three pieces of pasta in centimeters.
- Repeat the experiment with a new piece of pasta.

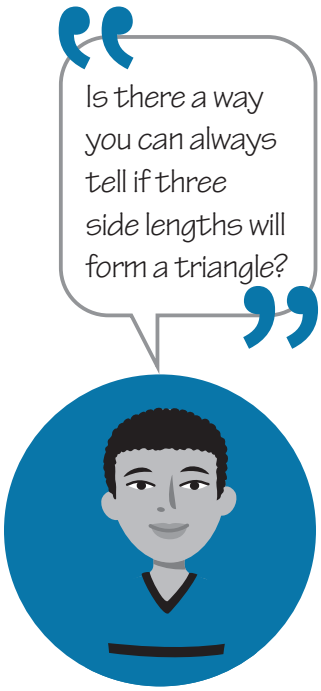
2. **Record your measurements and the measurements of your group members in the table provided.**

3. Collect and record your classmates' measurements.

Piece 1 (cm)	Piece 2 (cm)	Piece 3 (cm)	Forms a Triangle? (yes or no)

4. **What percent of the pasta pieces formed triangles when the pieces were connected end to end?**

5. **Examine the lengths of the pasta pieces that did form a triangle. Compare them with the lengths of the pasta pieces that did not form a triangle. Make a conjecture about the conditions under which it is possible to form a triangle.**



ACTIVITY
3.3

A Triangle Given Three Segments

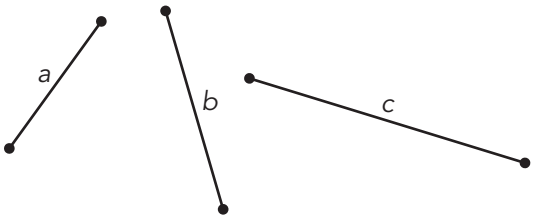
Let's continue to investigate Sarah's question and your conjecture with patty paper.

You and your partner should use different sets of segments for this investigation.

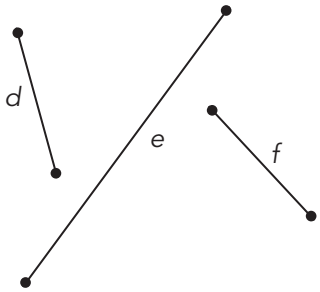
1. **Trace each of the three segments onto its own sheet of patty paper.**

   a. **Overlay the sheets to determine if you can create a triangle. If you can, record the triangle on its own sheet of patty paper.**

Set 1:



Set 2:



b. Now create as many different triangles as you can, using the given segments as sides of a possible triangle. Use a different sheet of patty paper to record each unique triangle.

c. What do you notice? How many different triangles were you able to create?

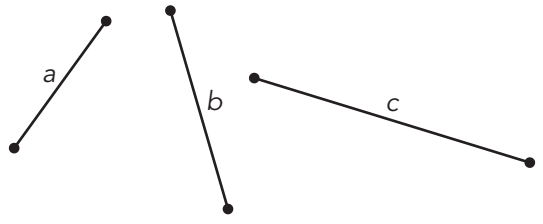
2. Use the patty paper examples from Set 1 and Set 2 to make a conjecture about when three segments can be used to create a triangle. Test your conjecture by creating additional triangles.

Now, use your construction tools (your compass and your straightedge) to attempt to construct the triangles. Make sure at least one person in your group starts with each of the different segments.

3. Construct a triangle given the three segments from Set 1.

a. Use the starter line provided.

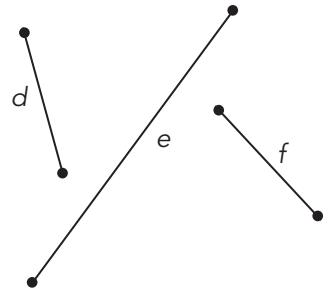
Triangles are *congruent* when all of their corresponding angle measures and corresponding side lengths are the same.



b. **Compare the triangle you constructed with the triangles the other students in your group constructed. What do you notice?**

4. **Construct a triangle given the three segments from Set 2.**

   a. **Use the starter line provided.**



   b. **Compare the triangle you constructed with the triangles the other students in your group constructed. What do you notice?**

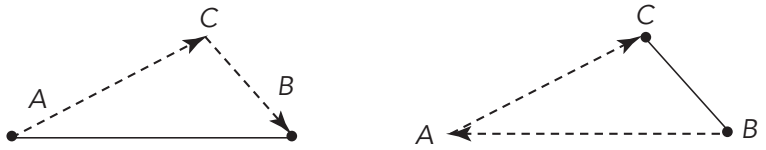
When the given information can be used to construct congruent triangles, the information is said to define a unique triangle.

ACTIVITY
3.4 Triangle Inequality Theorem

The measure of $\overline{AB}$ can be expressed in two different ways. *AB* is read as "the distance from point *A* to point *B*." $m\overline{AB}$ is read as "the measure of line segment *AB*."

In Euclidean geometry—the geometry of straight lines on flat planes—the shortest distance between two points is a straight line.

That means that any distance *AC* + *CB* will be greater than or equal to the distance *AB*. It also means that any distance *BA* + *AC* will be greater than or equal to the distance *BC*.



1. How could you use this fact to test whether three line segments can form a triangle? Explain your reasoning.

2. Provide examples of line segments that cannot possibly form a triangle.

3. **What would it mean for the distance *AC* + *CB* to be equal to the distance *AB*?**

4. **Based upon your observations, determine if it is possible to form a triangle using segments with the given measurements. Explain your reasoning.**

   a. **2 cm, 5.1 cm, 2.4 cm**

   b. **9.2 cm, 7 cm, 1.9 cm**

The rule that you have been using is known as the *Triangle Inequality Theorem.* The **Triangle Inequality Theorem** states that the sum of the lengths of any two sides of a triangle is greater than the length of the third side.

A theorem is a mathematical rule that can be formally proven.

NOTES

TALK the TALK

None, One, or Many?

Determine if the given information could be used to form a unique triangle, many different triangles, or no triangles. Explain your reasoning.

1. 3 in., 2.9 in., 5 in.

2. 112 mm, 300 mm

3. 5 yd, 10 yd, 21 yd

4. 8 ft, 9 ft, 11 ft

5. 13.8 km, 6.3 km, 7.5 km

Assignment

Write

Describe the Triangle Inequality Theorem in your own words.

Remember

When given two line segments, it is possible to construct an infinite number of triangles. When given three line segments, it is possible to construct 0 triangles, a unique triangle, or an infinite number of triangles.

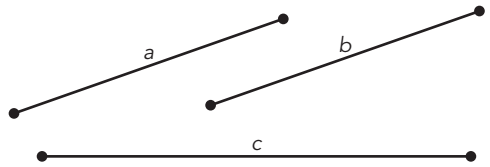
Practice

1. Determine if the given side lengths could be used to form a unique triangle, many different triangles, or no triangles. Explain your reasoning.

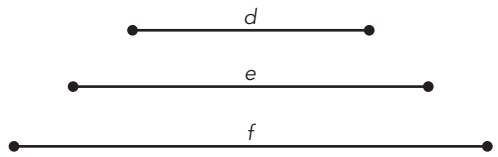
   a. 300 mm, 190 mm
   b. 4 m, 5.1 m, 12.5 m
   c. 7.4 cm, 8.1 cm, 9.8 cm
   d. 12 ft, 7 ft, 14 ft
   e. 20.2 in., 11 in., 8.2 in.

2. Analyze the given line segments. If the given information would create a unique triangle, multiple triangles, or no triangles. Then use the information to construct a triangle, if possible.

   a.



   b.

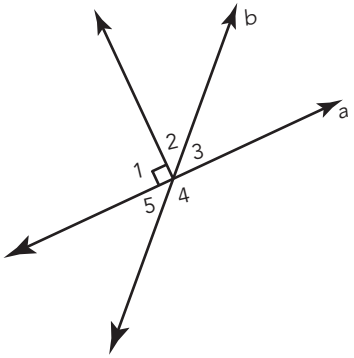


Stretch

Use your compass and straightedge to construct an equilateral triangle.

Review

1. Use the diagram to identify the specified angles given lines *a* and *b*.
   a. All adjacent angles
   b. All linear pairs
   c. All vertical angles
   d. All right angles
   e. All complementary angles



2. The average monthly rainfall in two Alaskan cities over a 30-year period is shown in the table. Use this data to answer the questions.
   a. Determine the five number summary describing the average monthly rainfall in each city.
   b. Construct a box-and-whisker plot to display the average monthly rainfall in each city.
   c. Describe the rainfall in each Alaskan city in terms of the median and IQR.

Month	Annette, Alaska Average Monthly Rainfall (inches)	Barrow, Alaska Average Monthly Rainfall (inches)
January	9.67	0.12
February	8.05	0.12
March	7.96	0.09
April	7.37	0.12
May	5.73	0.12
June	4.72	0.32
July	4.26	0.87
August	6.12	1.04
September	9.49	0.69
October	13.86	0.39
November	12.21	0.16
December	11.39	0.12

3. Number tiles containing the numbers 11–20 are in a bag. One tile is pulled from the bag. Determine each probability.
   a. *P*(prime) =
   b. *P*(multiple of 3) =