1 Function Overview

MATHia Unit	MATHia Workspace	Overview	CCSS	Problem Solving	Animation	Worked Examples	Classification Tools	Explore
Searching for Patterns	Exploring and Analyzing Patterns	Students watch a video about a well-known mathematician creating an expression for the sum of a sequence of numbers from a pattern he noticed and then answer questions that move them from a numerical expression to an algebraic one. Next, students analyze three different patterns to generate linear, exponential, and quadratic algebraic expressions.	A.SSE.A.1		•			
	Comparing Familiar Function Representations	Students review three familiar function families--linear, quadratic, and exponential. They practice matching the equation of a function and the graph of a function to one of these function families. Finally, the choose which of these function families represents the relationship given in the context of a real-world problem.	A.SSE.A.1				•	
Graphs of Functions	Identifying Key Characteristics of Graphs of Functions	Students will identify key characteristics from the graph of a function, such as the intercepts, minimum and maximum x-values, minimum and maximum y-values, domain, and range.	F.IF.B.4	•				
	Transforming Functions	Students use interactive Explore Tools to investigate transformations of linear, exponential, and quadratic functions, including horizontal and vertical translations and dilations. Students use the Explore Tools to solve real-world problems modeling changes to an exponential function describing doubling and to a quadratic function describing the height of a jump. In the final problems, students identify the graphs of transformations of quadratic functions based on their transformation equations.	F.BF.B.3 A.SSE.A.1b					•

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2 Quadratic Functions

MATHia Unit	MATHia Workspace	Overview	CCSS	Problem Solving	Animation	Worked Examples	Classification Tools	Explore
Forms of Quadratic Functions	Examining the Shape and Structure of Quadratic Functions	Students sort functions based upon whether they are written in standard, factored or vertex form. They deal with each form independently, where they are guided as to how to identify key characteristics of the graph from the function. They identify the concavity and y-intercept from functions in standard form, the concavity and x-intercepts from functions in factored form, and the concavity, vertex, and axis of symmetry from functions in vertex form. Given graphs, they use key characteristics to select the function that generates the graph.	A.SSE.A.1 A.SSE.A.2				•	
	Quadratic Modeling	Students complete a table of values and graph from a scenario represented by a quadratic model. Students construct the quadratic function for the scenario as a product of a monomial and a binomial, or as the product of two binomials.	A.REI.B.4b	•				
	Quadratic Equation Solving	Students solve quadratic equations by using factoring or the quadratic formula.	A.REI.B.4b	•				
	Quadratic Transformations	Given a representation of a transformed function, students determine how the basic quadratic function was transformed to create the new function.	F.BF.3	•				
Operations with Complex Numbers	Introduction to Complex Numbers	Students watch a video introducing them to the imaginary number line and its relation to the real number line. They then practice identifying real and imaginary numbers through the sorting tool. Finally, students are introduced to complex numbers and practice identifying them on the complex plane to help them understand that all numbers are complex, but some are real and some are purely imaginary.	N.CN.A.1		•		•	
	Simplifying Radicals with Negative Radicands	Students simplify radical expressions that result in complex numbers.	N.CN.A.1	•				
	Simplifying Powers of i	Students identify expressions that are equivalent to i, -1, -i, and 1. They use the definition of i to rewrite higher powers of i.	N.CN.A.1			•		
	Adding and Subtracting Complex Numbers	Students add and subtract complex numbers.	N.CN.A.2	•				
	Multiplying Complex Numbers	Students multiply complex numbers, including both problems where the two complex numbers are complex conjugates, and problems where they are not.	N.CN.A.2	•				
	Solving Quadratic Equations with Complex Roots	Students solve quadratic equations, some of which have real solutions and some of which have complex solutions.	N.CN.C.7	•				

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3 Polynomial Functions

MATHia Unit	MATHia Workspace	Overview	CCSS	Problem Solving	Animation	Worked Examples	Classification Tools	Explore
Graphs of Polynomial Functions	Modeling Polynomial Functions	Students solve two problems in context involving polynomial, specifically, cubic functions. The first half of each problem requires students to use the function to solve for the dependent variable. The second half of the problem requires students to use the graph to solve for the independent variable and an interpret a minimum or maximum point on the graph.	F.IF.B5			•		
	Analyzing Polynomial Functions	Students explore power functions, concentrating on cubics and quartics and the key characteristics of end behavior and extrema. The determine end behavior based upon whether the functions are even-degree or odd-degree and the sign of its leading term. They use graphs to determine extrema, relative maximum and minimums, and absolute maximums and minimums.	A.APR.1 F.BF.1b			•		•
	Classifying Polynomial Functions	Students classify graphs of polynomial functions by their key characteristics.	F.IF.B.4	•				
	Interpreting Key Features of Graphs in Terms of Quantities	Students are provided graphs in context. They interpret the meaning of relative maximum and minimum points, y-intercepts, x-intercepts, and increasing and decreasing intervals.	F.IF.B.4			•		
	Identifying Key Characteristics of Polynomial Functions	Students are provided graphs in context. They interpret the meaning of relative maximum and minimum points, y-intercepts, x-intercepts, and increasing and decreasing intervals.	F.IF.B.4	•				
	Identifying Zeros of Polynomials	Students are introduced to the term zeros, with clarification made among real zeros, imaginary zeros and zeros with multiplicity. They identify the number and types of zeros when given graphs of cubic and quartic functions. They sort graphs based upon their number of real zeros. When given graphs, they select the function that represents the graph based upon whether it is an even-degree or odd-degree function with an a-value that is positive or negative; in one case, differentiation between the number of zeros is also required.	A.APR.B.3				•	
	Using Zeros to Sketch a Graph of Polynomial	Students sketch a third- or fourth-order polynomial function.	A.APR.B.3	•				
	Understanding Average Rate of Change of Polynomial Functions	Students are provided the formula for the average rate of change for non-linear functions. The formula determines the rate of change of the line segment connecting the endpoints of a specified interval. Students are reminded that it is called an “average” because the rate of changes varies along a curve. Students estimate the average rate of change of polynomial functions for a specified interval when given a graph. They then use the formula to calculate the average rate of change of polynomial functions for a specified interval when given a graph.	F.IF.B.6			•		

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MATHia Unit	MATHia Workspace	Overview	CCSS	Problem Solving	Animation	Worked Examples	Classification Tools	Explore
Polynomial Operations	Adding Polynomials with Higher Orders	Students add higher order polynomials.	A.APR.A.1	•				
	Adding and Subtracting Higher Order Functions	Students add higher order polynomials.	A.APR.A.1	•				
	Multiplying Polynomials	Students determine which factor table is appropriate for a given problem, set up the table, and then use the table to multiply polynomials.	A.APR.A.1	•				
	Synthetic Division	Students use synthetic division as an efficient method to divide a higher-order polynomial by a linear divisor.	A.APR.D.6	•				
Solving Polynomials	Factoring Higher Order Polynomials	Students factor quadratic expressions using all known factoring methods.	A.SSE.A.2	•				
	Solving Polynomial Functions	For the first time, students see the algebraic representations that determine the graphs of polynomial functions. They make a connection between f(x) = 0 and a polynomial equation set equal to zero. They begin to solve polynomial equations by seeing both graphical and algebraic method(using factored form) for the same equation. Students then focus on cubic equations with multiple or imaginary roots, once again connecting graphical and algebraic solution methods. Next, they deal with cubic equations that are not originally set equal to zero, are not in factored form or require the use of the quadratic formula to determine some roots. Students then practice solving quartic equations using these same skills.	A.APR.B.2			•		
	Solving Polynomial Inequalities	Students solve polynomial inequalities graphically. In all cases, they solve inequalities written as greater than or less than zero. The worked example demonstrates to separate the left hand side and right hand side of the inequality into two separate equations. From there, the equations are graphed, and students visually inspect where the graph lies above or below the x-axis. Initially, students select the regions of the graph that make the inequality true directly on the graph. After some practice, they must also select the intervals for x that are solutions to the inequality.	A.CED.A.1			•		
	Comparing Polynomial Functions in Different Forms	Given two polynomial functions in different representations — equation, graph, table, or description — with a contextual or noncontextual scenario, students compare the functions' degrees, extrema, rates of change, or zeros over a specific interval.	F.IF.C.9	•				

4	Piecewise Functions			Problem Solving	Animation	Worked Examples	Classification Tools	Explore
MATHia Unit	**MATHia Workspace**	**Overview**	**CCSS**					
Graphs of Piecewise Functions	Introduction to Piecewise Functions	Students are introduced to a linear piecewise function through a real-world scenario and giving the definition of a piecewise function. They then sort sketches of graphs of linear piecewise functions to given scenarios. Finally, students identify the graph of a linear piecewise function after being given the function's equation.	F.IF.C.7b			•	•	
	Interpreting Piecewise Functions	Students identify the domain in both non-continuous and continuous piecewise functions given an equation and the graph of the function. They are then given a domain and a graph of a piecewise function and are asked to determine the equation the graph with that domain represents.	F.IF.C.7b			•		
	Analyzing Step Functions	Students are introduced to step functions in the first problem. They then identify the domain of a given equation of a step function using a problem situation and graph. Next students are asked to identify the step function that represents a given problem situation and graph.	F.IF.C.7b			•		

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5 Sequences and Series

MATHia Unit	MATHia Workspace	Overview	CCSS	Problem Solving	Animation	Worked Examples	Classification Tools	Explore
Finite Geometric Series	Introduction to Finite Geometric Series	Students review sequences and sort geometric sequences from all other types of sequences. Series are defined as the sum of the terms of a sequence, and finite and infinite series are defined as well. The focus for the remainder of the workspace is on finite geometric series. Students are introduced to summation notation. The formula to calculate the sum of a finite geometric series is developed using Euler's Method, simplified in words, and then provided formally. Students calculate sums of finite series when given a series or summation notation for a series. Students also solve problems in real-world contexts requiring the calculation of the sum of geometric series.	A.SSE.B.4			•	•	
	Problem Solving using Finite Geometric Series	Students solve problems given scenarios where the appropriate model is a finite geometric series.	A.SSE.B.4	•				

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6 Rational Expressions and Functions

MATHia Unit	MATHia Workspace	Overview	CCSS	Problem Solving	Animation	Worked Examples	Classification Tools	Explore
Rational Functions	Introduction to Rational Functions	Students are given the definition of a rational function and use the definition to sort given functions as rational functions or not. They are then shown the graph of a rational function and introduced to horizontal and vertical asymptotes. Students use a function and its graph to describe the asymptotes. Next they determine asymptotes of rational functions using only the equation.	(+)F.IF.7d			•	•	
	Modeling Ratios as Rational Functions	Students solve rational equation problems using a worksheet format, with separate columns for the independent quantity, the numerator, the denominator, and the rational expressions. Questions have either the independent quantity or a dependent quantity given, with the Solver available to solve for independent quantities.	A.REI.A.2	•				
Rational Expressions and Equations	Simplifying Rational Expressions	Students simplify simple rational expressions.	A.APR.D.6	•				
	Multiplying and Dividing Rational Expressions	Students simplify products and quotients of rational expressions.	A.APR.D.6	•				
	Adding and Subtracting Rational Expressions	Students simplify sums and differences of rational expressions.	A.APR.D.6	•				
	Solving Rational Equations that result in Linear Equations	Students solve rational equations, and classify the solutions as valid or extraneous.	A.REI.A.2	•				
Rational Models	Modeling Rational Functions	Students solve rational equation problems using a worksheet format, with separate columns for the independent quantity, the numerator, the denominator, and the rational expressions. All questions have the independent quantity given.	A.APR.D.6	•				
	Using Rational Models	Students solve contextual rational equation problems using a given equation. Questions have either the independent quantity or the dependent quantity given, with the Solver available to solve for independent quantities.	A.CED.A.1	•				
	Solving Work, Mixture, and Distance Problems	Introduction to solving work, mixture, and distance problems using a worksheet format. Students either write expressions for given problem entity descriptions, or select problem entity descriptions for given expressions. As students complete steps, the associated expressions are echoed in the worksheet. Students conclude by equating two expressions for the same entity which are used to solve for an unknown.	A.CED.A.1	•				
	Modeling and Solving with Rational Functions	Students solve work, mixture, and distance problems using a worksheet format. They write expressions for the given information, then use relationships with related problem entities to write expressions for them, and conclude by solving for an unknown by equating two expressions for the same entity.	A.CED.A.1	•				

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7 Inverse Functions

MATHia Unit	MATHia Workspace	Overview	CCSS	Problem Solving	Animation	Worked Examples	Classification Tools	Explore
Inverses of Functions	Investigating Inverses of Functions	Students watch an animation which demonstrates that the inverse of a point is formed by reversing the x- and y-coordinates of a point. Thus, the inverse of a function, the set of all x-y coordinates which satisfy an equation, is a reflection of the original function across the line y = x. Students use the Horizontal Line Test to identify the graphs of inverses of functions, and determine whether a function is invertible.	F.IF.C.7b		•	•		
	Graphing Square Root Functions	Students learn how to determine and graph the square root function as the inverse of the quadratic function $y = x^2$ with a domain restricted to $x \geq 0$. Students identify simple graphed transformations of the square root function and write equations for those transformations.	F.IF.C.7b			•		
	Graphs of Inverses	Students enter the inverse coordinate pairs, plot them on a graph, and reason about the domain and range.	F.BF.B.4	•				
	Calculating Inverses of Linear Functions	Given a function, students determine the equation of the inverse function and use composition of function to verify that the functions are inverses.	F.BF.B.4a	•				

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8 Radical Functions

MATHia Unit	MATHia Workspace	Overview	CCSS	Problem Solving	Animation	Worked Examples	Classification Tools	Explore
Simplification and Operations with Radicals	Simplifying Radicals	Students simplify numerical radical expressions.	N.RN.A.1	•				
	Adding and Subtracting Radicals	Students simplify and add and subtract numerical radical expressions.	N.RN.A.1	•				
	Multiplying Radicals	Students multiply and simplify numerical radical expressions.	N.RN.A.1	•				
	Dividing Radicals	Students divide and simplify numerical radical expressions.	N.RN.A.1	•				
Radical Expressions with Variables	Simplifying Radicals with Variables	Students simplify radical expressions with variables.	N.RN.A.2	•				
	Adding and Subtracting Radicals with Variables	Students simplify and add and subtract radical expressions with variables.	N.RN.A.2	•				

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9	Exponential Models							
MATHia Unit	**MATHia Workspace**	**Overview**	**CCSS**	**Problem Solving**	**Animation**	**Worked Examples**	**Classification Tools**	**Explore**
Exponential and Logarithmic Functions	Properties of Exponential Graphs	Students recall exponential functions and identify exponential growth and decay functions by their equation forms. Students watch an animation demonstrating how to build an exponential expression modeling an account balance earning compound interest. They use the formula $A = P(1 + r)^t$ to determine compound interest amounts. Students then learn that the constant e represents the base of an exponential when the interest is compounded continuously. Students solve real-world problems about changes in populations using the formula for continuous exponential growth or decay.	F.IF.C.7e		●	●		
	Introduction to Logarithmic Functions	Students watch an animation demonstrating that a logarithm is an expression equal to the exponent of a corresponding exponential expression and that a logarithmic function is the inverse of the corresponding exponential function. Students evaluate logarithms and generalize about forms such as log_a(a), log_a(1), and log_a(1/a). Students identify and analyze logarithmic functions of base 2, 10, and e, the natural logarithm.	F.IF.C.7e		●			
Solve Equations with Base 2, 10, or e	Solving Base 10 Equations (No Type In)	Students solve equations of the form A*B^x = C and A*log_B(x) = C, where B is either 2 or 10.	F.LE.A.4	●				
	Solving Base e Equations (No Type In)	Students solve equations of the form A*e^x = C and A*ln(x) = C.	F.LE.A.4	●				
	Solving Any Base Equations (No Type In)	Students solve equations of the form A*B^x = C and A*log_B(x) = C, where B is either 2, e, or 10.	F.LE.A.4	●				

10	Trigonometric Functions			Problem Solving	Animation	Worked Examples	Classification Tools	Explore
MATHia Unit	**MATHia Workspace**	**Overview**	**CCSS**					
Graphs of Trigonometric Functions	Understanding the Unit Circle	Students watch an animation demonstrating how to determine and use radian measures as distances around the unit circle. The animation shows how the sine and cosine functions are constructed, mapping radian measures as inputs to the outputs of the sine or cosine of a central angle measure on the unit circle. Students derive how to convert between radian measures and degree measures. Finally, students analyze the sine and cosine functions, as members of a family of periodic trigonometric functions. Students identify the amplitude, midline, and period of each function and use the period to evaluate each function for different radian measures.	F.TF.A.1 F.TF.A.2		•			
	Choosing Trigonometric Functions to Model Periodic Phenomena	Given a scenario that can be modeled by the sine function, students extract the values of A, B, C, and D to create the function using A*sin(2pi/B(x – C)) + D.	F.TF.B.5	•				
Pythagorean Identity	Proving the Pythagorean Identity	Students combine their knowledge of the Pythagorean Theorem and the unit circle to complete a proof of the Pythagorean identity $(\sin(x))^2 + (\cos (x))^2 = 1$. They use this identity to solve for solve problems where the value of sine or cosine in a specific quadrant is provided, and they must solve for the value of the other trigonometric function. A proof is provided for $(\tan(x))^2 + 1 = (\sec(x))^2$, and students duplicate the process to prove $1 + (\cot(x)^2) = (\csc(x))^2$. Students use the identity $(\tan(x))^2 + 1 = (\sec(x))^2$ to solve problems where the value of tangent or cosine in a specific quadrant is provided, and they must solve for the value of the other trigonometric function.	F.TF.C.8			•		
	Using the Pythagorean Identity	Students use the Pythagorean Identity to solve for sin or cos of an angle given sin, cos, or tan of that angle.	F.TF.C.8	•				