1 Searching for Patterns
Pacing: 29 Days

Topic 1: Quantities and Relationships

Students analyze scenarios and graphs representing the functions they will study throughout the course. They learn to write equations for functions in function notation. Students recognize that different function families have different key characteristics, and they use graphical behavior to classify functions according to their function families.

Standards: N.Q.1, N.Q.2, A.REI.10, F.IF.1, F.IF.4, F.IF.5   **Pacing:** 8 Days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
1	**A Picture Is Worth a Thousand Words** Understanding Quantities and Their Relationships	N.Q.1 N.Q.2 F.IF.1 F.IF.4	1	Students identify the independent and dependent quantities for various real-world scenarios, match a graph to the scenario, and interpret the scale of the axes. They observe similarities and differences in the graphs, and then focus on key characteristics, such as intercepts, increasing and decreasing intervals, and relative maximum and minimum points.	• There are two quantities that change in problem situations. • When one quantity depends on another, it is said to be the dependent quantity. The quantity that the dependent quantity depends upon is called the independent quantity. • The independent quantity is used to label the x-axis. The dependent quantity is used to label the y-axis. • The domain includes the values that make sense for the independent quantity. The range includes the values that make sense for the dependent quantity. • Graphs can be used to model problem situations.
2	**A Sort of Sorts** Analyzing and Sorting Graphs	F.IF.4	1	Students sort a variety of graphs based on their own rationale, compare their groupings with their classmates', and discuss the reasoning behind their choices. Next, four different groups of graphs are given and students analyze the groupings and explain possible rationales behind the choices made. Students explore different representations of relations.	• A relationship between two quantities can be graphed on the coordinate plane. • Graphical behaviors can reveal important information about a relationship. • A graph of a relationship can have a minimum or maximum or no minimum or maximum. A graph can pass through one or more quadrants. A graph can exhibit vertical or horizontal symmetry. A graph can be increasing, decreasing, neither increasing nor decreasing, or both increasing and decreasing.
3	**G of X** Recognizing Functions and Function Families	F.IF.1 F.IF.4 F.IF.5	2	Function notation is introduced. The terms *increasing function*, *decreasing function*, and *constant function* are defined. Students sort the graphs from the previous lesson into groups using these terms and match each graph with its appropriate equation written in function notation. The terms *function family*, *linear function*, and *exponential function* are then defined. Next, the terms *absolute minimum* and *absolute maximum* are defined. Students sort the remaining graphs into groups using these terms and match each graph with its appropriate equation written in function notation. The terms *quadratic function* and *linear absolute value function* are then defined. Linear piecewise functions are defined, and students match the remaining graphs to their appropriate functions. In the final activity, students demonstrate how the families differ with respect to their intercepts.	• A function is a relation that assigns to each element of the domain exactly one element of the range. • The family of linear functions includes functions of the form $f(x) = mx + b$, where a and b are real numbers. • The family of exponential functions includes functions of the form $f(x) = abx + c$, where a, b, and c are real numbers, and b is greater than 0 but is not equal to 1. • The family of quadratic functions includes functions of the form $f(x) = ax2 + bx + c$, where a, b, and c are real numbers, and a is not equal to 0. • The family of linear absolute value functions includes functions of the form $f(x) = a\|x + b\| + c$, where a, b, and c are real numbers, and a is not equal to 0. • Linear piecewise functions include functions that have equation changes for different parts, or pieces, of the domain.

*Pacing listed in 45-minute days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
4	**Function Families for 200, Alex** Recognizing Functions by Characteristics	F.IF.4	2	Given characteristics describing the graphical behavior of specific functions, students name the possible function family/families that fit each description. Students revisit the scenarios and graphs from the first lesson, name the function family associated with each scenario, identify the domain, and describe the graph. Students then write equations and sketch graphs to satisfy a list of characteristics. They conclude by determining that a function or equation, not just a list of characteristics, is required to generate a unique graph.	• The graph of an exponential or quadratic function is a curve. • The graph of a linear or linear absolute value function is a line or pair of lines, respectively. • The graph of a linear or exponential function is either increasing or decreasing. • The graph of a quadratic function or a linear absolute value function has intervals where it is increasing and intervals where it is decreasing. Each function also has an absolute maximum or absolute minimum. • Key characteristics of graphs help to determine the function family to which it belongs.
	Learning Individually with MATHia or Skills Practice	N.Q.2 F.IF.1	2	Students answer questions related to two animations—one discussing dependent and independent quantities and slope in a real-world context, and the other investigating the shapes of graphs of functions, which show the linear and non-linear relationships between different quantities in real-world contexts. They study numberless graphs of functions and match the graphs to various situations. Students then answer questions related to an animation describing different function families, their graphs, equations, and general characteristics.	

*Pacing listed in 45-minute days

Topic 2: Sequences

Students explore sequences represented as lists of numbers, as tables of values, as equations, and as graphs modeled on the coordinate plane. Students recognize that all sequences are functions. They also recognize the characteristics of arithmetic and geometric sequences and learn to write recursive and explicit formulas for both. They learn to use a modeling process as a structure for approaching real-world mathematical problems.

Standards: F.IF.3, F.IF.5, F.BF.1a, F.BF.2   **Pacing:** 10 Days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
1	**Is There a Pattern Here?** Recognizing Patterns and Sequences	F.IF.3 F.IF.5 F.BF.1a	2	Given ten contexts or geometric patterns, students write a numeric sequence to represent each problem. They represent each sequence as a table of values, state whether each sequence is increasing or decreasing, and describe the sequence using a starting value and operation. They determine that all sequences are functions and have a domain that includes only positive integers. Infinite sequence and finite sequence are defined.	• A sequence is a pattern involving an ordered arrangement of numbers, geometric figures, letters, or other objects. • A term of a sequence is an individual number, figure, or letter in the sequence. • A sequence can be written as a function. The domain includes only positive integers. • An infinite sequence is a sequence that continues forever, or never ends. • A finite sequence is a sequence that terminates, or has an end term.
2	**The Password Is ... Operations!** Arithmetic and Geometric Sequences	F.BF.2	2	Given 16 numeric sequences, students generate several additional terms for each sequence and describe the rule they used for each sequence. They sort the sequences into groups based upon common characteristics of their choosing and explain their rationale. The terms arithmetic sequence, common difference, geometric sequence, and common ratio are then defined, examples are provided, and students respond to clarifying questions. They then categorize the sequences from the beginning of the lesson as being arithmetic, geometric or neither arithmetic or geometric and identify the common difference or common ratio where appropriate. Students begin to create graphic organizers identifying four different representations for each arithmetic and geometric sequence; in the first activity, they glue each arithmetic and geometric sequence to a separate graphic organizer and label them, and in the second activity, the corresponding graph is added to each graphic organizer. The remaining representations are completed in the following lessons. The lesson concludes with students writing sequences given a first term and a common difference or common ratio and identifying whether the sequences are arithmetic or geometric.	• An arithmetic sequence is a sequence of numbers in which the difference between any two consecutive terms is a positive or negative constant. This constant is called the common difference and is represented by the variable d. • A geometric sequence is a sequence of numbers in which the ratio between any two consecutive terms is a constant. This constant is called the common ratio and is represented by the variable r. • The graph of a sequence is a set of discrete points. • The points of an arithmetic sequence lie on a line. When the common difference is a positive, the graph is increasing, and when the common difference is a negative, the graph is decreasing. • The points of a geometric sequence do not lie on a line. When the common ratio is greater than 1, the graph is increasing; when the common ratio is between 0 and 1, the graph is decreasing; and when the common ratio is less than 0, the graph alternates between increasing and decreasing between consecutive points.

*Pacing listed in 45-minute days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
3	**Did You Mean:** *Recursion*? Determining Recursive and Explicit Expressions from Contexts	F.BF.1a	1	Scenarios are presented that can be represented by arithmetic and geometric sequences. Students determine the value of different terms in each sequence. As the term number increases it becomes more time-consuming to generate the term value, which sets the stage for explicit formulas to be defined and used. Students practice using these formulas to determine the values of terms in both arithmetic and geometric sequences.	• A recursive formula expresses each new term of a sequence based on a preceding term of the sequence. • An explicit formula for a sequence is a formula for calculating each term of the sequence using the term's position in the sequence. • The explicit formula for determining the nth term of an arithmetic sequence is $a_n = a_1 + d(n-1)$, where n is the term number, a_1 is the first term in the sequence, a_n is the nth term in the sequence, and d is the common difference. • The explicit formula for determining the nth term of a geometric sequence is $g_n = g_1 \cdot r^{n-1}$, where n is the term number, g_1 is the first term in the sequence, g_n is the nth term in the sequence, and r is the common ratio.
4	**3 Pegs, N Discs** Modeling Using Sequences	F.BF.2	2	Students are introduced to the process of mathematical modeling, with each of the four activities representing a specific step in the process. Students are invited to play a puzzle game, observe patterns, and think about a mathematical question. Students then organize their information and pursue a given question by representing the patterns they noticed using mathematical notation. As a third step, students analyze their recursive and explicit formulas and use them to make predictions. Finally, students test their predictions and interpret their results.	• Mathematical modeling involves noticing patterns and formulating mathematical questions, organizing information and representing this information using appropriate mathematical notation, analyzing mathematical representations and using them to make predictions, and then testing these predictions and interpreting the results. • Both recursive and explicit formulas can be used for sequences that model situations. • Sequence formulas can be used to make predictions for real-world situations.
Learning Individually with MATHia or Skills Practice		F.IF.3 F.BF.1a	3	In the MATHia Software, students determine and classify the patterns in sequences and identify the next terms. They then determine the recursive and explicit formulas for given sequences.	

*Pacing listed in 45-minute days

Topic 3: Linear Regressions

Students formalize their understanding of how to use lines of best fit to model bivariate data. Students use technology to generate linear regressions. To determine the appropriateness of a line to model a given data set, they analyze the shape of the scatterplot, calculate and assess the correlation coefficient, and build and evaluate residual plots. Students differentiate between correlation and causation, recognizing that a correlation between two quantities does not necessarily mean that there is also a causal relationship.

Standards: N.Q.3, S.ID.6, S.ID.6a, S.ID.6b, S.ID.6c, S.ID.7, S.ID.8, S.ID.9   **Pacing:** 11 Days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
1	**Like a Glove** Least Squares Regressions	N.Q.3 S.ID.6a S.ID.6c S.ID.7	2	Students informally approximate a line of best fit for a given data set, write an equation for their line, and then use their function to make predictions, learning about interpolation and extrapolation. They are then introduced to a formal method to determine the linear regression line of a data set via graphing technology.	• Interpolation is the process of using a regression equation to make predictions within the data set. • Extrapolation is the process of using a regression equation to make predictions beyond the data set. • A least squares regression line is the line of best fit that minimizes the squares of the distances of the points from the line.
2	**Gotta Keep It Correlatin'** Correlation	N.Q.3 S.ID.6a S.ID.6c S.ID.8 S.ID.9	2	Students analyze graphs and estimate a reasonable correlation coefficient based on visual evidence. They then use technology to determine a linear regression and interpret the correlation coefficient. Next, students analyze several problem situations to determine whether correlation is always connected to causation.	• A correlation is a measure of how well a regression model fits a data set. • The correlation coefficient, r, is a value between 21 and 1 that indicates the type (positive or negative) of association and the strength of the relationship. Values close to 1 or 21 demonstrate a strong association, while a value of 0 signifies no association. • Causation is when one event causes a second event. A correlation is a necessary condition for causation, but not a sufficient condition for causation. • Two relationships are that are often mistaken for causation are a common response, when some other reason may cause the same result, and a confounding variable, when there are other variables that are unknown or unobserved.
3	**The Residual Effect** Creating Residual Plots	S.ID.6 S.ID.6a S.ID.6b S.ID.6c	2	Students calculate a linear regression for a real-world problem and analyze the correlation coefficient to conclude whether the linear model is a good fit. The terms residual and residual plot are defined. Students calculate the residuals, construct a residual scatter plot, and conclude by its shape that there may be a more appropriate model. They are given a second data plot. Students create a scatter plot and determine the equation for the least squares regression line and the correlation coefficient with respect to the problem situation. They then calculate residuals and create a residual plot to conclude how the data are related.	• A residual is the distance between an observed data value and its predicted value using the regression equation. • Analyzing residuals is a method to determine whether a linear model is appropriate for a data set. • A residual plot is a scatter plot of the independent variable on the x-axis and the residuals on the y-axis. • The shape of a residual plot can be useful to determine whether there may be a more appropriate model than a line of best fit for a data set.

*Pacing listed in 45-minute days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
4	**To Fit or Not to Fit? That Is the Question!** Using Residual Plots	N.Q.3 S.ID.6 S.ID.6a S.ID.6b S.ID.6c	2	Students construct a scatter plot, determine a linear regression equation, compute the correlation coefficient, determine the residuals, and create a residual plot for a data set with one variable. They use all of the given information to decide whether a linear model is appropriate. A quadratic function is given and students conclude that this type of function appears to be a better fit. Finally, students summarize how the shape of a scatter plot, the correlation coefficient, and the residual plot help determine whether a linear model is an appropriate fit for the data set. The lesson emphasizes the importance of using more than one measure to determine if a linear model is a good fit.	• Scatter plots, regression functions, correlation coefficients, residuals, and residual plots are used to determine the appropriate model of best fit. • It is important to use several measures to determine the appropriate model of best fit.
	Learning Individually with MATHia or Skills Practice		3	Students investigate linear regression functions. They enter data related to various real-world contexts and use the Explore Tool to analyze the linear trend present in the data set, as given by the regression function. Students investigate how moving the points of the data set affects the slope of the regression line, and they analyze the effect of outliers on the regression function. Students are given a table of data and a linear regression equation that represents the line of best fit, which they use to interpolate and extrapolate values. Students solve problems in context, giving rough estimates of the value of r, stating how the estimate is reflected in the table of values, and determining whether the linear regression equation is appropriate for the data set. Students analyze a scatter plot and line of best fit, a table comparing the data with the residuals, and a residual plot.	

*Pacing listed in 45-minute days

2 Exploring Constant Change
Pacing: 47 Days

Topic 1: Linear Functions

Students connect arithmetic sequences to linear functions, proving that the constant difference, d, is always equal to the slope, m, of the corresponding linear function. Students examine the structure of equations representing functions and compare the graphs to determine what their differences indicate about the functions and the scenarios they model. Students are introduced to transformation notation, $y = A \cdot f(B(x - C)) + D$, although only the A-value (vertical dilations and/or reflections across the x-axis) and the D-value (vertical translations) are explicitly addressed. The topic concludes with students comparing key characteristics of linear functions presented in different forms.

Standards: N.Q.1, A.CED.1, F.IF.1, F.IF.2, F.IF.3, F.IF.4, F.IF.6, F.IF.7a, F.IF.9, F.BF.3, F.LE.1a, F.LE.1b, F.LE.2   **Pacing:** 14 Days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
1	**Connecting the Dots** Making Connections Between Arithmetic Sequences and Linear Functions	F.IF.1 F.IF.3 F.IF.6 F.LE.1a F.LE.1b F.LE.2	2	The lesson builds from what students know about arithmetic sequences to a general understanding of linear functions. Students connect an arithmetic sequence written in explicit form to a linear function in slope-intercept form. They compare the terms of each equation and prove that the common difference and the slope are always constant and equal. First differences is defined as a strategy to determine whether a table represents a linear relationship. Average rate of change is defined and presented graphically. Finally, students use what they know about arithmetic sequences to complete a graphic organizer to summarize the characteristics and representations of linear functions.	• The explicit formula of an arithmetic sequence can be rewritten as the slope-intercept form of a linear function using algebraic properties. • The explicit formula of an arithmetic sequence, $a_n = a_1 + d(n - 1)$, includes the first term of the sequence, $f(1)$, and the common difference. The slope-intercept form of a linear function, $f(x) = mx + b$, includes $f(0)$ and the slope. • Both the average rate of change formula and slope formula calculate the unit rate over a given interval. The average rate of change refers to the dependent variable as $f(x)$, while the slope formula uses y. • First differences is a strategy to determine whether a table of values can be modeled by a linear function. First differences are the values determined by subtracting consecutive output values when the input values have an interval of 1. If the first differences of a table of values are constant, the relationship is linear. • The domain of an arithmetic sequence is consecutive integers beginning with 1, while the domain of a linear function includes all real numbers.
2	**Fun with Functions, Linear Ones** Making Sense of Different Representations of a Linear Function	N.Q.1 A.SSE.1a A.CED.1 A.REI.10 F.IF.2 F.IF.4 F.BF.3	2	Students determine whether functions represented as scenarios, equations, or graphs are linear functions. They extend what they learned about first differences to analyze tables with input values that are not consecutive integers. Students then analyze a scenario and graph that can be represented by a function in the form $f(x) = ax$. A new scenario requires an equation in the form $f(x) = a(x - c)$. They analyze the meaning of this shift in the graph in terms of the context and compare the structure to that of $f(x) = ax + b$. The scenario changes a second time, and the students explore an equation in the form $f(x) = a(x - c) + d$.	• If a table represents a linear function, the slope, or average rate of change, is constant between all given points. • Using an equation to solve for the independent value given the dependent value always results in an exact answer. Using a graph or a table to determine the independent value sometimes results in an exact answer. • The graph of an equation plotted on the coordinate plane represents the set of all its solutions. • The general form of a linear function is $f(x) = ax + b$, where a and b are real numbers and $a \neq 0$. In this form, the a-value is the leading coefficient, which describes the steepness and direction of the line. The b-value describes the y-intercept. • The factored form of a linear function is $f(x) = a(x - c)$, where a and c are real numbers and $a \neq 0$. When a polynomial is in factored form, the value of x that makes each factor equal to zero is the x-intercept. This value is called the zero of the function. • A linear function is a polynomial with a degree of one.

*Pacing listed in 45-minute days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
3	**Move It!** Transforming Linear Functions	F.IF.4 F.IF.7a F.BF.3 G.GPE.5	3	Students identify key characteristics of several linear functions. A graph and a table of values for the basic linear function $f(x) = x$ is given, and students investigate $f(x) + D$ and $A \cdot f(x)$. Given a function $g(x)$ in terms of $f(x)$, students graph $g(x)$ and describe each transformation on $f(x)$ to produce $g(x)$. They prove algebraically that a line and its translation are parallel to one another and write equations of lines parallel to a given line through a given point. Finally, students use their knowledge of linear function transformations to test a video game that uses linear functions to shoot targets. They write the function transformations several ways and identify the domains, ranges, slopes, and y-intercepts of the new functions.	• For the basic function $f(x) = x$, the transformed function $y = f(x) + D$ affects the output values of the function. For $D > 0$, the graph vertically shifts up. For $D < 0$, the graph vertically shifts down. The amount of shift is given by $|D|$. • For the basic function $f(x) = x$, the transformed function $y = A \cdot f(x)$ affects the output values of the function. For $|A| > 1$, the graph stretches vertically by a factor of A units. For $0 < |A| < 1$, the graph compresses vertically by a factor of A units. For $A < 0$, the graph reflects across the x-axis. • A line and its translation are parallel to one another.
4	**Amirite?** Determining Slopes of Perpendicular Lines	G.GPE.5	2	Students explore the relationship between perpendicular lines. They use rotations to prove that if two lines are perpendicular, then the slopes of the lines are negative reciprocals. Students write the equation of a line perpendicular to a given line that passes through a given point.	• Transformations can be used to create perpendicular lines. By rotating a line 90°, the pre-image and image form perpendicular lines. • If two lines are perpendicular, their slopes are negative reciprocals. • All horizontal lines have a slope of zero, are parallel to one another, and are perpendicular to vertical lines. All vertical lines have a slope that is undefined, are parallel to one another, and are perpendicular to horizontal lines.
5	**Making a Connection** Comparing Linear Functions in Different Forms	N.Q.1 F.IF.9 G.GPE.5	2	Students compare linear functions represented in different forms to answer questions about real-world scenarios. They also identify the scale and origin on the graph of a function given a situation description. Finally, students generate and compare their own linear functions using tables, graphs, and equations.	• Functions can be represented using tables, equations, graphs, and with verbal descriptions. • Features of linear functions such as y-intercepts, slope, independent quantities, and dependent quantities can be determined from different representations of functions. • Lines that are parallel have the same slope. Lines that are perpendicular have slopes that are negative reciprocals.
	Learning Individually with MATHia or Skills Practice	A.REI.10 F.IF.1 F.IF.2 F.IF.6 F.IF.9 F.BF.3	3	Students use an Explore Tool to investigate linear functions. They analyze and compare the x- and y-intercepts, domains, ranges, and slopes of linear functions. They evaluate functions using function notation. Students use an interactive function machine and graph to identify transformations of functions.	

*Pacing listed in 45-minute days

Topic 2: Solving Linear Equations and Inequalities

Students use the Properties of Equality to justify the steps to solve one-variable equations. They examine and compare the structure of equations that have one, zero, and infinitely many solutions. Students use what they know about solving equations to solve literal equations for variables of interest and to convert between common formulas. When constraints are put on a scenario, students connect what they know about equations to solve linear inequalities. They learn how a negative coefficient on the variable affects the inequality sign and solve more complex inequalities. Finally, students solve compound inequalities and represent their solutions on number lines.

Standard: N.Q.1, N.Q.3, A.CED.1, A.CED.3, A.CED.4, A.REI.1, A.REI.3 **Pacing:** 9 Days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
1	**Strike a Balance** Solving Linear Equations	A.CED.1 A.REI.1 A.REI.3	1	Students are given a mathematical sentence that is always true and one that is always false. They choose any variable or constant and use the Properties of Equality to investigate ways to change the outcome of the given number sentences. Students reason that the mathematical sentence that is always true is still always true and that one that is false is still false. The terms no solution and infinite solutions are defined. Finally, students play Tic-Tac-Bingo as they work together to create equations with given solution types from assigned expressions.	• A solution to an equation is any variable value that makes that equation true. • Solving equations requires the use of number properties and properties of equality. • The Properties of Equality state that if an operation is performed on both sides of the equation, to all terms of the equation, the equation maintains its equality. • When the Properties of Equality are applied to an equation, the transformed equation will have the same solution as the original equation. • Equations with infinite solutions are created by equating two equivalent expressions. • Equations with no solution are created by equating expressions of the form $ax + b$ with the same value for a and different values for b. • Equations with a solution $x = 0$ are created by equation expressions of the form $ax + b$ with different values for a and the same value for b.
2	**It's Literally About Literal Equations** Literal Equations	N.Q.1 A.CED.4	1	Students identify the slope and intercepts of functions in general, factored, and standard form. They determine the same characteristics for the equation $Ax + By = C$. They then explain which form is more efficient in determining the slope and the x- and y-intercepts. Next, the term literal equation is defined. Students rewrite different literal equations to solve for given variables.	• The general form of a linear equation is $y = ax + b$, where a and b are real numbers; a represents the slope, and b represents the y-intercept. • The factored form of a linear equation is $y = a(x - c)$, where a and b are real numbers; a represents the slope, and c represents the x-intercept. • The standard form of a linear equation is $Ax + By = C$ where A is a positive integer, B and C are integers and both A and $B \neq 0$. It can be rewritten in general form as $y = (-A/B)x + C/B$; $-A/B$ represents the slope, C/B represents the y-intercept, and C/A represents the x-intercept. • General form of a linear equation is most useful form to identify the slope and y-intercept. Factored form of a linear equation is the most useful form to identify the slope and x-intercept. • Literal equations can be rewritten to highlight a specific variable.

*Pacing listed in 45-minute days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
3	**Not All Statements Are Made Equal** Modeling Linear Inequalities	N.Q.3 A.CED.1 A.CED.3 A.REI.3	2	Students use the graph of a function modeling a scenario with a positive rate of change to determine solutions to linear inequalities. The term solve an inequality is defined. Students write and solve two-step inequalities algebraically, choosing the most accurate solution in the context of the problem situation. Students solve linear inequalities for a scenario with a negative rate of change that affects the sign of the inequality. Finally, they solve linear inequalities that require more than two steps to solve.	• A linear inequality context can be modeled with a table of values, a graph on a coordinate plane, a graph on a number line, and with an inequality statement. • Solutions to linear inequalities can be determined both graphically and algebraically; they can be expressed using a number line or inequality statement. • The steps to solving a linear inequality algebraically are the same steps to solve a linear equation, except that when solving a linear inequality with a negative rate of change, the inequality sign of the solution must be reversed to accurately reflect the relationship.
4	**Don't Confound Your Compounds** Solving and Graphing Compound Inequalities	N.Q.3 A.CED.1 A.REI.3	2	The term compound inequality is defined. Students determine the inequality symbols that complete statements about a scenario represented by compound inequalities and express them in compact form. Given a scenario, they express the inequalities using symbols, then solve and graph the inequalities. The terms solution of a compound inequality, conjunction, and disjunction are defined. Students solve and graph compound inequalities, including those written in compact form.	• A compound inequality is an inequality that is formed by the union, "or," or the intersection, "and" of two simple inequalities. • Certain compound inequalities can be written in compact form. • The solution of a compound inequality (conjunction) written in the form $a < x < b$, where a and b are any real numbers, is the part or parts of the solutions that satisfy both of the inequalities. • The solution of a compound inequality (disjunction) written in the form $x < a$ or $x > b$, where a and b are any real numbers, is the part or parts of the solutions that satisfy either of the inequalities.
	Learning Individually with MATHia or Skills Practice	A.CED.1 A.REI.3	3		

*Pacing listed in 45-minute days

Topic 3: Systems of Equations and Inequalities

Students build on their current tools for solving systems of equations. They first use equations in standard form to solve systems of equations using linear combinations. Students analyze the structure of equations to select an efficient method to solve: inspection, graphing, substitution, or linear combinations. Students are introduced to two-variable inequalities. They recognize that just as the solutions to a one-variable inequality are a set of numbers, the solutions to a two-variable inequality are a set of ordered pairs. Students solve systems of linear inequalities graphically. They write systems of equations and inequalities for real-world situations and use function notation to solve linear programming problems.

Standards: A.CED.2, A.CED.3, A.REI.5, A.REI.6, A.REI.11, A.REI.12, F.IF.2   **Pacing:** 13 Days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
1	**Double the Fun** Introduction to Systems of Equations	A.CED.2 A.REI.6 A.REI.10 A.REI.11 F.IF.7a	1	Students explore a scenario that can be modeled with a system of linear equations in standard form. They graph the equations using the intercepts. They determine the intersection of the lines graphically and algebraically using substitution. Finally, students write a system of equations for given scenarios and analyze the slopes and y-intercepts and their relevance to the problem situation. They solve each system of equations graphically and algebraically, concluding that for any system there is no solution, one solution, or an infinite number of solutions.	• The standard form of a linear equation is $Ax + By = C$ where A, B, and C are constants and A and B are not both zero. Linear functions written in standard form can be graphed using the x- and y-intercepts. • Understand that the graph of an equation in two variables is the set of all its solutions plotted on the coordinate plane. • A linear system of equations is two or more linear equations that define a relationship between quantities. The solution of a linear system is an ordered pair that makes both equations in the system true. • Lines that do not intersect describe a system of equations in which each linear equation has the same slope and there is no solution. • Lines intersecting at a single point describe a system of equations in which each linear equation has a different slope and there is one solution. • Lines intersecting at an infinite number of points describe a system of equations in which each linear equation is the same equation and there are an infinite number of solutions. • Consistent systems of equations are systems that have one or many solutions. Inconsistent systems of equations are systems that have no solutions.
2	**The Elimination Round** Using Linear Combinations to Solve a System of Linear Equations	A.CED.2 A.REI.5 A.REI.6	2	Students explore a system of equations with opposite y-coefficients that is solved for x by adding the equations together. The term linear combinations method is defined, and students analyze systems that are solved by multiplying either one or both equations by a constant to rewrite the system with a single variable. Students analyze different systems of equations to determine how they would rewrite the equations to solve for one variable. Next, they apply the linear combinations method to two real-world problems, one with fractional coefficients.	• The linear combinations method is a process to solve a system of linear equations by adding two equations together, resulting in an equation in one variable. • When using the linear combinations method, it is often necessary to multiply one or both equations by a constant to create two equations in which the coefficients of one of the variables are additive inverses.

*Pacing listed in 45-minute days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
3	**Throwing Shade** Graphing Inequalities in Two Variables	A.CED.2 A.CED.3 A.REI.12	2	Scenarios are used that are represented by two-variable inequalities. Students write the inequality, complete a table of values, and use the table of values to graph the situation. The terms half-plane and boundary line are defined. Students use shading and solid or dashed lines to indicate which regions on the coordinate plane represent solution sets to the problem situation. Multiple representations such as equations, tables, and graphs are used to represent inequalities and their solutions.	• The graph of a linear inequality is a half-plane, or half of a coordinate plane. • Shading is used to indicate which half-plane describes the solution to the inequality. • Dashed and solid lines are used to indicate if the line itself is included in the solution set of an inequality. • Linear inequalities and their graphs can be used to represent and solve problems in context.
4	**Working with Constraints** System of Linear Inequalities	A.CED.3 A.REI.12	2	The term constraints is defined. Students write a system of linear inequalities to model a scenario, and graph the system, determining that overlapping shaded regions identify the possible solutions to the system. They practice graphing several systems of inequalities and determining the solution set. Finally, students match systems, graphs, and possible solutions of systems.	• In a system of linear inequalities, the inequalities are known as constraints because the values of the expressions are constrained to lie within a certain region. • The solution of a system of linear inequalities is the intersection of the solutions to each inequality. Every point in the intersecting region satisfies all inequalities in the system.
5	**Working the System** Solving Systems of Equations and Inequalities	A.CED.3 A.REI.6 A.REI.12	1	Students write a system of linear equations for each of three different scenarios: one in the form $y = ax + b$, one in the form $y = a(x - c) + b$, and one in the form $y = a(cx) + b$. They use any method to solve the system before reasoning about the solution in terms of the problem context. Students write a system composed of four linear inequalities to model a scenario and graph the system. Students determine the correct region that contains the solution set that satisfies all of the inequalities in the system.	• Contexts about choosing between two options can sometimes be modeled by a system of linear equations or inequalities. • The point of intersection of two lines separates the input values, with x-values less than and x-values greater than the x-value of the point of intersection. The solution to a problem in context may be dependent upon where the input values lie relative to the point of intersection. • Based upon a context, the solution of a system may be represented by inequalities rather than a single coordinate pair.
6	**Take It to the Max … or Min** Linear Programming	A.CED.3 A.REI.11 A.REI.12 F.IF.2	1	Students are introduced to function notation for two variables and the term linear programming is defined. They define variables and identify the constraints as a system of linear inequalities for different scenarios. Students then graph the solution region of the system and label all points of intersection of the boundary lines, identifying the vertices of the solution region. They write a function to represent the profit or cost and substitute each of the four vertices into the equation of the function to determine a maximum profit or a minimum cost.	• Linear programming is a branch of mathematics that determines the maximum and minimum value of linear expressions on a region produced by a system of linear inequalities. • Real-world problems that involve determining maximum profit or minimum costs may be solved using linear programming. • Linear programming involves determining the solution to a system of linear inequalities, identifying the vertices of its solution region, and substituting the coordinates of each vertex into an algebraic expression to determine a maximum or minimum value.
	Learning Individually with MATHia or Skills Practice	A.REI.5 A.REI.6 A.REI.11 A.REI.12	4		

*Pacing listed in 45-minute days

Topic 4: Shapes on a Coordinate Plane

Students derive the Distance Formula from the Pythagorean Theorem. They calculate the lengths of sides of triangles and quadrilaterals and use the slope criteria to determine whether opposite sides are parallel and/or adjacent sides are perpendicular. They use this information to classify the shapes according to their properties. They are introduced to the Midpoint Formula and discover the pattern when connecting the midpoints of consecutive sides of quadrilaterals. Students then calculate the perimeter and area of triangles and quadrilaterals, investigating how rigid motions can make the calculations more efficient. They must use the slope criteria for perpendicular lines to write the equation for the altitude of triangles. Finally, students consider real-world situations requiring them to calculate the perimeter and area of polygons that lie on a coordinate plane.

Standards: G.GPE.4, G.GPE.5, G.GPE.7   **Pacing:** 11 Days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
1	**The Shape of Things** Classifying Shapes on the Coordinate Plane	G.GPE.4￼G.GPE.5	3	Students sort triangles and quadrilaterals based on properties. They are introduced to the Distance Formula and use it to calculate the lengths of sides of triangles and quadrilaterals on the coordinate plane. Students also use the slope formula to determine whether opposite sides of a quadrilateral are parallel and whether consecutive sides of a quadrilateral are perpendicular. They use these skills to classify triangles and quadrilaterals that lie on a coordinate plane and to determine the fourth point of a quadrilateral when given three points. Students are then introduced to the Midpoint Formula and use it to classify secondary figures formed when connecting the midpoints of consecutive sides of quadrilaterals. Finally, students consider translations as a strategy for identifying the coordinates that create quadrilaterals with parallel sides.	• The Distance Formula states that the distance d between points (x_2, x_1) and (y_2, y_1) on a coordinate plane is given by the equation $d = \sqrt{((x_2 - x_1)^2 + (y_2 - y_1)^2)}$ • The Distance Formula can be used to classify triangles and quadrilaterals based on side lengths. • The slope formula can be used to determine whether opposite sides are parallel or consecutive sides are perpendicular in a quadrilateral on the coordinate plane. • The Midpoint Formula states that if (x_1, y_1) and (x_2, y_2) are two points on the coordinate plane, then the midpoint of the line segment that joins these two points is $((x_1 + x_2)/2, (y_1 + y_2)/2)$. • The use of translations is an effective strategy when determining endpoints of parallel segments on a coordinate plane.
2	**Know It Inside Out** Area and Perimeter of Triangles and Rectangles on the Coordinate Plane	G.GPE.5￼G.GPE.7	3	Students calculate the perimeter and area of rectangles and triangles on the coordinate plane. They double dimensions of figures and explain how this affects the area of the figure. Students translate rectangles and triangles on the coordinate plane to more efficiently determine the perimeter and area of the figures. Finally, students algebraically determine the non-vertical height of a triangle as they treat each side as the base; they then use the height to calculate the area of the triangle. They conclude that the area of a triangle remains the same regardless of the side of the triangle considered as the base and the triangle's height determined by that base.	• Rigid motion transformations (translations, rotations, and reflections) can be used to change the position of figures on the coordinate plane. • Performing translations on figures can help to compute perimeter and area more efficiently. • Non-vertical heights of a figure can be calculated algebraically using formulas, writing equations, and solving a system of equations. • The area of a triangle is the same regardless of what base and height of the triangle are used in the calculation.

*Pacing listed in 45-minute days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
3	**In All Shapes and Sizes** Area and Perimeter of Polygons on the Coordinate Plane	G.GPE.7	2	Students consider real-world situations requiring them to use the Distance Formula to calculate the perimeter and area of polygons that lie on a coordinate plane. The term composite figure is defined, and students divide a composite figure into various known polygons to compute its area. A velocity-time graph is used to model a real-world scenario. Student determine distances represented as the area under the curve of these graphs.	• A composite figure is a figure that is formed by combining different shapes. • Polygons can be divided into a combination of triangles and rectangles to help determine their area. • The area of a composite figure is determined by dividing the figure into familiar shapes and using the area formulas associated with those shapes. • The Distance Formula, slope formula, and the Pythagorean Theorem can be used to determine the area of polygons and composite figures on the coordinate plane. • A velocity-time graph can model acceleration, and distance can be determined by calculating the area under a curve.
	Learning Individually with MATHia or Skills Practice		3		

*Pacing listed in 45-minute days

3 Investigating Growth and Decay
Pacing: 20 Days

Topic 1: Introduction to Exponential Functions

Students recall geometric sequences and explore their graphs. They learn that some geometric sequences belong to the exponential function family. They learn to rewrite geometric sequences as exponential functions in the form $f(x) = a \cdot b^x$. They examine the structure of exponential expressions and compare exponential functions represented in different forms. Finally, students expand their experiences with function transformations, now including horizontal transformations and dilations in their repertoire. They generalize the effect of these translations on points, sketch graphs of transformed functions, and write equations based on a given sequence of transformations.

Standards: A.CED.1, A.REI.3, A.REI.10, F.IF.4, F.IF.7e, F.IF.9, F.BF.1a, F.BF.3, F.LE.1a, F.LE.2, F.LE.5   **Pacing:** 14 Days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
1	**Constant Ratios** Geometric Sequences and Exponential Functions	A.SSE.1a A.REI.10 F.BF.1a F.LE.1a F.LE.2 F.LE.5	2	Students learn through investigation that while all geometric sequences are functions, only some geometric sequences can be represented as exponential functions. They identify the constant ratio in different representations of exponential functions and then show algebraically that the constant ratio between output values of an exponential function is represented by the variable b in the function form $f(x) = a \cdot b^x$. Students also identify the a-value of that form as the y-intercept of the graph of the function. Students learn the term horizontal asymptote and explore this concept on different graphs, analyzing end behavior, particularly as the x-values approach negative infinity. The lesson concludes with a comparison of the base of the power in the equation $f(x) = a \cdot b^x$, the expression $f(x + 1) / f(x)$, and the common ratio of the corresponding geometric sequence.	• All geometric sequences are functions; however, only geometric sequences with a positive constant ratio are exponential functions. • If a geometric sequence represents an exponential function, then the Product of Powers Rule and the definition of negative exponents can be used to rewrite the explicit formula for the sequence as an exponential function. • The form of an exponential function is $f(x) = a \cdot b^x$, where b represents the constant ratio and $(0, a)$ represents the y-intercept. • For an exponential function in the form $f(x) = a \cdot b^x$, the ratio $f(x + 1) / f(x)$ is constant and equal to b. • A horizontal asymptote is a horizontal line that a function gets closer and closer to, but never intersects.
2	**To the What?** Comparing Exponential Functions	A.CED.1 A.REI.3 F.IF.9 F.BF.1a F.LE.2	2	Students solve a problem in context that is represented by a decreasing exponential function. They then write an exponential function to represent a given table or graph. Students analyze a worked example that demonstrates how to solve an exponential equation using common bases and then use this method to solve several exponential equations. Students then compare characteristics of pairs of exponential functions represented in different forms.	• Multiple representations such as tables, equations, and graphs are used to represent exponential problem situations. • Exponential functions represented in different forms reveal characteristics such as the y-intercept, b-value or constant multiplier, horizontal asymptote, and whether the function is increasing or decreasing. • A decreasing exponential function may be represented by a negative a-value or a b-value that lies between 0 and 1. • Common bases and properties of exponents are used to solve simple exponential equations.

*Pacing listed in 45-minute days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
3	**My A, B, C, Ds** Transformations of Exponential Functions	F.IF.4 F.IF.7e F.BF.3	2	Students explore a variety of different transformations of exponential functions, including vertical translations, horizontal translations, vertical reflections and dilations, and horizontal reflections and dilations. For each transformation, students sketch graphs of the transformation, compare characteristics of the transformed graphs with the graph of the parent function, including the horizontal asymptote when appropriate, and write the transformations using coordinate notation. They also consider different ways to rewrite and interpret equations of function transformations. Finally, students summarize the effects of the different transformations at the end of the lesson.	• A reflection of a graph is a mirror image of the graph about a line. The line that the graph is reflected about is called the line of reflection. • Reflections across the x-axis can be expressed using the notation $(x, y) \rightarrow (x, -y)$. It affects the y-coordinate of each point on the graph. • Reflections across the y-axis can be expressed using the notation $(x, y) \rightarrow (-x, y)$. It affects the x-coordinate of each point on the graph.
Learning Individually with MATHia or Skills Practice			8		

*Pacing listed in 45-minute days

Topic 2: Using Exponential Equations

In this topic, students use exponential equations to solve problems. They solve equations of the form $a \cdot b^x = d$ by graphing $y = a \cdot b^x$ and $y = d$ and locating the point of intersection. Students then solve real-world problems that can be modeled by exponential functions. They use technology to calculate regression equations and use them to make predictions. Students are reminded of the modeling process for problem solving.

Standards: N.Q.2, A.SSE.1b, A.CED.1, A.CED.2, A.REI.10, A.REI.11, F.BF.1b, F.LE.1c, F.LE.3, F.LE.5, S.ID.6a   **Pacing:** 6 Days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
1	**Downtown and Uptown** Exponential Equations for Growth and Decay	A.SSE.1b A.CED.1 A.CED.2 F.IF.6 F.LE.1c F.LE.3 F.LE.5	2	Students compare linear and exponential functions in the context of simple interest and compound interest situations. They identify the values in the exponential function equation that indicate whether an exponential function is a growth or decay function, and they apply this reasoning in context.	• Simple interest can be represented by a linear function. Compound interest can be represented by an exponential function. • An exponential growth function is of the form $y = a(1 + r)^x$, where r is the rate of growth. • An exponential decay function is of the form $y = a(1 - r)^x$, where r is the rate of decay.
2	**The Horizontal Line and Powers** Interpreting Parameters in Context	A.CED.1 A.CED.2 A.REI.10 A.REI.11 F.BF.1b F.LE.5	1	Students match exponential equations to their graphs to discern that the horizontal asymptote is always represented by $y = D$. For exponential growth and decay scenarios, students complete tables of values, graph the functions, and write exponential equations using function notation. Students use graphs to estimate the solutions to equations by graphing both sides of the equation and locating the point of intersection.	• Given an exponential function, $f(x) = A \cdot b^{B(x - C)} + D$, the horizontal asymptote is always represented by the equation $y = D$, and the y-intercept is the A-value plus any vertical translation. • Multiple representations such as tables, equations, and graphs can be used to represent and compare exponential problem situations. • Graphs can be used to solve exponential equations by graphing both sides of the equation and estimating the point of intersection. • A quantity increasing exponentially eventually exceeds a quantity increasing linearly. • Properties of exponential functions can be compared using different representations. • An exponential function and a constant function can be added to create a third function that is the sum of the two functions, resulting in a graph that is a vertical translation of the original exponential function.
3	**Tea and Carbon Dioxide** Modeling Using Exponential Functions	N.Q.2 S.ID.6a	2	Given a data set, students create a scatter plot, write a regression equation, use the function to make predictions, and interpret the reasonableness of a prediction. The lesson concludes with students generalizing about the common features of scenarios that are modeled by exponential functions. They also describe the shape of a scatter plot representing an exponential function and sketch possible graphs of exponential functions.	• Technology can be used to determine exponential regression equations to model real-world situations. The regression equation can then be used to make predictions. • Sometimes referring to the scenario or obtaining further information may be required to determine whether a scatter plot is best modeled by a linear or exponential function.

*Pacing listed in 45-minute days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
4	**BAC Is BAD News** Choosing a Function to Model BAC	N.Q.2 S.ID.6a	1	Students are given a context involving the blood alcohol content (BAC) of a driver and the driver's likelihood of causing an accident. Students are then given data from a study connecting BAC and the relative probability of causing an accident. They apply the relationship from the data to create a model predicting the likelihood of a person causing an accident based on their BAC. They summarize their learning by writing an article for a newsletter about the seriousness of drinking and driving. The lesson concludes with students connecting their process for solving the problem to the steps in the mathematical modeling process.	• Determining and using a regression equation is sometimes a step in the process of solving a more complex mathematical problem, rather than the final solution. • The mathematical modeling process is an effective structure to solve complex mathematical problems.
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*Pacing listed in 45-minute days

4 Describing Distributions
Pacing: 16 Days

Topic 1: One-Variable Statistics

Students create and analyze dot plots, histograms, and box-and-whisker plots to analyze different sets of data. They learn that different data displays are useful with different types of data sets. They learn formal notation for the mean and the formula for standard deviation. Students recognize and extract outliers in skewed data sets, noting that the outliers do not have much impact on the mean and interquartile range of a data set. Students have multiple opportunities to use the statistical process to compare two data sets using shape and measures of center and spread.

Standards: S.ID.1, S.ID.2, S.ID.3   **Pacing:** 9 Days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
1	**Represent!** Graphically Representing Data	S.ID.1 S.ID.3	2	A scenario presents students with twenty data points. Students analyze the data by creating a dot plot. The terms dot plot, discrete data, and data distribution (including symmetric, skewed right, and skewed left) are defined. In the second activity, the terms histogram, bin, frequency, and continuous data are defined. Students use a much larger data set for the same scenario in the previous activity, presented in a frequency table. They then construct and analyze a histogram for the data. In the third activity, the terms box-and-whisker plot and five number summary are defined. Students are given the five number summary of the data from the previous activity and use it to construct and analyze a box-and-whisker plot. Finally, students are presented with two box-and-whisker plots and asked to write an analysis comparing the two data sets.	• Discrete data are data that have a finite number of values, or data that can be counted, while continuous data are data that can take any numerical value within a range. • A dot plot is best used to organize and display a small amount of discrete data points. • A histogram is effective in displaying large amounts of continuous data using vertical bars, or bins, representing intervals of data. • A box-and-whisker plot displays the spread of data based on a five-number summary consisting of the minimum value, the first quartile (Q1), the median, the third quartile (Q3), and the maximum value. • A box-and-whisker plot is helpful when comparing two large data sets.
2	**A Skewed Reality** Determining the Better Measure of Center and Spread for a Data Set	S.ID.2 S.ID.3	2	Students are presented with various data displays and predict the location of the mean and median in each display. A worked example presents the formula for calculating the arithmetic mean and introduces students to the formal notation. Students construct a box-and-whisker plot that overlays a given dot plot to analyze the spread of the data points. The term interquartile range (IQR) is introduced, and students calculate the IQR for the same data set. They remove any outliers and reanalyze the IQR of the data set. Next, students compare two new data sets displayed in a table and in box-and-whisker plots, removing possible outliers. They then calculate and interpret the standard deviation to compare three symmetric data sets. At the end of the lesson, students know when and how to use mean and standard deviation vs. mean and IQR to describe the center and spread of a data set.	• Extremely high or low data values have a greater effect on the mean than the median of a data set. • The data distribution is the overall shape of a graph. • The median and IQR are the most appropriate measures of center and spread when data have a skewed distribution. • Removing outliers from a data set has a minimal effect, in any, on the IQR. • The mean and standard deviation are the most appropriate measures of center and spread when data have a symmetric distribution.

*Pacing listed in 45-minute days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
3	**Daring to Compare** Comparing Data Sets	S.ID.2 S.ID.4	2	Students conclude that when comparing two data sets, if one data set is skewed, then the median and IQR should be used to compare the sets. Next, students are provided with three scenarios that each compare two different data sets. In the each, students are provided with a table comparing two data sets and must decide which measure of center and spread to use in their comparison.	• When analyzing data, it must first be determined whether the data distribution is symmetric or skewed in order to know what measure of center and measure of spread is appropriate. • The median and IQR are the appropriate measures of center and spread when comparing data sets where at least one distribution is skewed. • Depending upon the context, a smaller or larger measure of center may be desired; however, a smaller measure of spread, either the IQR or standard deviation, is always preferred. • If the distribution of a data set appears symmetric once the outliers are removed, it is appropriate to use the mean and standard deviation to analyze the center and spread of the data.
Learning Individually with MATHia or Skills Practice			3		

*Pacing listed in 45-minute days

Topic 2: Two-Variable Categorical Data

Students display and analyze categorical data using a number of different distribution types. They begin with frequency and marginal frequency tables. To better compare the joint frequencies, students create relative frequency tables and relative marginal frequency tables. They answer questions about the values in the tables and use the tables to make decisions about the relationship between the variables. To help discern associations between categorical data sets, students create conditional relative frequency tables. Finally, students are presented with a statistical question and data set that they need to organize, analyze, and summarize for a final report.

Standard: S.ID.5   **Pacing:** 7 Days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
1	**It Takes Two** Creating and Interpreting Frequency Distributions	S.ID.5	2	Students differentiate between questions that are answered with numeric data from those answered with categorical data. They are presented with data expressed as categories rather than numerical values. The terms two-way frequency table, frequency distribution, joint frequency, and marginal frequency distribution are defined. Students organize data into a two-way frequency table and create a marginal frequency distribution and bar graphs to answer questions related to the given scenario. Finally, students interpret the data analyzed in the context of the scenario.	• Data sets can be categorical or numerical. • A frequency distribution displays the joint frequencies for categorical data in a two-way table. • A marginal frequency distribution displays the total of the joint frequencies of the rows and columns of a frequency distribution. • A bar graph displays the frequency of categorical data.
2	**Relatively Speaking** Relative Frequency Distribution	S.ID.5	1	Students construct a relative frequency distribution and marginal relative frequency distribution using data for a scenario. They analyze the distributions and answer questions about the problem situation. Next, students are shown stacked bar graphs that represent the relative frequency distribution in two different ways. They compare the graphs to the tables in the previous activity and explain the advantages of graphing the data each way. Finally, students analyze and interpret the data represented by the stacked bar graphs in terms of the problem situation.	• A relative frequency distribution provides the ratio of occurrences for each category to the total number of occurrences. • A marginal relative frequency distribution includes the ratio of total occurrences for each category to the total number of occurrences. • A stacked bar graph is a bar graph in which the bars are stacked on top of each other as opposed to sitting next to each other.
3	**On One Condition ... or More** Conditional Relative Frequency Distribution	S.ID.5	1	Students consider what different joint frequencies in a marginal relative frequency distribution represent. They construct a stacked bar graph and analyze the percentages shown in the graph before the term conditional relative frequency distribution is introduced. Students construct a conditional relative frequency distribution and use it to answer questions related to the given scenario. They construct a second conditional relative frequency distribution in terms of the other variable. Finally, students construct a conditional relative frequency distribution and interpret the data in terms of the problem situation.	• A conditional relative frequency distribution is the percent of proportion of occurrences of a category given the specific value of another category.

*Pacing listed in 45-minute days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
4	**Data Jam** Drawing Conclusions from Data	S.ID.5	1	Students synthesize what they know about analyzing and interpreting two-variable categorical data to make a recommendation in a real-world scenario. They organize a given data set by creating a frequency distribution and a stacked bar graph, and then use conditional relative frequency distributions to determine whether there is an association between the two categories. Students formulate conclusions for specified subsets of the data and use statistics to support their conclusions.	• A marginal frequency distribution is used to formulate and support conclusions to a real-world problem. • A stacked bar graph is a visual display used to formulate and support conclusions to a real-world problem. • A conditional relative frequency distribution compares occurrences within a category is used to formulate and support conclusions to a real-world problem.
Learning Individually with MATHia or Skills Practice			2	Students construct frequency distributions and relative frequency distributions. They use these distributions to answer questions about joint frequencies and marginal frequencies. Students then create conditional relative frequency tables and discern any possible associations.	

*Pacing listed in 45-minute days

5 Analyzing Geometric Functions
Pacing: 35 Days

Topic 1: Constructions

Students learn the tools of formal geometric construction. They complete three major constructions: inscribing a square, a regular hexagon, and an equilateral triangle in a circle. As they learn how to make these constructions, students learn the basic constructions necessary, including duplicating and bisecting lines and angles, constructing perpendicular lines, and constructing parallel lines through points on and off the given line.

Standard: G.CO.1, G.CO.12, G.CO.13   **Pacing:** 10 Days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
1	**Construction Ahead** Constructing a Square	G.CO.1 G.CO.12 G.CO.13	3	Students are introduced to geometric construction as a method of reasoning exactly about geometric objects without the aid of measuring tools such as rulers, protractors, and coordinate planes. Construction is distinguished from sketching and drawing. Students construct segments, rays, lines, circles, as well as perpendicular lines through a point on a line and not on a line using patty paper folding and compass and straightedge. In the final activity, students construct a square inscribed in a circle using both rotations of a right triangle on the plane and a compass and straightedge construction.	• When you construct geometric figures, you create exact figures using only patty paper or a compass and straightedge. • The midpoint of a segment is a point that divides the segment into two congruent segments. A segment bisector is a line, line segment, or ray that divides a line segment into two line segments of equal length. A perpendicular bisector is a line, line segment, or ray that bisects a line segment and is also perpendicular to the line segment. • Any point on a perpendicular bisector is equidistant to the endpoints of the original segment which it bisects. • The perpendicular bisector of any chord of a circle passes through the center of the circle. • The diagonals of a square are congruent, bisect each other, are perpendicular to one another, and bisect the angles of the square. • When a square is inscribed in a circle, a segment that is a diagonal of the square is also a diameter of the circle. When a circle is inscribed in a square, the diameter of the circle is the same length as a side of the square.
2	**Copycats** Constructing a Regular Hexagon Inscribed in a Circle	G.CO.12 G.CO.13	2	Students use patty paper and construction tools (a compass and straightedge) to duplicate line segments and angles. Students use these constructions to create angles twice the measure of a given angle and line segments twice the length of a given line segment. Students use what they know about line segment duplication to construct a regular hexagon inscribed in a circle. They verify that all the angles of the hexagon are congruent by using the angle duplication construction. In the final activity, students verify a geometric theorem—the Inscribed Angle Theorem—using an angle duplication construction.	• Constructions can be used to duplicate a given angle. • A 60° angle can be constructed by creating an equilateral triangle with a circle. • A regular hexagon can be inscribed in a circle by duplicating 60° angles to create six equilateral triangles sharing the center of the circle as a vertex. • A regular hexagon can be inscribed in of a circle by constructing six adjacent congruent chords the same length as the radius of the circle around the circumference of the circle. • The central angle of a circle is twice the measure of an inscribed angle which intercepts the same arc of the circle. • Constructions can be used to verify geometric theorems.

*Pacing listed in 45-minute days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
3	**A Regular Triangle** Constructing an Equilateral Triangle	G.CO.12 G.CO.13	2	Students use patty paper and construction tools (a compass and straightedge) to bisect angles. Students use these constructions to create angles one-fourth and one-eighth the measure of a given angle. Students use what they know about constructions to construct an equilateral triangle. In the final activity, students use what they know about bisecting an angle and constructing an equilateral triangle to construct a 75-degree angle and present their constructions to the class.	• An angle bisector is a line, segment, or ray that is drawn through the vertex of an angle and divides the angle into two congruent angles. Angle bisectors can be constructed using patty paper or a compass and straightedge. • Both an equilateral triangle with a given side length and an equilateral triangle inscribed in a circle can be created using construction tools. • Constructions of 90° angles, 60° angles, and angle bisectors can be used to construct angles of other measures.
	Learning Individually with MATHia or Skills Practice		3		

*Pacing listed in 45-minute days

Topic 2: Rigid Motions on a Plane

Using the intuitive understandings of rigid motions built in middle school, students learn the formal definitions of translations, reflections, and rotations. They define translations in terms of equal distances along directed line segments, reflections in terms of perpendicular lines, and rotations in terms of equal arcs around concentric circles. They use rigid motions to solve problems and identify a sequence of rigid motions that maps a given figure onto another. Finally, students consider reflectional and rotational symmetry — which rotations and reflections map a plane figure onto itself. They identify the lines of reflection and angles of rotation for given plane figures.

Standards: G.CO.1, G.CO.2, G.CO.3, G.CO.4, G.CO.5   **Pacing:** 14 Days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
1	**Put Your Input In, Take Your Output Out** Geometric Components of Rigid Motions	G.CO.1 G.CO.4	3	Students use a transformation machine and patty paper to translate shapes along line segments, rotate shapes on figures around points, and reflect shapes across lines of reflection. Students then analyze the component parts of the transformation machine throughout the rest of the lesson, including constructing parallel line segments and identifying and defining lines, angles, and rotation angles. Students recall that an image transformed by rigid motions such as translations, reflections, and/or rotations is congruent to its pre-image.	• Pre-images transformed by rigid motions such as translations, reflections, and rotations are congruent to their images. • Two lines perpendicular to a third line are parallel to each other. • If two lines intersected by a transversal have corresponding angles, alternate interior angles and alternate exterior angles congruent, then the lines are parallel. • A line is a geometric object such that if any part of the line is translated to another part of the line so that the two parts have two points in common, then the first part will lie exactly on top of the second part. • Translations can be described using lines and line segments. Reflections can be described using lines. Rotations can be described using rotation angles.
2	**Bow Thai** Translations as Functions	G.CO.2 G.CO.4	2	Students recall that they used lines and line segments as "transformation machines" to translate plane figures. They identify different ways of drawing the same transformation machine and investigate a transformation machine created with multiple line segments. Students use the context of designing an animated website banner to investigate translations as functions. Students learn that parallel lines can be used for translations. They distinguish between rigid motions, or isometries, and transformations, such as dilations, which are not isometries.	• Translations along parallel lines are rigid motions and always produce images that are congruent to the pre-image. • A translation is a function, represented as $T_{AB}(P) = P'$ which takes as its input the location of a point P and translates it a distance AB in the direction AB. • Isometries are rigid motion transformations that preserve size and shape.
3	**Staring Back at Me** Reflections as Functions	G.CO.2 G.CO.4 G.CO.5	2	Students construct a perpendicular bisector of a line segment and then use patty paper to recognize that the perpendicular bisector is a line of symmetry for the line segment, which allows a reflection across the line to match up its endpoints. Students are then encouraged to attempt the impossible task of drawing two points in the plane that cannot be reflected one onto the other. Students investigate reflections as functions using the context from the previous lesson of animating objects on a website, and they describe the points of reflection as equidistant from the line of reflection, which is the perpendicular bisector of the segment connecting the points. Students prove the Perpendicular Bisector Theorem and its converse and then use sequences of isometries to demonstrate that two plane figures are congruent.	• The perpendicular bisector of a line segment is a line of reflection between the two endpoints of the segment. • Reflections are isometries. • A reflection is a function, R_ℓ, which takes as its input, P, the location of a point with respect to some line of reflection, ℓ, and outputs $R_\ell(P)$ or the opposite of the location of P with respect to the line of reflection. • The Perpendicular Bisector Theorem states: "If two points are equidistant from a third point, the third point lies on the perpendicular bisector of the segment connecting the two points."

*Pacing listed in 45-minute days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
4	**Turn Yourself Around** Relations as Functions	G.CO.2 G.CO.4 G.CO.5	3	Students draw concentric circles and build a rotation of a triangle of 75°. They investigate the rotation and determine that it is an isometry, producing a figure with congruent corresponding sides. Students formally define a rotation as a function which takes as input the location of a point with respect to a center of rotation and outputs the rotation of the point about the center through a rotation angle. Students draw rotation transformations given functions. Students then use what they know about the Perpendicular Bisector Theorem to determine the center of rotation and angle of rotation given only the pre-image and image figures. Students complete the lesson by describing sequences of translations, reflections, and rotations which map congruent figures onto each other.	• Rotations are isometries. • A rotation is a function, $R_{E,t}(P) = P'$ that maps its input, a point P, to another location, P'. This movement to a new location is defined by a center of rotation, E, and a rotation angle, t. • The center of rotation lies on the perpendicular bisector of each pair of corresponding points of a pre-image and its rotated image. For this reason, the center of rotation is the point of intersection of any two of these perpendicular bisectors.
5	**OKEECHOBEE** Reflectional and Rotational Symmetry	G.CO.3	1	Students use patty paper to create and fold shapes into two matching parts and rotate shapes so that they match exactly to their starting position. They then investigate given cutout shapes to see if they have these folding and rotating properties. Students define reflectional symmetry and rotational symmetry and identify shapes with these properties. They determine the number of lines of symmetry for a given shape and the angles a shape can be rotated through to match the original shape. Students identify and draw the sequences of reflections and rotations that carry a figure onto itself and investigate how a figure's lines of symmetry relate to these properties.	• A plane figure has reflectional symmetry if you can draw a line so that the figure to one side of the line is a reflection of the figure on the other side. • A plane figure has rotational symmetry if you can rotate the figure more than 0 degrees and less than 360 degrees and the resulting figure is the same as the original figure. • An individual figure may have horizontal symmetry, vertical symmetry, and/or rotational symmetry. • A regular polygon of n-sides has n lines of symmetry. • The measure of the angle of rotation of a regular polygon with n sides is 360 degrees / n, which is the supplement of the measure of each of its interior angles.
Learning Individually with MATHia or Skills Practice			3		

*Pacing listed in 45-minute days

Topic 3: Congruence Through Transformations

This topic builds on the work of the previous topic and the work with triangles in middle school. Students use the definitions of congruence through rigid motions to determine the minimum criteria for triangle congruence. First, they are grounded in formal geometric reasoning. Students consider counterexamples, conditional statements, truth values, and truth tables They consider terms used in formal geometry proof, postulate and theorem, and investigate the Linear Pair Postulate and the Segment Addition Postulate. This preparation readies students to prove by construction the Side-Side-Side, Side-Angle-Side, and Angle-Side-Angle Congruence Theorems. Students close the topic by solving problems using these theorems.

Standards: G.CO.6, G.CO.7, G.CO.8   **Pacing:** 11 Days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
1	**The Elements** Formal Reasoning in Euclidean Geometry		2	Conditional statements are analyzed and associated with truth values. Truth tables are used to help students organize information. Euclid's first five postulates and Euclid's Elements are mentioned. The terms postulate and theorem are defined, and students use the Linear Pair Postulate, the Segment Addition Postulate, and the Angle Addition Postulate to answer related questions.	• The two reasons why a conclusion may be false is either the assumed information is false or the conclusion does not follow from the hypothesis. • A counterexample is used to show a general statement is not true. • A conditional statement is a statement that can be written in the form "If p, then q." The variable p represents the hypothesis and the variable q represents the conclusion. • A truth value is whether or not a conditional statement is true or false; it is true if the conditional statement could be true, and it is false if the conditional statement could not be true. • Truth tables are used to organize truth values of conditional statements. • A postulate is a statement that is accepted without proof. A theorem is a statement that can be proven. • The Linear Pair Postulate states: "If two angles form a linear pair, then the angles are supplementary." • The Segment Addition Postulate states: "If point B is on line $\overline{AC}$ and between points A and C, then $AB + BC = AC$." • The Angle Addition Postulate states: "If point D lies in the interior of $\angle ABC$, then $m\angle ABD + m\angle DBC = m\angle ABC$."
2	**ASA, SAS, and SSS** Proving Triangle Congruence Theorems	G.CO.7 G.CO.8	3	Students use what they have learned in the previous topic: (1) isometries preserve distances and angle measures, (2) any point in the plane can be reflected across a line to map to another point in the plane, and (3) a point is equidistant from two other points if and only if it lies on their perpendicular bisector. They use these facts to create and verify proofs of the SSS, SAS, and ASA Congruence Theorems using rigid motion transformations. Students then explore some non-examples of congruence theorems (AAA and SSA). Students explore a problem at the beginning of the lesson which can be solved by creating congruent triangles at the end of the lesson.	• A proof is a series of statements and corresponding reasons forming a valid argument that starts with a hypothesis and arrives at a conclusion. • The Side-Side-Side (SSS) Congruence Theorem states: "If three sides of one triangle are congruent to the corresponding sides of another triangle, then the triangles are congruent." • Corresponding parts of congruent triangles are congruent, abbreviated as CPCTC, is often used as a reason for stating congruences in geometric proofs after triangles have been proven congruent. • The The Side-Angle-Side (SAS) Congruence Theorem states: "If two sides and the included angle of one triangle are congruent to the corresponding sides and included angle of another triangle, then the triangles are congruent." • The Angle-Side-Angle (ASA) Congruence Theorem states: "If two angles and the included side of one triangle are congruent to the corresponding angles and included side of another triangle, then the triangles are congruent." • Triangle congruence theorems such as SSS, SAS, and ASA can be proven using rigid motion transformations.

*Pacing listed in 45-minute days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
3	**I Never Forget a Face** Using Triangle Congruence to Solve Problems	G.CO.6	3	Students use the criteria for triangle congruence they proved in the previous lesson--Side-Side-Side, Side-Angle-Side, and Angle-Side-Angle--to solve real-world and mathematical problems. Students learn that triangle congruence has been an important factor in developing computer face recognition techniques. They apply these techniques to a few problems and then use the triangle congruence criteria to determine whether two triangles are congruent—both for triangles presented on a coordinate plane and for triangles not on a coordinate plane.	• The SSS, SAS, and ASA Congruence Theorems can be applied to solve real-world and mathematical problems. • Congruent parts of triangles can be depicted from a diagram rather than stated. These can be instances where two triangles share a common side or angle. • The SSS, SAS, and ASA Congruence Theorems can be applied to triangles on or off the coordinate plane.
	Learning Individually with MATHia or Skills Practice		3		

Total Days: 147

Learning Together: 104
Learning Individually: 43

*Pacing listed in 45-minute days