CARNEGIE LEARNING

1 Relating Quantities and Reasoning with Equations

MATHia Unit	MATHia Workspace	Overview	CCSS	Problem Solving	Animation	Worked Examples	Classification Tools	Explore
Function Overview	Identifying Quantities	Students answer questions related to two animations — one discussing dependent and independent quantities and slope in a real-world context, and the other investigating the shapes of graphs of functions which show the linear and non-linear relationships between different quantities in real-world contexts. Students study numberless graphs of functions and match the graphs to various situations.	N.Q.A.2		•			
	Introduction to Function Families	Students answer questions related to an animation describing different function families (linear, quadratic, exponential, absolute value), their graphs, equations, and general characteristics. Students then investigate the graphs and characteristics of linear, exponential, quadratic, and linear absolute functions in more detail.	F.IF.A.1		•			
	Understanding Linear Functions	Students use an interactive Explore tool to investigate linear functions in the context of a plane's ascent and descent. Students analyze the different functions' x-intercepts, y-intercepts, domains, ranges, and slopes. Students then solve problems in context by using the Explore tool and solving for the slope (rate of change of descent/ascent) and the initial height (y-intercept).	F.IF.A.1 F.IF.A.6					•
	Evaluating Linear Functions	Given a function in function notation, students determine input and output values.	F.IF.A.2	•				
	Exploring Graphs of Linear Functions	Students use an interactive function machine and a graph to identify and analyze function equations and graphs. Students identify intercepts of the graphs.	A.REI.D.10 F.IF.A.1 F.BF.3					•
	Identifying Key Characteristics of Graphs of Functions	Students will identify key characteristics from the graph of a function, such as the intercepts, minimum and maximum x-values, minimum and maximum y-values, domain, and range.	F.IF.A.1	•				
Linear Equations	Modeling Rates of Change	Students will determine linear expressions with integer coefficients that represent real-world contexts. They will use these expressions to solve problems.	A.CED.A.1	•				
	Modeling Linear Equations Given Two Points	Students are given the ordered pairs for two points, either mathematically or in context and are asked to identify the equation of the line that connects the points.	A.CED.A.1	•				
	Modeling Linear Equations Given an Initial Point	Students define variables and write expressions and relations to describe linear contexts.	A.CED.A.1	•				
	Modeling Linear Functions using Multiple Representations	Students model problems using expressions, tables, and graphs. Students use number properties to evaluate and solve one-step and two-step equations.	A.CED.A.1	•				
	Comparing Linear Functions in Different Forms	Given two linear functions in different representations — equation, graph, table, or description — with a contextual or noncontextual scenario, students compare the functions' slopes or y-intercepts.	F.IF.C.9	•				

MATHia Unit	MATHia Workspace	Overview	CCSS	Problem Solving	Animation	Worked Examples	Classification Tools	Explore
Linear Inequalities	Graphing Inequalities	Students graph simple inequalities involving rational numbers on a number line.	A.REI.B.3	•				
	Solving Two-Step Linear Inequalities	Students solve linear inequalities	A.REI.B.3	•				
	Representing Compound Inequalities	Students make a distinction between two types of compound inequalities, conjunctions and disjunctions. They interpret the meaning of compound inequality statements written in inequality notation. Students write compound inequalities to represent problems in context. Students interpret graphs by selecting the compound inequality that represents a given graph.	A.REI.B.3				•	
Absolute Value Equations	Graphing Simple Absolute Value Equations using Number Lines	Students write a simple absolute value equation from a verbal statement, determine the number of solutions, and then represent the solution on a number line.	A.CED.A.3	•				
	Solving Absolute Value Equations	Students solve multi-step absolute value equations, determine the number of solutions, and then represent the solution on a number line.	A.CED.A.3	•				
	Reasoning About Absolute Value Inequalities	Students use graphical representations to solve absolute value inequalities. They learn to write equivalent compound inequalities for absolute value inequalities.	A.CED.A.3			•		

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2 Linear and Exponential Relationships

MATHia Unit	MATHia Workspace	Overview	CCSS	Problem Solving	Animation	Worked Examples	Classification Tools	Explore
Systems of Linear Equations	Representing Systems of Linear Functions	Students reason with linear functions and their graphs to solve systems of two linear functions in real-world contexts. Students use graphs, situations, and equations to solve for both the independent and dependent variables in problems.	A.REI.C.6 A.REI.D.11			•		
	Solving Linear Systems using Linear Combinations	Students solve systems of linear equations using linear combinations and compare the algebraic and graphical solutions.	A.REI.C.5	•				
	Solving Linear Systems using Any Method	Students choose to solve systems of linear equations using substitution or linear combinations.	A.REI.C.6	•				
Linear Inequalities in Two Variables	Graphing Linear Inequalities in Two Variables	Students graph and solve linear inequalities in two variables graphically by determining the correct half-planes for the solution sets.	A.REI.D.12	•				
	Systems of Linear Inequalities	Students determine the intersections between two inequalities, graph the inequalities, and shade the regions representing the solutions and their intersections.	A.REI.D.12	•				
Sequences	Describing Patterns in Sequences	Students determine the patterns in sequences and determine the next terms in sequences.	F.IF.A.3	•				
	Writing Recursive Formulas	Students determine if sequences are arithmetic or geometric and determine recursive formulas for the sequences.	F.IF.A.3 F.BF.A.1a	•				
	Writing Explicit Formulas	Students determine if sequences are arithmetic or geometric and develop the explicit formulas for the sequences.	F.BF.A.1a	•				
	Writing Sequences as Functions	Students classify sequences as arithmetic, geometric, or neither based on their graphs. Students then determine the function family for the sequence, write an explicit formula for the sequence, and finally rewrite it in linear, exponential, or quadratic form as appropriate.	F.IF.A.3	•				

MATHia Unit	MATHia Workspace	Overview	CCSS	Problem Solving	Animation	Worked Examples	Classification Tools	Explore
Exponential Functions	Introduction to Exponential Functions	Students view an animation explaining an exponential function in context. They compare linear and exponential sequences, functions, tables, and graphs, and then sort examples of these based upon whether they show linear or exponential growth. Students identify key characteristics of exponential functions (asymptotes, x-intercepts, y-intercepts, domain, range, and intervals of increase or decrease) from a function table, or graph.	F.IF.B.4		•			
	Relating the Domain to Exponential Functions	Students determine the domain of exponential functions. Scenarios are provided, and in light of the context, two factors must be considered: the lowest and highest values for the independent variable and the types of numbers that make sense for the independent variable. Several examples are provided to model the process of selecting an appropriate domain prior to students completing problems independently.	F.IF.B.5			•		
	Using Properties of Exponents	Students review the properties of powers and identify simplified versions of expressions with numeric powers. They deal with more complex exponential expressions with variables, sorting them into groups that are equivalent expressions.	F.IF.C.8b				•	
	Calculating and Interpreting Average Rate of Change	A formula is provided to calculate the average rate of change for a specified interval of an exponential function, and the reason it is considered an average is explained. Students are provided contexts along with either a graph or table. They must calculate the average rate of change for specified intervals of the function, and make an estimation for another interval based upon their results.	F.IF.B.6			•		
	Comparing Exponential Functions in Different Forms	Given two exponential functions in different representations — equation, graph, table, or description — with a contextual or noncontextual scenario, students compare the functions, y-intercepts, x-intercepts, or average rates of change over a specific interval.	F.IF.C.9	•				
Compare Linear and Exponential Models	Recognizing Linear and Exponential Models	Students compare linear and exponential functions and their graphs in the context of simple interest (linear) and compound interest (exponential). Students solve problems related to the independent and dependent variables of both linear and exponential functions using the graphs and equations.	S.ID.C.7 S.ID.C.8			•		
	Recognizing Growth and Decay	Students watch two different animations: one shows a model of exponential growth and one shows a model of exponential decay. They analyze how to recognize the difference between the two exponential models before interpreting exponential functions using scenarios of population increase and decrease.	F.LE.1c		•			

MATHia Unit	MATHia Workspace	Overview	CCSS	Problem Solving	Animation	Worked Examples	Classification Tools	Explore
Linear and Exponential Transformations	Introduction to Transforming Exponential Functions	Students use four animations, demonstrating the different ways of transforming an exponential function, to investigate how changing the equation for an exponential function changes the graph of the function. Students answer questions related to horizontal and vertical translations and dilations of exponential functions.	F.BF.B.3		•			
	Shifting Vertically	Students vertically shift graphs of linear and exponential functions. Students use verbal descriptions, graphs, and algebraic representations.	F.BF.B.3	•				
	Reflecting and Dilating using Graphs	Students reflect and dilate graphs of linear and exponential functions. Students use verbal descriptions, graphs, and algebraic representations.	F.BF.B.3	•				
	Shifting Horizontally	Students horizontally shift graphs of linear and exponential functions. Students use verbal descriptions, graphs, and algebraic representations.	F.BF.B.3	•				
	Transforming using Tables of Values	Given a table of values and a table of transformed values, students determine how the basic linear and exponential functions were transformed to create the new functions.	F.BF.B.3	•				
	Using Multiple Transformations	Given a representations of a transformed function, students determine how the basic linear and exponential functions were transformed to create the new functions.	F.BF.B.3	•				
Rational Exponents	Properties of Rational Exponents	Students learn the names of the components of radical notation (radical, radicand, index and nth root). They use the properties of powers to make sense of the fact that x to the one-half power and the square root of x are equivalent. Students practice rewrite expressions with radical notation using rational exponents, and then reverse the process and rewrite expressions with rational exponents using radical notation. In these problems, all rational exponents are positive fractions with one as a numerator.	N.RN.1			•		
	Rewriting Expressions with Radical and Rational Exponents	Students expand their understanding of rational exponents to include making sense of fractional exponents with a numerator other than one and negative exponents. Given various expressions with exponents with fractions, exponents with negative values and powers raised to a power, they select an equivalent radical expression. The process is then reversed, and students convert radical expressions to expressions with positive or negative fractional exponents.	N.RN.2			•		

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3 Descriptive Statistics

MATHia Unit	MATHia Workspace	Overview	CCSS	Problem Solving	Animation	Worked Examples	Classification Tools	Explore
Numerical Summary Statistics	Determining Appropriate Measures	Students use their understanding of mean, median, and mode to determine which was used as the measure of central tendency.	S.ID.A.2	•				
	Measuring the Effects of Changing Data Sets	Students calculate mean and median, with and without an additional data value, and compare the original and adjusted measures.	S.ID.A.3	•				
	Comparing and Interpreting Measures of Center	Students calculate the mean and median of data set and determine which is the better measure of center based on the distribution of the data. They also examine when a mean or median cannot be determined from a given display as well as the effect that outliers have on the mean. Finally, students compare two data sets according to each set's better measure of center.	S.ID.A.1 S.ID.A.2 S.ID.A.3			•		
Lines of Best Fit	Exploring Linear Regression	Students use an Interactive Explore Tool to investigate linear regression functions. Students enter data related to various real-world contexts and use the Explore Tool to analyze the linear trend present in the data set, as given by the regression function. Students investigate how moving the points of the data set affects the slope of the regression line, and they analyze the effect of outliers on the regression function. Students also explore Anscombe's Quartet — a group of four data sets which are used to show that data sets which have strikingly different graphical shapes can be described by the same linear regression function.	S.ID.B.6a S.ID.B.6c S.ID.B.7					•
	Using Linear Regression	Students are given a table of data and a linear regression equation that represents the line of best fit. They calculate values of the dependent variable using the linear regression equation. Students compare the values of the dependent variable from both representations, stating whether the question called for interpolation or extrapolation, and whether the linear regression answer was reasonable or not based upon the table of data. The worked example and practice problems are provided in a context.	S.ID.B.6a			•		
	Interpreting Liens of Best Fit	Students are introduced to the terms correlation coefficient, positive association, and negative association through examples of scatter plots. They select the possible correlation coefficients for given scatter plots from a range of choices using their conceptual understanding. They complete problems in context, giving rough estimates of the value of r, stating how the estimate is reflected in the table of values, and determining whether the linear regression equation is appropriate for the data set.	S.ID.7 S.ID.8			•		
	Analyzing Residuals of Lines of Best Fit	Students view an animation demonstrating the least squares regression method. Given a scatter plot and possible lines of best fit, they must complete the least squares regression method for each line to determine which is the line of best fit. The term residual is defined and a formula provided. Students analyze a scatter plot and line of best fit, a table comparing the data with the residuals, and a residual plot. They practice making residual tables and analyzing residual plots to decide whether a line of best fit is a good fit.	S.ID.B.6b		•			

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4 Tools of Geometry

MATHia Unit	MATHia Workspace	Overview	CCSS	Problem Solving	Animation	Worked Examples	Classification Tools	Explore
Lines, Rays, Segments, and Angles	Naming Lines, Rays, Segments, and Angles	Students practice identifying geometric entities from their names, writing names for various geometric entities, and identifying when an entity has multiple possible names.	G.CO.A.1	•				
	Working with Measures of Segments and Angles	Students practice writing measure statements for segments and angles using appropriate notation.	G.CO.A.1	•				
Distances on the Coordinate Plane	Deriving the Distance Formula	Students answer questions related to an animation demonstrating how the Distance Formula is derived using the Pythagorean Theorem. Students then use Interactive Explore Tools and the Distance Formula to solve mathematical problems about the distances between two points on the coordinate plane.	G.GPE.B.7		•			
	Calculating Distances using the Distance Formula	Students use the distance formula to determine distances between points.	G.GPE.B.7	•				
	Partitioning Segments Proportionately	Students determine the coordinates of points that partition given line segments into different ratios.	G.GPE.B.6	•				
	Calculating Perimeter and Area using the Distance Formula	Students use the distance formula to determine perimeters and areas of different shapes.	G.GPE.B.7	•				
Parallel and Perpendicular Lines	Introduction to Parallel and Perpendicular Lines	Students answer questions related to an animation demonstrating that the rotation of a point (x, y) 90 degrees counterclockwise on the coordinate plane is given by the coordinates (-y, x). Students answer questions to discover that the slopes of perpendicular lines are negative reciprocals of each other. Students then use graphs of functions to understand that the slopes of parallel lines are equal. Finally, students use their knowledge of parallel and perpendicular lines as graphs of functions to solve problems in a real-world context.	G.GPE.B.5		•			
	Modeling Parallel and Perpendicular Lines	Students determine the equations of lines parallel or perpendicular to given lines.	G.GPE.B.5	•				

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5	Congruence							
MATHia Unit	**MATHia Workspace**	**Overview**	**CCSS**	**Problem Solving**	**Animation**	**Worked Examples**	**Classification Tools**	**Explore**
Rigid Motion	Developing Definitions of Rigid Motions	Students learn the formal definitions for translation, reflection, and rotation as rigid motions. Students then apply these formal definitions to other situations involving mathematical transformations.	G.CO.A.4			•		
	Rotations and Reflections on the Plane	Students apply the formal definitions of reflection and rotation to identify rigid motions that carry rectangles, non-rectangular parallelograms, trapezoids, and regular polygons onto themselves. Students learn that figures which can be reflected or rotated onto themselves have reflectional or rotational symmetry.	G.CO.A.3			•		
	Specifying a Sequence of Transformations	Students will select multiple transformations from translation, rotation, dilation, and reflection about any line to match a pre-image to a target image, given a reference point.	G.CO.A.5	•				

6	Parallelograms							
MATHia Unit	**MATHia Workspace**	**Overview**	**CCSS**	**Problem Solving**	**Animation**	**Worked Examples**	**Classification Tools**	**Explore**
Properties of Parallelograms	Understanding Parallelograms	Students are given the properties of parallelograms and use the information to determine the side parallel to a given side of a parallelogram as well as the sides or angles that are congruent to a given side or angle of a parallelogram. They then determine a missing statement to prove a quadrilateral is a parallelogram using the Parallelogram/Congruent-Parallel Side Theorem. Finally, students identify quadrilaterals by properties of their sides, angles, and diagonals.	G.CO.C.11			•		
	Properties of Parallelograms	Students will be given a diagram of a parallelogram with the measures of some sides, diagonals and/or angles given, and asked to determine the measures of the remaining sides/diagonals/angles.	G.CO.C.11	•				