LEARNING GOALS

• Apply the Pythagorean Theorem to determine unknown side lengths of right triangles in mathematical and real-world problems.
• Apply the Pythagorean Theorem to determine the lengths of diagonals of two- and three-dimensional figures.

KEY TERM

• diagonal

You have learned about the Pythagorean Theorem and its converse. How can you apply the Pythagorean Theorem to determine lengths in geometric figures?
Getting Started

Diagonally

Draw all of the sides you cannot see in each rectangular solid using dotted lines. Then draw a three-dimensional diagonal using a solid line.

1. 

2. 

How many three-dimensional diagonals can be drawn in each figure?
Previously, you have drawn or created many right triangles and used the Pythagorean Theorem to determine side lengths. In this lesson, you will explore the diagonals of various shapes.

1. Rectangle $ABCD$ is shown.

   a. Draw diagonal $AC$ in Rectangle $ABCD$. Then, determine the length of diagonal $AC$.

   b. Draw diagonal $BD$ in Rectangle $ABCD$. Then, determine the length of diagonal $BD$.

   c. What can you conclude about the diagonals of this rectangle?
2. Square $ABCD$ is shown.

![Diagram of square ABCD]

a. Draw diagonal $AC$ in Square $ABCD$. Then, determine the length of diagonal $AC$.

b. Draw diagonal $BD$ in Square $ABCD$. Then, determine the length of diagonal $BD$.

c. What can you conclude about the diagonals of this square?

All squares are also rectangles, so does your conclusion make sense?
3. Graph and label the coordinates of the vertices of Trapezoid $ABCD$: $A (1, 2)$, $B (7, 2)$, $C (7, 5)$, $D (3, 5)$.


- b. What right triangle can be used to determine the length of diagonal $AC$?

- c. Determine the length of diagonal $AC$.

- d. Draw diagonal $BD$ in Trapezoid $ABCD$.

- e. What right triangle can be used to determine the length of diagonal $BD$?

- f. Determine the length of diagonal $BD$.

- g. What can you conclude about the diagonals of this trapezoid?
4. Graph and label the coordinates of the vertices of isosceles Trapezoid ABCD: A (1, 2), B (9, 2), C (7, 5), D (3, 5).

How is this trapezoid different from the first trapezoid you drew?

a. Draw diagonal AC in Trapezoid ABCD.

b. What right triangle can be used to determine the length of diagonal AC?
c. Determine the length of diagonal $AC$.

d. Draw diagonal $BD$ in Trapezoid $ABCD$.

e. What right triangle can be used to determine the length of diagonal $BD$?

f. Determine the length of diagonal $BD$.

g. What can you conclude about the diagonals of this isosceles trapezoid?
Use your knowledge of right triangles, the Pythagorean Theorem, and area formulas.

1. Determine the area of each shaded region. Use 3.14 for $\pi$ and round to the nearest tenth.
   a. A rectangle is inscribed in a circle as shown.

   ![Diagram of a rectangle inscribed in a circle with dimensions 10 cm and 6 cm]

   b. The figure is composed of a right triangle and a semi-circle.

   ![Diagram of a right triangle and semi-circle with dimensions 8 mm and 5 mm]
A rectangular box of long-stem roses is 18 inches in length, 6 inches in width, and 4 inches in height.

Without bending a long-stem rose, you are to determine the maximum length of a rose that will fit into the box.

1. What makes this problem different from all of the previous applications of the Pythagorean Theorem?

2. Compare a two-dimensional diagonal to a three-dimensional diagonal. Describe the similarities and differences.

3. Which diagonal represents the maximum length of a rose that can fit into a box?
4. Consider the rectangular solid shown.
   a. Draw all of the sides in the rectangular solid you cannot see using dotted lines.

   b. Draw a three-dimensional diagonal in the rectangular solid.

   c. Let’s consider that the three-dimensional diagonal you drew in the rectangular solid is also the hypotenuse of a right triangle. If a vertical edge is one of the legs of that right triangle, where is the second leg of that same right triangle?

   d. Draw the second leg using a dotted line. Then lightly shade the right triangle.

   e. Determine the length of the second leg you drew.

   f. Determine the length of the three-dimensional diagonal.

   g. What does the length of the three-dimensional diagonal represent in terms of this problem situation?

5. Describe how the Pythagorean Theorem was used to solve this problem.
Determine the length of the diagonal of each rectangular solid.

1. \(6 \text{ in.} \times 4 \text{ in.} \times 10 \text{ in.}\)

2. \(4 \text{ m} \times 8 \text{ m} \times 7 \text{ m}\)

3. \(10 \text{ cm} \times 6 \text{ cm} \times 15 \text{ cm}\)

4. \(5 \text{ yd} \times 7 \text{ yd} \times 7 \text{ yd}\)

5. \(3 \text{ in.} \times 12 \text{ ft} \times 2 \text{ ft}\)

6. \(2 \text{ ft} \times 3 \text{ in.} \times 15 \text{ in.}\)
TALK the TALK

The Ant and the Fly Again

A rectangular room is 10 ft × 16 ft × 8 ft.

An ant crawls from point A to point B taking the shortest path.

A fly flies from point A to point B taking the shortest path.

1. Whose path was shorter?

2. How much shorter is the shorter path?