Legend tells us that when the inventor of the game of chess showed his work to the emperor, the emperor was so pleased that he allowed the inventor to choose any prize he wished. So the very wise inventor asked for the following: 1 gold coin for the first square on the chess board, 2 gold coins for the second square, 4 coins for the third, and so on up to the 64th square. The emperor, not as wise as the inventor, quickly agreed to such a cheap prize. Unfortunately, the emperor could not afford to pay even the amount for just the 32nd square: 4,294,967,295 gold coins!

How many gold coins would the emperor have to pay for just the 10th square? 20th square? What pattern did you use to calculate your answers?
Problem 1  Sequences

The inventor from the story used his knowledge of sequences to his advantage to gain riches.

A sequence is a pattern involving an ordered arrangement of numbers, geometric figures, letters, or other objects. A term in a sequence is an individual number, figure, or letter in the sequence.

Here are some examples of sequences.

Sequence A: 2, 4, 6, 8, 10, 12,…

Sequence B: △, □, △, □, △, □, …

Sequence C: A, B, C, D, E, F, G,…

Sequence D: →, ↓, ←, ↑, →, …

Often, only the first few terms of a sequence are listed, followed by an ellipsis. An ellipsis is three periods, which stand for “and so on.”

1. What is the next term in Sequence A?

2. What is the third term in Sequence B?

3. What is the twenty-fifth term in Sequence C?

4. What is the twelfth term in Sequence D?
Problem 2  Designing a Bead Necklace

Emily is designing a necklace by alternating black and green beads. To create her necklace, she performs the following steps.

Step 1: She starts with one black bead.

Step 2: Next, she places one green bead on each side of the black bead.

Step 3: Then, she places two black beads on each side of the green beads.

Step 4: Then, she places three green beads on each side of the black beads.

Step 5 and 6: She continues this pattern two more times, alternating between black and green sets of beads.

1. Write the first six terms in the sequence that represents this situation. Make sure each term indicates the total number of beads on the necklace after Emily completes that step. Finally, explain how you determined the sequence.
Problem 3  Crafting Toothpick Houses

Ross is crafting toothpick houses for the background of a diorama. He creates one house and then adds additional houses by adjoining them as shown.

A diorama is a three-dimensional natural scene in which models of people, animals, or plants are seen against a background.

1. Write the first eight terms in the sequence that represents this situation. The first term should indicate the number of toothpicks used for one house. The second term should indicate the total number of toothpicks needed for two houses, and so on. Explain your reasoning.

2. How is the number of toothpicks needed to build each house represented in the sequence?
Problem 4  Taking Apart a Card Trick

Matthew is performing a card trick. It is important that he collect the cards shown in a particular order. Each turn, he collects all of the cards in the right-most column, and all the cards in the bottom row.

1. Write a sequence to show the number of cards removed during each of the first five turns.

2. Write a sequence to show the number of cards remaining after each of the first five turns.

3. What pattern is shown in each sequence?
Problem 5  Arranging Pennies

Lenny is making arrangements with pennies. He has made three penny arrangements and now he wants to make five more arrangements. Each time he adds another arrangement, he needs to add one more row to the base than the previous row in the previous arrangement.

1. Write the first eight terms in the sequence that represents this situation. Each term should indicate the total number of pennies in each arrangement. Explain your reasoning.

2. Explain why the pattern does not increase by the same amount each time.
Problem 6  Building Stairs

Dawson is stacking cubes in configurations that look like stairs. Each new configuration has one additional step.

1. Write the first five terms in the sequence that represents this situation. Each term should indicate the number of faces shown from the cubes shown. The bottom faces are not shown. The first cube has 5 shown faces. Explain your reasoning.

2. Predict the number of shown faces in a stair configuration that is 7 cubes high. Show your work.
Problem 7  Arranging Classroom Tables

Some schools purchase classroom tables that have trapezoid-shaped tops rather than rectangular tops. The tables fit together nicely to arrange the classroom in a variety of ways. The number of students that can fit around a table is shown in the first diagram. The second diagram shows how the tables can be joined at the sides to make one longer table.

1. Write the first 5 terms in the sequence that represents this situation. Each term should indicate the total number of students that can sit around one, two, three, four, and five tables. Explain your reasoning.

2. The first trapezoid table seats five students. Explain why each additional table does not have seats for five students.
Problem 8  Drawing Flower Petals

Draw a flower in a series of stages. The figure shows a pair of flower petals as the starting point, Stage 0. In each stage, draw new petal pairs in the middle of every petal pair already drawn.

• In Stage 1, you will draw _____ petals.
• In Stage 2, you will draw _____ petals.
• In Stage 3, you will draw _____ petals.

1. Write the first 5 terms in the sequence that represents this situation. Each term should indicate the number of new petals drawn in that stage. Explain your reasoning.
Problem 9  Babysitting

Every Friday, Sarah earns $14 for babysitting. Every Saturday, Sarah spends $10 going out with her friends.

1. Write a sequence to show the amounts of money Sarah has every Friday after babysitting and every Saturday after going out with her friends for five consecutive weeks. The sequence should have 10 terms. Explain your reasoning.

Problem 10  Recycling

The first week of school, Ms. Sinopoli asked her class to participate in collecting cans for recycling. The students started bringing in cans the second week of school. They collected 120 cans per week.

1. Write a sequence to show the running total of cans collected through the first nine weeks of school. Explain your reasoning.
Problem 11  Selling Tickets

Sam is working at the ticket booth during a basketball game. His cash box has two $10 bills, five $5 bills, and twenty $1 bills. Tickets cost $3.

1. How much money does Sam have at the beginning of the basketball game?

2. Write a sequence to show the amount of cash Sam has available to start selling tickets, and the amounts available after selling one ticket, two tickets, three tickets, four tickets, and five tickets. Explain your reasoning.

Talk the Talk

There are many different patterns that can generate a sequence. Some possible patterns are:

- adding or subtracting by the same number each time,
- multiplying or dividing by the same number each time,
- adding by a different number each time, with the numbers being part of a pattern,
- alternating between adding and subtracting.

The next term in a sequence is calculated by determining the pattern of the sequence and then using that pattern on the last known term of the sequence.
Look back at Problems 2 through 11.

1. Describe the pattern of each sequence by completing the table shown.

<table>
<thead>
<tr>
<th>Sequence Name</th>
<th>Increases or Decreases</th>
<th>Describe the Pattern</th>
</tr>
</thead>
<tbody>
<tr>
<td>Designing a Bead Necklace</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Crafting a Toothpick House</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Taking Apart a Card Trick (1)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Arranging Pennies</td>
<td></td>
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<tr>
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<td></td>
<td></td>
</tr>
</tbody>
</table>

2. Which sequences are similar? Explain your reasoning.

Be prepared to share your solutions and methods.