

# Quantities and Relationships

## Topic 1 Overview

8 Days



### How is *Quantities and Relationships* organized?

In *Quantities and Relationships*, students encounter different scenarios representing the functions they will study throughout the course. The intent is merely to introduce these new functions, providing an overview but not a deep understanding at this point. The topic is designed to help students recognize that different function families have different key characteristics. In later study—both in this course and in future courses—they will formalize their understanding of the defining characteristics of each type of function.

Students begin by analyzing real-world scenarios. These scenarios move beyond the linear relationships familiar from middle school to include various nonlinear functions. Students connect the scenarios to corresponding graphs. They examine the graphical behavior of different function types by exploring a wide variety of graphs. Students search for patterns in the graphs' shape and structure, and then sort them according to defined characteristics.

Students review the definition of *function*, *domain*, and *range*. Building on their knowledge from grade 8, they formalize their representations of functions by writing equations in function notation. They use graphical behavior and the structure of the corresponding equations to classify each function according to its function family. Finally, with a more thorough understanding of the key characteristics of graphs of functions, students

return to the scenarios from the first lesson and define each in terms of function family and graphical behavior.

At the end of *Quantities and Relationships*, students create a graphic organizer with the equation representation and graphical behavior of each function family studied in this course: linear, exponential, quadratic, linear absolute value, and linear piecewise functions.



### What is the entry point for students?

Throughout middle school, students explored relationships between quantities. In grade 6, students defined independent and dependent variables and used them to write equations and create tables and graphs for various relationships. In grade 8, students defined a function and used linear functions to model the relationship between two quantities. Students have written linear functions in slope-intercept form and should be able to identify the slope and  $y$ -intercept in the equation. *Quantities and Relationships* introduces formal function notation as a natural extension of the informal expression evaluation mastered in grades 6 through 8. In grade 8, students learned that a graph of a function is the set of ordered pairs consisting of an input and the corresponding output. They characterized graphs as functions using the terms *increasing*, *decreasing*, *constant*, *discrete*, *continuous*, *linear*, and *nonlinear*. In *Quantities and Relationships*, students build on these characteristics to define new function families.



## How does a student demonstrate understanding?

Students will demonstrate understanding of the standards in *Quantities and Relationships* if they can:

- Choose appropriate scale and origin for graphs.
- Identify the appropriate unit of measure for each variable or quantity.
- Analyze a graph and state the key characteristics of the graph.
- Use a problem situation to explain what the characteristics of a graph mean in context.
- Define a function as a relation in which each input has exactly one output.
- Recognize a linear, exponential, quadratic, linear absolute value, or linear piecewise function by its equation or graph.
- Recognize that not all graphs are straight lines or functions.
- Use function notation to identify the output  $f(x)$  for a corresponding input  $x$ .
- Identify the domain and range and the independent and dependent quantities in a relationship.
- Recognize an equation, table, or graph as a function or non-function.
- Relate the domain of a function to its graph.



## Why is *Quantities and Relationships* important?

The study of functions is a main focus of high school mathematics. This topic builds the foundation for future, more in-depth study by familiarizing students with the concept of a function. With a function approach, students see functions as objects that can be represented by scenarios, equations, tables, and graphs.

Recognizing patterns and structure in these representations will allow them to connect different representations and to generalize patterns across function families. As students explore linear, piecewise, absolute value, exponential, and quadratic functions in future topics, the background acquired in *Quantities and Relationships* will help them to recognize and differentiate among mathematical relationships, and to compare and contrast the key characteristics of graphs and equations. Students will continue to use formal function notation throughout this course and in higher-level math courses. Facility with this notation helps students to see relationships between a function's input,  $x$ , and its corresponding output,  $f(x)$ .

Ultimately, searching for patterns in graphs is critical as students learn to recognize, generalize, and use patterns that exist in numbers, in shapes, and in the world around them. This work helps students to become better problem solvers and make sense of the mathematics.



## How do the activities in *Quantities and Relationships* promote student expertise in the mathematical practice standards?

All Carnegie Learning topics are written with the goal of creating mathematical thinkers who are active participants in class discourse, so elements of habits of mind should be evident in all lessons. Students are expected to make sense of problems and work towards solutions, reason using concrete and abstract ideas, and

communicate their thinking while providing a critical ear to the thinking of others.

Throughout *Quantities and Relationships*, students search for patterns in tables, equations, and scenarios. They examine the structure of these function representations to identify common characteristics of function types. They should notice that the equations of graphs in the same family all take the same general form.

## Materials Needed

Glue  
Graphing technology  
Scissors

## Digital Access

For all digital files aligned to this topic, log in to your account at [CarnegieLearning.com](https://www.CarnegieLearning.com).



## Learning Together: 6 Days

Lesson	Lesson Name	Standards	Days	Highlights	Spaced Review
1.1	<b>A Picture Is Worth a Thousand Words</b> Understanding Quantities and Their Relationships	N.Q.1 N.Q.2 A.REI.10 F.IF.1 F.IF.4	1	Students identify the independent and dependent quantities for various real-world scenarios, match a graph to the scenario, and interpret the scale of the axes. They observe similarities and differences in the graphs, and then focus on key characteristics, such as intercepts, increasing and decreasing intervals, and relative maximum and minimum points.	7.EE.1 8.EE.7b
1.2	<b>A Sort of Sorts</b> Analyzing and Sorting Graphs	F.IF.4	1	Students sort a variety of graphs based on their own rationale, compare their groupings with their classmates', and discuss the reasoning behind their choices. Next, four different groups of graphs are given and students analyze the groupings and explain possible rationales behind the choices made. Students explore different representations of relations.	7.EE.1 8.EE.7b N.Q.2

Lesson	Lesson Name	Standards	Days	Highlights	Spaced Review
1.3	<b>F of X</b> Recognizing Functions and Function Families	F.IF.1 F.IF.4 F.IF.5	2	Function notation is introduced. The terms <i>increasing function</i> , <i>decreasing function</i> , and <i>constant function</i> are defined. Students sort the graphs from the previous lesson into groups using these terms and match each graph with its appropriate equation written in function notation. The terms <i>function family</i> , <i>linear function</i> , and <i>exponential function</i> are then defined. Next, the terms <i>absolute minimum</i> and <i>absolute maximum</i> are defined. Students sort the remaining graphs into groups using these terms and match each graph with its appropriate equation written in function notation. The terms <i>quadratic function</i> and <i>linear absolute value function</i> are then defined. Linear piecewise functions are defined, and students match the remaining graphs to their appropriate functions. In the final activity, students demonstrate how the families differ with respect to their intercepts.	7.EE.1 8.EE.7b F.IF.4
1.4	<b>Function Families for 200, Alex</b> Recognizing Functions by Characteristics	F.IF.4	2	Given characteristics describing the graphical behavior of specific functions, students name the possible function family/families that fit each description. Students revisit the scenarios and graphs from the first lesson, name the function family associated with each scenario, identify the domain, and describe the graph. Students then write equations and sketch graphs to satisfy a list of characteristics. They conclude by determining that a function or equation, not just a list of characteristics, is required to generate a unique graph.	7.EE.1 8.EE.7b F.IF.1

## New Notation

An equation representing a function can be written using function notation. The function notation  $f(x)$  is read as “ $f$  of  $x$ ” and indicates that  $x$  is the independent variable.

$$f(x) = 8x + 15$$

name of function

independent variable

In the function shown, the cost, defined by  $f$ , is a function of  $x$ , the number of shirts ordered. The name of the function is  $f$ . It is not a variable.



### Learning Individually with MATHia: Approximately 2 Days

In the MATHia software, students answer questions related to two animations—one discussing dependent and independent quantities and slope in a real-world context, and the other investigating the shapes of graphs of functions, which show the linear and non-linear relationships between different quantities in real-world contexts. They study numberless graphs of functions and match the graphs to various situations. Students then answer questions related to an animation describing different function families, their graphs, equations, and general characteristics.

At Carnegie Learning, we believe it is our responsibility to continuously enhance MATHia to better support your students' learning needs. We leverage the learning data in MATHia to regularly improve both the learning experience and content. To get the latest MATHia content alignment map, log in to your account at [CarnegieLearning.com](https://CarnegieLearning.com).



## Learning Individually with Skills Practice: Approximately 2 Days

Problem Set	Overview
Identifying Independent and Dependent Quantities	Students determine the independent and dependent quantities in real-world scenarios. They then label axes of graphs with the corresponding quantities.
Identifying Domain and Range	Students identify the domain and range of real-world situations using words and inequalities.
Identifying Characteristics of Graphs	From a given set, students determine which graphs are functions, have a maximum or minimum, are increasing only, and/or are decreasing only.
Identifying Functions	Students determine whether given relations represented as tables, graphs, or mappings are functions. They then rewrite functions using function notation.
Recognizing Function Families	From a graph, students determine which function family it belongs to: linear functions, quadratic functions, exponential functions, linear absolute value functions, or linear piecewise functions.
Identifying Function Characteristics	Student sketch graphs of functions based on given characteristics.

### Assessments

There are six assessments aligned to this topic: a Pre-test, a Post-test, End of Topic Test (Form A and Form B), Standardized Test Practice, and a Performance Task.

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