Matching, Sorting & Exploring:

Discovering Function Families

Sami Briceño
Texas Lead Manager of School Partnerships
sbriceno@carnegielearning.com
How interesting would a website be without pictures or illustrations? Does an inviting image on a magazine cover make you more likely to buy it? Pictures and images aren’t just for drawing your attention, though. They also bring life to text and stories.

There is an old proverb that states that a picture is worth a thousand words. There is a lot of truth in this saying—and images have been used by humans for a long time to communicate. Just think: would you rather post a story of your adventure on a social media site, or post one picture to tell your thousand-word story in a glance?
3. Read each scenario and then determine the independent and dependent quantities. Be sure to include the appropriate units of measure for each quantity.

**Something’s Fishy**

Candice is a building manager for the Crowley Enterprise office building. One of her responsibilities is cleaning the office building’s 200-gallon aquarium. For cleaning, she must remove the fish from the aquarium and drain the water. The water drains at a constant rate of 10 gallons per minute.

- independent quantity:
- dependent quantity:

**Smart Phone, but Is It a Smart Deal?**

You have had your eye on an upgraded smart phone. However, you currently do not have the money to purchase it. Your cousin will provide the funding, as long as you pay him interest. He tells you that you only need to pay $1 in interest initially, and then the interest will double each week after that. You consider his offer and wonder: is this really a good deal?

- independent quantity:
- dependent quantity:
Can't Wait to Hit the Slopes!

Andrew loves skiing—he just hates the ski lift ride back up to the top of the hill. For some reason the ski lift has been acting up today. His last trip started fine. The ski lift traveled up the mountain at a steady rate of about 83 feet per minute. Then all of a sudden it stopped and Andrew sat there waiting for 10 minutes! Finally, the ski lift began to ascend up the mountain to the top.

- independent quantity:

- dependent quantity:

It's Magic

The Amazing Aloysius is practicing one of his tricks. As part of this trick, he cuts a rope into many pieces and then magically puts the pieces of rope back together. He begins the trick with a 20-foot rope and then cuts it in half. He then takes one of the halves and cuts that piece in half. He repeats this process until he is left with a piece so small he can no longer cut it. He wants to know how many total cuts he can make and the length of each remaining piece of rope after the total number of cuts.

- independent quantity:

- dependent quantity:
Baton Twirling

Jill is a drum major for the Altadena High School marching band. She has been practicing for the band’s halftime performance. For the finale, Jill tosses her baton in the air so that it reaches a maximum height of 22 feet. This gives her 2 seconds to twirl around twice and catch the baton when it comes back down.

- independent quantity:
- dependent quantity:

Music Club

Jermaine loves music. He can lip sync almost any song at a moment’s notice. He joined Songs When I Want Them, an online music store. By becoming a member, Jermaine can purchase just about any song he wants. Jermaine pays $1 per song.

- independent quantity:
- dependent quantity:
A Trip to School

On Monday morning, Myra began her 1.3-mile walk to school. After a few minutes of walking, she walked right into a spider's web—and Myra hates spiders! She began running until she ran into her friend Tanisha. She stopped and told Tanisha of her adventurous morning and the icky spider's web! Then they walked the rest of the way to school.

• independent quantity:

• dependent quantity:

Jelly Bean Challenge

Mr. Wright judges the annual Jelly Bean Challenge at the summer fair. Every year, he encourages the citizens in his town to guess the number of jelly beans in a jar. He keeps a record of everyone's guesses and the number of jelly beans that each person's guess was off by.

• independent quantity:

• dependent quantity:
While a person can describe the monthly cost to operate a business, or talk about a marathon pace a runner ran to break a world record, graphs on a coordinate plane enable people to see the data. Graphs relay information about data in a visual way. If you read almost any newspaper, especially in the business section, you will probably encounter graphs.

Points on a coordinate plane that are or are not connected with a line or smooth curve model, or represent, a relationship in a problem situation. In some problem situations, all the points on the coordinate plane will make sense. In other problem situations, not all the points will make sense. So, when you model a relationship on a coordinate plane, it is up to you to consider the situation and interpret the meaning of the data values shown.

1. Cut out each graph on the following pages. Then, analyze each graph, match it to a scenario, and tape it next to the scenario it matches. For each graph, label the x- and y-axes with the appropriate quantity and unit of measure. Then, write the title of the problem situation on each graph.

What strategies will you use to match each graph with one of the eight scenarios?
1.1 Understanding Quantities and Their Relationships

Graph A

Graph B

Graph C

Graph D
1.1 Understanding Quantities and Their Relationships
5. Compare the graphs for each scenario given and describe any similarities and differences you notice.

a. *Smart Phone, but Is It a Smart Deal?* and *Music Club*

b. *Something’s Fishy* and *It’s Magic*

c. *Baton Twirling* and *Jelly Bean Challenge*

6. Consider the scenario *A Trip to School*.

a. Write a scenario and sketch a graph to describe a possible trip on a different day.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Graph</th>
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b. Compare your scenario and sketch with your classmates’ scenarios and sketches. What similarities do you notice? What differences do you notice?

Be prepared to share your solutions and methods.
Are you getting the urge to start driving? Chances are that you'll be studying for your driving test before you know it. But how much will driving cost you? For all states in the U.S., auto insurance is a must before any driving can take place. For most teens and their families, this more than likely means an increase in auto insurance costs.

So how do insurance companies determine how much you will pay? The fact of the matter is that auto insurance companies sort drivers into different groups to determine their costs. For example, they sort drivers by gender, age, marital status, and driving experience. The type of car is also a factor. A sports vehicle or a luxury car is generally more expensive to insure than an economical car or a family sedan. Even the color of a car can affect the cost to insure it!

Do you think it is good business practice to group drivers to determine auto insurance costs? Or do you feel that each individual should be reviewed solely on the merit of the driver based on driving record? Do you think auto insurance companies factor in where a driver lives when computing insurance costs?
Problem 1 Let's Sort Some Graphs

Mathematics is the science of patterns and relationships. Looking for patterns and sorting objects into different groups can provide valuable insights. In this lesson, you will analyze many different graphs and sort them into various groups.

1. Cut out the twenty-two graphs on the following pages. Then analyze and sort the graphs into different groups. You may group the graphs in any way you feel is appropriate. However, you must sort the graphs into more than one group!

In the space provided, record the following information for each of your groups.

- Name each group of graphs.
- List the letters of the graphs in each group.
- Provide a rationale why you created each group.
1.2 Analyzing and Sorting Graphs
A relation is the mapping between a set of input values called the **domain** and a set of output values called the **range**.

A relation can be represented in the following ways.

### Ordered Pairs

\[ \{(-2, 2), (0, 2), (3, -4), (3, 5)\} \]

### Equation

\[ y = \frac{2}{3}x - 1 \]

### Verbal

The relation between students in your school and each student's birthday.

### Mapping

![Mapping Diagram]

### Graph

![Graph Diagram]

### Table

<table>
<thead>
<tr>
<th>Domain</th>
<th>Range</th>
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<tr>
<td>-1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>-5</td>
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<tr>
<td>6</td>
<td>-5</td>
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<tr>
<td>7</td>
<td>-8</td>
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</table>
A function is a relation between a given set of elements, such that for each element in the domain there exists exactly one element in the range.

The Vertical Line Test is a visual method used to determine whether a relation represented as a graph is a function. To apply the Vertical Line Test, consider all of the vertical lines that could be drawn on the graph of a relation. If any of the vertical lines intersect the graph of the relation at more than one point, then the relation is not a function.

A discrete graph is a graph of isolated points. A continuous graph is a graph of points that are connected by a line or smooth curve on the graph. Continuous graphs have no breaks.

The Vertical Line Test applies for both discrete and continuous graphs.

Use the Vertical Line Test to sort the graphs in Problem 1 into two groups: functions and non-functions. Record your results by writing the letter of each graph in the appropriate column in the table shown.

<table>
<thead>
<tr>
<th>Functions</th>
<th>Non-Functions</th>
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</table>
Problem 2 Up, Down, or Neither?

In the previous lesson, you determined which of the given graphs represented functions. Gather all of the graphs from the previous lesson that you identified as functions.

A function is described as increasing when the dependent variable increases as the independent variable increases. If a function increases across the entire domain, then the function is called an increasing function.

A function is described as decreasing when the dependent variable decreases as the independent variable increases. If a function decreases across the entire domain, then the function is called a decreasing function.

If the dependent variable of a function does not change or remains constant over the entire domain, then the function is called a constant function.

1. Analyze each graph from left to right. Sort all the graphs into one of the four groups:
   - increasing function,
   - decreasing function,
   - constant function,
   - a combination of increasing, decreasing, or constant.

<table>
<thead>
<tr>
<th>Increasing Function</th>
<th>Decreasing Function</th>
<th>Constant Function</th>
<th>Combination of Increasing, Decreasing, or Constant</th>
</tr>
</thead>
</table>
2. Each function shown represents one of the graphs in the increasing function, decreasing function, or constant function categories. Enter each function into a graphing calculator to determine the shape of its graph. Then match the function to its corresponding graph by writing the function directly on the graph that it represents.

- \( f(x) = x \)
- \( f(x) = \left(\frac{1}{2}\right)^x - 5 \)
- \( f(x) = 2^x \), where \( x \) is an integer
- \( f(x) = \frac{2}{3} x + 5 \)
- \( f(x) = -x + 3 \), where \( x \) is an integer
- \( f(x) = \left\lfloor \frac{1}{2} \right\rfloor \)
- \( f(x) = 2 \), where \( x \) is an integer

3. Consider the seven graphs and functions that are increasing functions, decreasing functions, or constant functions.
   a. Sort the graphs into two groups based on the equations representing the functions and record the function letter in the table.

<table>
<thead>
<tr>
<th>Group 1</th>
<th>Group 2</th>
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</table>

   b. What is the same about all the functions in each group?
Congratulations! You have just sorted the graphs into their own function families. A function family is a group of functions that share certain characteristics.

The family of linear functions includes functions of the form \( f(x) = mx + b \), where \( m \) and \( b \) are real numbers.

The family of exponential functions includes functions of the form \( f(x) = a \cdot b^x + c \), where \( a, b, \) and \( c \) are real numbers, and \( b \) is greater than 0 but is not equal to 1.

4. Go back to your table in Question 3 and identify which group represents linear and constant functions and which group represents exponential functions.

5. If \( f(x) = mx + b \), represents a linear function, describe the \( m \) and \( b \) values that produce a constant function.

**PROBLEM 3** Least, Greatest, or Neither?

A function has an **absolute minimum** if there is a point that has a \( y \)-coordinate that is less than the \( y \)-coordinates of every other point on the graph. A function has an **absolute maximum** if there is a point that has a \( y \)-coordinate that is greater than the \( y \)-coordinates of every other point on the graph.

1. Sort the graphs from the Combination category in Problem 2 into three groups:
   - those that have an absolute minimum value,
   - those that have an absolute maximum value, and
   - those that have no absolute minimum or maximum value.

Then record the function letter in the appropriate column of the table shown.

<table>
<thead>
<tr>
<th>Absolute Minimum</th>
<th>Absolute Maximum</th>
<th>No Absolute Minimum or Absolute Maximum</th>
</tr>
</thead>
<tbody>
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</table>

Think about the graphical behavior of the function over its entire domain.
2. Each function shown represents one of the graphs with an absolute maximum or an absolute minimum value. Enter each function into your graphing calculator to determine the shape of its graph. Then match the function to its corresponding graph by writing the function directly on the graph that it represents.

- \( f(x) = x^2 + 8x + 12 \)
- \( f(x) = |x - 3| - 2 \)
- \( f(x) = x^2 \)
- \( f(x) = |x| \)
- \( f(x) = -|x| \)
- \( f(x) = -3x^2 + 4 \), where \( x \) is integer
- \( f(x) = -\frac{1}{2}x^2 + 2x \)
- \( f(x) = -2|x + 2| + 4 \)

3. Consider the graphs of functions that have an absolute minimum or an absolute maximum. (Do not consider Graphs A and C yet.)

a. Sort the graphs into two groups based on the equations representing the functions and record the function letter in the table.

<table>
<thead>
<tr>
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<th>Group 2</th>
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</table>

b. What is the same about all the functions in each group?
Congratulations! You have just sorted functions into two more function families.

The family of **quadratic functions** includes functions of the form $f(x) = ax^2 + bx + c$, where $a$, $b$, and $c$ are real numbers, and $a$ is not equal to 0.

The family of **linear absolute value functions** includes functions of the form $f(x) = a|x + b| + c$, where $a$, $b$, and $c$ are real numbers, and $a$ is not equal to 0.

4. Go back to your table in Question 3 and identify which group represents quadratic functions and which group represents linear absolute value functions.

**PROBLEM 4** Piecing Things Together

Analyze each of the functions shown. These functions represent the last three graphs of functions from the no absolute minimum and no absolute maximum category.

- $f(x) = \begin{cases} -2x + 10, & x < 3 \\ 4, & 3 \leq x < 7 \\ -2x + 18, & x \geq 7 \end{cases}$

- $f(x) = \begin{cases} -2, & x < 0 \\ \frac{1}{2}x - 2, & x \geq 0 \end{cases}$

- $f(x) = \begin{cases} \frac{1}{2}x + 4, & x < 2 \\ -3x + 11, & 2 \leq x < 3 \\ \frac{1}{2}x + \frac{1}{2}, & x \geq 3 \end{cases}$

These functions are part of the family of **linear piecewise functions**. **Linear piecewise functions** include functions that have equation changes for different parts, or pieces, of the domain.

Because these graphs each contain compound inequalities, there are additional steps required to use a graphing calculator to graph each function.
Let’s graph the piecewise function:

\[ f(x) = \begin{cases} 
-2x + 10, & x < 3 \\
4, & 3 \leq x < 7 \\
-2x + 18, & x \geq 7 
\end{cases} \]

You can use a graphing calculator to graph piecewise functions.

**Step 1:** Press Y=. Enter the first section of the function within parentheses. Then press the division button.

**Step 2:** Press the ( key and enter the inequality representing the domain for the first section of the function within parentheses. For a compound inequality representing the domain, press the ( key twice, and enter the first part of the compound inequality within parentheses. Enter the second part of the compound inequality within parentheses and then type two closing parentheses.

**Step 3:** Enter the remaining sections of the piecewise functions as Y₂ and Y₃.

By completing the first piecewise function, you can now choose the graph that matches your graphing calculator screen.

1. Enter the remaining functions into your graphing calculator to determine the shapes of their graphs.
2. Match each function to its corresponding graph by writing the function directly on the graph that it represents.
Congratulations! You have just sorted the remaining functions into one more function family.

The family of linear piecewise functions includes functions that have equation changes for different parts, or pieces, of the domain.

You will need these graphs again in Problem 5. Wait for it...
### PROBLEM 5  We Are Family!

You have now sorted each of the graphs and equations representing functions into one of five function families: linear, exponential, quadratic, linear absolute value, and linear piecewise.

1. Glue your sorted graphs and functions to the appropriate function family Graphic Organizer on the pages that follow. Write a description of the graphical behavior for each function family.

You’ve done a lot of work up to this point! You’ve been introduced to linear, exponential, quadratic, linear absolute value, and linear piecewise functions. Don’t worry—you don’t need to know everything there is to know about all of the function families right now. As you progress through this course, you will learn more about each function family.

Be prepared to share your solutions and methods.
The family of linear functions includes functions of the form $f(x) = mx + b$, where $m$ and $b$ are real numbers.
**Graphical Behavior**

**Increasing / Decreasing:**

**Domain / Range of Continuous / Discrete Functions:**

**Maximum / Minimum:**

**Curve / Line:**

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**Exponential Functions**

The family of **exponential functions** includes functions of the form \( f(x) = a \cdot b^x + c \), where \( a, b, \) and \( c \) are real numbers, and \( b \) is greater than 0 but not equal to 1.

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**Examples**
Graphical Behavior

Increasing / Decreasing: 

Maximum / Minimum: 

Curve / Line: 

Domain / Range of Continuous / Discrete Functions:

Quadratic Functions

The family of quadratic functions includes functions of the form, 
\[ f(x) = ax^2 + bx + c \]
where \(a, b,\) and \(c\) are real numbers, and \(a\) is not equal to 0.

Examples
Graphical Behavior

Increasing / Decreasing:

Maximum / Minimum:

Curve / Line:

Domain / Range of Continuous / Discrete Functions:

Linear Absolute Value Functions

The family of linear absolute value functions includes functions of the form $f(x) = a|x + b| + c$, where $a$, $b$, and $c$ are real numbers, and $a$ is not equal to 0.

Examples
Graphical Behavior

Increasing / Decreasing:

Maximum / Minimum:

Curve / Line:

Domain / Range of Continuous / Discrete Functions:

Linear Piecewise Functions

The family of linear piecewise functions includes functions that have equation changes for different parts, or pieces, of the domain.

Examples
2. Create your own function. Describe certain characteristics of the function and see if your partner can sketch it. Then try to sketch your partner’s function based on characteristics provided.

![Graph of a function](image)

**Talk the Talk**

Throughout this chapter, you were introduced to five function families: linear, exponential, quadratic, linear absolute value, and linear piecewise. Let’s revisit the first lesson in this chapter: *A Picture Is Worth a Thousand Words*. Each of the scenarios in this lesson represents one of these function families.

1. Describe how each scenario represents a function.

2. Complete the table to describe each scenario.
   a. Identify the appropriate function family.
   b. Based on the problem situation, identify whether the graph of the function should be discrete or continuous.
   c. Create a sketch of the mathematical model.
   d. Identify the graphical behavior.
<table>
<thead>
<tr>
<th>Scenario</th>
<th>Function Family</th>
<th>Domain of the Real-World Situation: Discrete or Continuous</th>
<th>Sketch of the Mathematical Model</th>
<th>Graphical Behavior</th>
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